Past-paper walk-through

Volume with Disc Method: Revolving Around the x- or y-Axis ▪ 8.9

eXercise

Questions in this set

- 2026 Q2
- 2021 Q3
- 2016 Q5

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2026 ▪ Q2

The function f is defined by $f(x) = 1.43^x + 0.57$, and the function g is defined by $g(x) = \dfrac{14x + 12}{x + 12}$. The graphs of f and g intersect at the points $(1, 2)$ and (a, b), as shown in Figure 1.

[Figure 1: A graph with x-axis from 0 to 3+ and y-axis from 0 to 5+. The curve $y = g(x)$ and the curve $y = f(x)$ both start near $(0, 1)$–$(0, 1.5)$, rise, and intersect at the labeled point $(1, 2)$ and again at the labeled point (a, b) (near $x = 3$, $y \approx 3.8$), with f above g just after the second intersection. The region R is shaded/bounded between the curve $y = g(x)$, the x-axis, the y-axis, and the vertical line $x = 1$ (region labeled R near the origin below g). The shaded region between the two curves from $x = 0$ to $x = a$ is shown shaded gray between $(1, 2)$ and (a, b).]

Question 2

For $x \geq 0$, the equation $y = g(x)$ can be rewritten as $x = h(y) = \dfrac{12y - 12}{14 - y}$, where h is a function of y. The graph of $x = h(y)$ and the horizontal line $y = 3.5$ are shown in Figure 2.
[Figure 2: A graph with x-axis from 0 to 3+ and y-axis from 0 to 5+. The curve $x = h(y)$ starts near $(0, 1)$ and increases, crossing the horizontal line $y = 3.5$ near $x = 3$. The region T is the region bounded by the curve $x = h(y)$, the y-axis, and the line $y = 3.5$, labeled T to the left of the curve.]
END OF PART A
(a) Let R be the region bounded by the graph of g, the x-axis, the y-axis, and the vertical line $x = 1$, as shown in Figure 1. Find the area of region R. Show the setup for your calculations.

Question 2

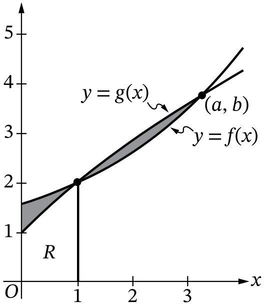

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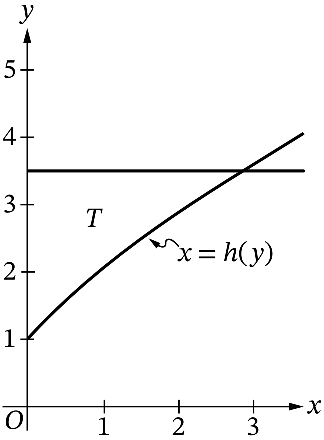
Figure 2

Question 2



Question 2

2026 ▪ Q2

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[Figure 2: A graph with x-axis from 0 to 3+ and y-axis from 0 to 5+. The curve $x = h(y)$ starts near $(0, 1)$ and increases, crossing the horizontal line $y = 3.5$ near $x = 3$. The region T is the region bounded by the curve $x = h(y)$, the y-axis, and the line $y = 3.5$, labeled T to the left of the curve.]

END OF PART A

(b) Region R, described in part A, is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is $\frac{1}{3}$ times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

Question 2

2026 ▪ Q2

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For $x \geq 0$, the equation $y = g(x)$ can be rewritten as $x = h(y) = \dfrac{12y - 12}{14 - y}$, where h is a function of y. The graph of $x = h(y)$ and the horizontal line $y = 3.5$ are shown in Figure 2.

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[Figure 2: A graph with x-axis from 0 to 3+ and y-axis from 0 to 5+. The curve $x = h(y)$ starts near $(0, 1)$ and increases, crossing the horizontal line $y = 3.5$ near $x = 3$. The region T is the region bounded by the curve $x = h(y)$, the y-axis, and the line $y = 3.5$, labeled T to the left of the curve.]

END OF PART A

(c) The shaded region in Figure 1 is bounded by the graphs of f and g on the interval from $x = 0$ to $x = a$. Find the area of the shaded region. Show the setup for your calculations.

