6.7 The Fundamental Theorem and Evaluating Integrals

Name: ____________ Class: ________ Date: __________ **Total: 13 marks**

KEY WORDS net change 净变化 • antiderivative 原函数 • fundamental theorem of calculus 微积分基本定理 • definite integral 定积分

Objective

Build the skills to answer exam questions on the **Fundamental Theorem of Calculus (evaluation form)** — using an antiderivative to evaluate a definite integral.

You must be able to:

- apply $\displaystyle\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$
- evaluate integrals of powers, exponentials, and basic trig functions
- interpret $\displaystyle\int_a^b f'(x)\,dx = f(b) - f(a)$ as a **net change 净变化**

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Evaluating with an antiderivative

$\displaystyle\int_1^3 2x\,dx$: an antiderivative of $2x$ is x^2, so

$$\int_1^3 2x\,dx = \left[x^2\right]_1^3 = 3^2 - 1^2 = 8.$$

■ A power and a constant

$\displaystyle\int_0^2 (3x^2+1)\,dx = \left[x^3 + x\right]_0^2 = (8+2) - 0 = 10$. Antidifferentiate term by term, then subtract the values at the limits.

> ■ **Net change from a rate**
>
> If f' is a rate of change, then $\displaystyle\int_a^b f'(x)\,dx = f(b) - f(a)$ is the **net change** in f. If a tank's level changes at rate $r(t)$ and $\int_0^{10} r\,dt = -4$, the level **fell** by 4 over those 10 units.
>
> ■ **Trig and exponential**
>
> $\displaystyle\int_0^{\pi} \sin x\,dx = \big[-\cos x\big]_0^{\pi} = -\cos\pi + \cos 0 = 1 + 1 = 2$, and $\displaystyle\int_0^1 e^x\,dx = \big[e^x\big]_0^1 = e - 1$.

2 Practice

Now apply the methods above.

2.1 Evaluate $\displaystyle\int_0^4 x\,dx$. [2]

2.2 Evaluate $\displaystyle\int_1^2 (4x^3)\,dx$. [2]

2.3 Evaluate $\displaystyle\int_0^{\pi/2} \cos x\,dx$. [2]

3 Exam-style questions

3.1 $\displaystyle\int_a^b f'(x)\,dx =$ [1]

- **A** $f'(b) - f'(a)$
- **B** $f(b) - f(a)$
- **C** $f(b) + f(a)$
- **D** $\frac{1}{2}(f(a) + f(b))$

3.2 A particle's velocity is $v(t) = 3t^2 - 4$ m/s.

(a) Evaluate $\displaystyle\int_0^2 v(t)\,dt$. [2]

(b) State what this value represents physically. [1]

3.3 Water enters a tank at rate $r(t) = 6t$ L/min. Find the total volume added between $t = 1$ and $t = 4$ minutes. [3]

Solutions

2.1 $\left[\frac{1}{2}x^2\right]_0^4 = 8 - 0 = 8.$

2.2 $\left[x^4\right]_1^2 = 16 - 1 = 15.$

2.3 $\left[\sin x\right]_0^{\pi/2} = 1 - 0 = 1.$

3.1 **B** — by the FTC, $\int_a^b f' = f(b) - f(a).$

3.2 (a) $\left[t^3 - 4t\right]_0^2 = (8 - 8) - 0 = 0.$ (b) The particle's **displacement** over $0 \leq t \leq 2$ is 0 — it returns to its start.

3.3 $\displaystyle\int_1^4 6t\,dt = \left[3t^2\right]_1^4 = 48 - 3 = 45$ L.