

# 6.8 Antiderivatives and Indefinite Integrals

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

**Total: 13 marks**

**KEY WORDS** indefinite integral 不定积分 • constant of integration 积分常数 • antiderivative 原函数  
• definite integral 定积分

## Objective

Build the skills to answer exam questions on **antiderivatives and indefinite integrals** — reversing differentiation.

**You must be able to:**

- apply the **power rule for integration**  $\int x^n dx = \frac{x^{n+1}}{n+1} + C$  (for  $n \neq -1$ )
- integrate  $\frac{1}{x}$ ,  $e^x$ , and basic trig functions
- always include the **constant of integration** 积分常数  $C$

## 1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

### ■ The power rule for integration

Add one to the power and divide by the new power:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1).$$

So  $\int x^3 dx = \frac{x^4}{4} + C$  and  $\int 5 dx = 5x + C$ .

### ■ The special case $n = -1$

The power rule fails for  $n = -1$ ; instead  $\int \frac{1}{x} dx = \ln|x| + C$ .

### ■ Standard antiderivatives

$$\int e^x dx = e^x + C, \quad \int \cos x dx = \sin x + C, \quad \int \sin x dx = -\cos x + C, \quad \int \sec^2 x dx = \tan x + C.$$

■ **Term by term (and never forget  $+C$ )**

Integrate a sum piece by piece:  $\int (6x^2 - 4x + 1) dx = 2x^3 - 2x^2 + x + C$ . The  $+C$  is essential — an indefinite integral is a **family** of functions.

## 2 Practice

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Now apply the methods above.

**2.1** Find  $\int x^5 dx$ . [1]

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**2.2** Find  $\int (4x^3 - 2x) dx$ . [2]

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**2.3** Find  $\int \left( \frac{1}{x} + e^x \right) dx$ . [2]

### 3 Exam-style questions

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**3.1**  $\int \cos x \, dx =$  [1]

- **A**  $-\sin x + C$
  - **B**  $\sin x + C$
  - **C**  $-\cos x + C$
  - **D**  $\tan x + C$
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**3.2** (a) Find the general antiderivative of  $f(x) = 3x^2 + 4x$ . [2]

(b) Find the particular antiderivative  $F$  with  $F(0) = 5$ . [2]

**3.3** Find  $\int \left( \sqrt{x} + \frac{2}{x} \right) dx$ . Write  $\sqrt{x}$  as  $x^{1/2}$  first. [3]

## Solutions

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**2.1**  $\frac{x^6}{6} + C.$

**2.2**  $x^4 - x^2 + C.$

**2.3**  $\ln|x| + e^x + C.$

**3.1 B** —  $\int \cos x \, dx = \sin x + C.$

**3.2** (a)  $F(x) = x^3 + 2x^2 + C.$  (b)  $F(0) = C = 5$ , so  $F(x) = x^3 + 2x^2 + 5.$

**3.3**  $\int (x^{1/2} + 2x^{-1}) \, dx = \frac{x^{3/2}}{3/2} + 2 \ln|x| + C = \frac{2}{3}x^{3/2} + 2 \ln|x| + C.$