Question 2

2026 ▪ Q2

The function f is defined by $f(x) = 1.43^x + 0.57$, and the function g is defined by $g(x) = \dfrac{14x+12}{x+12}$. The graphs of f and g intersect at the points $(1,2)$ and (a,b), as shown in Figure 1.

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Question 2

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END OF PART A

(d) Let T be the region bounded by the graph of $x = h(y)$, the y-axis, and the horizontal line $y = 3.5$, as shown in Figure 2. Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region T is revolved about the y-axis.

Question 3

2021 ▪ Q3

[Graph showing a curve in the first quadrant starting at the origin O, rising to a rounded peak, then curving back down to meet the x-axis; x-axis and y-axis labeled, no numeric scale marks shown.]

A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c. Each spinning toy is in the shape of the solid generated when such a region is revolved about the x-axis. Both x and y are measured in inches.

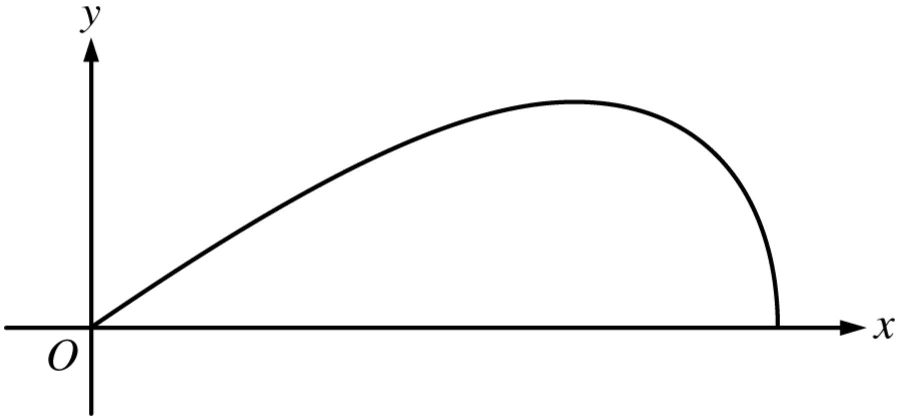
(a) Find the area of the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.

Question 3

(b) It is known that, for $y = cx\sqrt{4-x^2}$, $\dfrac{dy}{dx} = \dfrac{c(4-2x^2)}{\sqrt{4-x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?

(c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

Question 3



Solution 3

(a) Mark scheme

- For $c = 6$: $6x\sqrt{4-x^2} = 0 \Rightarrow x = 0, 2$, so Area $= \displaystyle\int_0^2 6x\sqrt{4-x^2}\,dx$. Using $u = 4 - x^2$: $\int_0^2 6x\sqrt{4-x^2}\,dx = 3\int_0^4 u^{1/2}\,du = 2u^{3/2}\Big|_0^4 = 2 \cdot 8 = 16$. The area of the region is 16 square inches. Point 1: presenting $cx\sqrt{4-x^2}$ or $6x\sqrt{4-x^2}$ as the integrand of a definite integral (limits need not be correct). Point 2: a correct antiderivative of the form $Ax\sqrt{4-x^2}\ldots$ [presented as $Au^{3/2}$-type expression] for any nonzero constant (independent of point 1). Point 3: the final answer 16 (or equivalent), only earned if point 2 was earned, using the correct limits.

Solution 3

(b) Mark scheme

- The largest cross-sectional radius occurs where $\dfrac{dy}{dx} = 0$:
  $\dfrac{dy}{dx} = \dfrac{c(4-2x^2)}{\sqrt{4-x^2}} = 0 \Rightarrow x = \sqrt{2}$. Then $y = c\sqrt{2}\sqrt{4-(\sqrt{2})^2} = 2c$. Setting $2c = 1.2$ gives $c = 0.6$. Point 1: setting $\frac{dy}{dx} = 0$ (or the equivalent $c(4-2x^2) = 0$); an unsupported $x = \sqrt{2}$ alone does not earn it. Point 2: the final answer $c = 0.6$ with supporting work (can be earned without point 1).

