

## 7.3 Sketching Slope Fields

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 10 marks

KEY WORDS slope field 斜率场 • differential equation 微分方程 • solution 解

### Objective

Build the skills to answer exam questions on **slope fields** — drawing and reading the little segments of  $\frac{dy}{dx}$ .

**You must be able to:**

- compute the slope  $\frac{dy}{dx}$  at grid points and draw short segments (a **slope field** 斜率场)
- match a differential equation to its slope field
- sketch a solution curve that **follows** the segments through a given point

### 1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

#### ■ Computing a slope at a point

For  $\frac{dy}{dx} = x + y$ , the slope at  $(1, 2)$  is  $1 + 2 = 3$ : draw a short segment of slope 3 there. Repeating at many points gives the slope field.

#### ■ Slopes that depend only on $x$

If  $\frac{dy}{dx} = 2x$ , the slope depends on  $x$  **only**, so every segment in a vertical column has the same slope — the field looks the same as you move up or down.

#### ■ Slopes that depend only on $y$

If  $\frac{dy}{dx} = y$ , segments in a horizontal row match. Where  $y = 0$  the slope is 0 (horizontal segments along the  $x$ -axis).

#### ■ Following the field

A solution curve is tangent to the segments everywhere. Starting at a point, draw a curve that always runs **along** the nearby segments — like a boat following a current.

## 2 Practice

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Now apply the methods above.

**2.1** For  $\frac{dy}{dx} = x - y$ , find the slope at  $(3, 1)$ . [1]

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**2.2** For  $\frac{dy}{dx} = xy$ , find the slope at  $(2, -1)$ . [1]

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**2.3** For  $\frac{dy}{dx} = y - 1$ , state where all the segments are **horizontal**. [1]

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## 3 Exam-style questions

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**3.1** In the slope field for  $\frac{dy}{dx} = 2x$ , along the line  $x = 0$  the segments are [1]

- **A** vertical
  - **B** horizontal
  - **C** slope +1
  - **D** slope +2
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**3.2** A slope field is given by  $\frac{dy}{dx} = x + 1$ .

(a) Find the slope at the points  $(-1, 0)$ ,  $(0, 2)$ , and  $(2, -3)$ . [3]

(b) Along which vertical line are all segments horizontal? [1]

**3.3** A slope field has horizontal segments only along the  $x$ -axis ( $y = 0$ ) and gets steeper as  $|y|$  grows, with the same pattern in every vertical column. Which differential equation

fits:  $\frac{dy}{dx} = x$ ,  $\frac{dy}{dx} = y$ , or  $\frac{dy}{dx} = xy$ ? Explain.

[2]

## Solutions

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**2.1**  $3 - 1 = 2$ .

**2.2**  $(2)(-1) = -2$ .

**2.3** Where  $y - 1 = 0$ , i.e. along the line  $y = 1$ .

**3.1 B** — at  $x = 0$  the slope is  $2(0) = 0$ , horizontal.

**3.2** (a)  $(-1, 0)$ : 0;  $(0, 2)$ : 1;  $(2, -3)$ : 3. (b)  $x = -1$  (where  $x + 1 = 0$ ).

**3.3**  $\frac{dy}{dx} = y$  — the slope is 0 where  $y = 0$  and grows with  $|y|$ , and depends on  $y$  only, so it is the same in every column.