

5.4 The First Derivative Test

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS local minimum 极小值 • local maximum 极大值 • local extremum 局部极值 • critical point 临界点

Objective

Build the skills to answer exam questions on **the First Derivative Test**.

You must be able to:

- use the sign change of f' to classify a **local extremum** 局部极值

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ First Derivative Test

At a critical point, look at how f' changes:

- $+$ to $-$ $\rightarrow$ local **maximum**;
- $-$ to $+$ $\rightarrow$ local **minimum**;
- no sign change $\rightarrow$ neither.

■ Example

$f'(x) = 2x - 4$ at $x = 2$: f' goes $-$ (for $x < 2$) to $+$ (for $x > 2$), so $x = 2$ is a local minimum.

2 Practice

2.1 State the sign change for a local maximum. [1]

2.2 State the sign change for a local minimum. [1]

2.3 For $f'(x) = 2x - 4$, classify $x = 2$. [2]

3 Exam-style questions

3.1 A local maximum occurs where f' changes [1]

- **A** $-$ to $+$
 - **B** $+$ to $-$
 - **C** stays $+$
 - **D** stays $-$
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3.2 A local minimum occurs where f' changes [1]

- **A** $+$ to $-$
 - **B** $-$ to $+$
 - **C** stays 0
 - **D** never
-

3.3 $f'(x) = 2x - 4$, critical at $x = 2$.

(a) State the sign of f' for $x < 2$. [1]

(b) State the sign for $x > 2$. [1]

(c) Classify $x = 2$. [1]

Solutions

2.1 f' changes from $+$ to $-$.

2.2 f' changes from $-$ to $+$.

2.3 f' goes $-$ to $+$, so $x = 2$ is a local minimum.

3.1 B.

3.2 B.

3.3 (a) negative. (b) positive. (c) local minimum.