Solution 3

(c) Mark scheme

- Volume $= \int_0^2 \pi \left(cx\sqrt{4-x^2}\right)^2 dx = \pi c^2 \int_0^2 x^2(4-x^2)\,dx =$ $\pi c^2 \int_0^2 (4x^2 - x^4)\,dx = \pi c^2 \left(\frac{4}{3}x^3 - \frac{1}{5}x^5\right)\Big|_0^2 = \pi c^2 \left(\frac{32}{3} - \frac{32}{5}\right) = \frac{64\pi c^2}{15}$. Setting $\frac{64\pi c^2}{15} = 2\pi$ gives $c^2 = \frac{15}{32}$, so $c = \sqrt{15/32}$. Point 1: integrand of the form $A(x\sqrt{4-x^2})^2$ in a definite integral, any limits, any nonzero constant A (mishandling c makes it ineligible for point 4). Point 2: correct limits $x = 0, 2$ and constant π (not 2π) as part of an integral — earnable without point 1. Point 3: correct antiderivative of the presented integrand, form $A(x\sqrt{4-x^2})^2$-derived

Solution 3

polynomial, for any nonzero A — needed for point 4. Point 4: final answer $c = \sqrt{15/32}$ (need not be simplified); cannot be earned without point 3.

Question 5

2016 ▪ Q5

[Diagram of a funnel with circular cross sections: a wide circular opening at the top tapering down through a cone shape to a narrow spout at the bottom. At a height *h* up from the bottom, a horizontal cross-sectional circle of radius *r* is indicated, with *r* labeled as the horizontal distance from the central vertical axis to the funnel's inner wall, and *h* labeled as the vertical distance from the bottom of the funnel up to that cross section.]

The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \leq h \leq 10$. The units of r and h are inches.

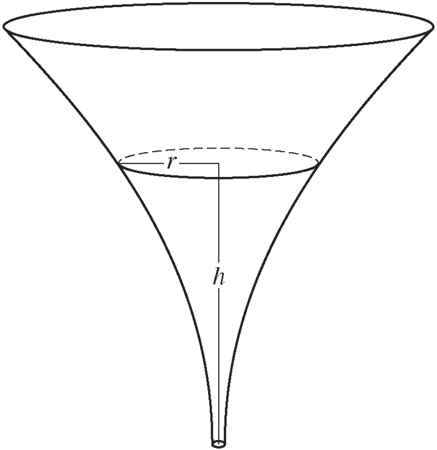
(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

Question 5

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

Question 5



Solution 5

(a) Mark scheme

- Average radius $= \frac{1}{10}\int_0^{10}\frac{1}{20}(3+h^2)\,dh = \frac{1}{200}\left[3h+\frac{h^3}{3}\right]_0^{10} = \frac{1}{200}\left((30+\frac{1000}{3})-0\right) = \frac{109}{60} inches. Points: 1 forthecorrectintegralexpression (average-valueformula), 1 forthecorrectantiderivative, 1 forthecorrectfinalanswer (109/60 in).$

Solution 5

(b) Mark scheme

- Volume $= \pi \int_0^{10} \left[\frac{1}{20}(3 + h^2)\right]^2 dh = \frac{\pi}{400} \int_0^{10} (9 + 6h^2 + h^4)\, dh = \frac{\pi}{400}\left[9h + 2h^3 + \frac{h^5}{5}\right]_0^{10} = \frac{\pi}{400}\left((90 + 2000 + \frac{100000}{5}) - 0\right) = \frac{2209\pi}{40}$ in^3$. Points: 1 for the correct integrand (\pi r^2 expanded), 1 for the correct antiderivative, 1 for the correct final volume($

Solution 5

(c) Mark scheme

- Since r=$\frac{1}{20}(3+h^2), differentiateimplicitlywithrespecttot: \frac{dr}{dt} = \frac{1}{20}(2h)\frac{dh}{dt} = \frac{h}{10}\frac{dh}{dt}.Ath = 3, dr/dt = -1/5 : -\frac{1}{5} = \frac{3}{10}\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = -\frac{1}{5}\cdot\frac{10}{3} = -\frac{2}{3}$ in/sec.$Points: 2forcorrectlyapplyingthechainruletorelatedr/dtanddh/dt, 1forthecorrectfinalanswer(-2/3in/se