Past-paper walk-through

Volumes with Cross Sections: Triangles and Semicircles ▪ 8.8

eXercise

Questions in this set

- 2014 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2014 ▪ Q2

[Graph of $y = f(x)$ for $x \geq 0$: a curve starting near the origin, dipping down below the x-axis to a minimum, then rising steeply back up and crossing above $y = 0$; the region R is labelled in the dip between the curve and the x-axis/line $y = 4$, roughly between the two crossing points of $f(x) = 4$.]

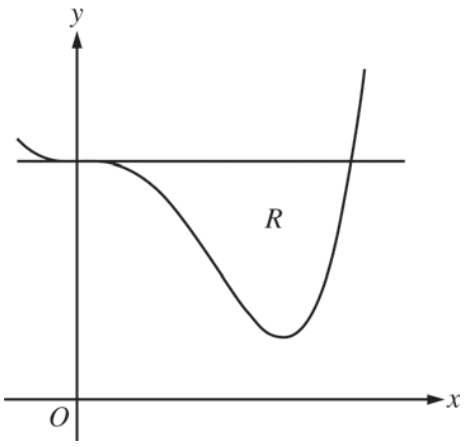
Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

Question 2



Solution 2

(a)

Mark scheme

- f(x) = 4 at x = 0 and x = 2.3 (the intersection points bounding R). Volume = $\pi \int_0^{2.3} \left[(4+2)^2 - (f(x)+2)^2\right] dx$ = $98.868 (or 98.867)$, $using washers with outer radius 6 (from y = -2 to y = 4) and inner radius f(x) + 2 (from y = -2 to y = f(x))$. $Points$: $2 for the correct integrand, 1 for correct limits of integration (0 to 2.3), 1 for the correct numerical answer$

Solution 2

(b)

Mark scheme

- Volume $= \int_0^{2.3} \frac{1}{2}(4-f(x))^2\,dx = 3.574 (or 3.573)$, *using that an isosceles right triangle with legs* $= 4 - f(x)$ *(the vertical distance in R) has area* $(1/2)s^2$. *Points* : *2 for the correct integrand, 1 for the correct numerical answer.*

Solution 2

(c)

Mark scheme

- Set the area of the left piece equal to the area of the right piece: $\int_0^k (4 - f(x))\,dx = \int_k^{2.3} (4 - f(x))\,dx$. (*Equation only* $--$ *do not solve for k.*) *Points* : 1 *for a correct expression for the area of one region*, 1 *for a correct equation setting the two regions' areas*