

Past-paper walk-through

Volumes with Cross Sections: Squares and Rectangles ■ 8.7

eXercise

Questions in this set

- 2026 Q2
- 2025 Q2
- 2024 Q6
- 2022 Q2
- 2019 Q5
- 2015 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- **//** separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 2

2026 ■ Q2

The function f is defined by $f(x) = 1.43^x + 0.57$, and the function g is defined by $g(x) = \frac{14x + 12}{x + 12}$. The graphs of f and g intersect at the points $(1, 2)$ and (a, b) , as shown in Figure 1.

[Figure 1: A graph with x -axis from 0 to $3+$ and y -axis from 0 to $5+$. The curve $y = g(x)$ and the curve $y = f(x)$ both start near $(0, 1)$ – $(0, 1.5)$, rise, and intersect at the labeled point $(1, 2)$ and again at the labeled point (a, b) (near $x = 3$, $y \approx 3.8$), with f above g just after the second intersection. The region R is shaded/bounded between the curve $y = g(x)$, the x -axis, the y -axis, and the vertical line $x = 1$ (region labeled R near the origin below g). The shaded region between the two curves from $x = 0$ to $x = a$ is shown shaded gray between $(1, 2)$ and (a, b) .]

Question 2

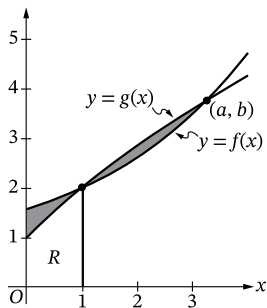
For $x \geq 0$, the equation $y = g(x)$ can be rewritten as $x = h(y) = \frac{12y - 12}{14 - y}$, where h is a function of y . The graph of $x = h(y)$ and the horizontal line $y = 3.5$ are shown in Figure 2.

[Figure 2: A graph with x -axis from 0 to $3+$ and y -axis from 0 to $5+$. The curve $x = h(y)$ starts near $(0, 1)$ and increases, crossing the horizontal line $y = 3.5$ near $x = 3$. The region T is the region bounded by the curve $x = h(y)$, the y -axis, and the line $y = 3.5$, labeled T to the left of the curve.]

END OF PART A

(a) Let R be the region bounded by the graph of g , the x -axis, the y -axis, and the vertical line $x = 1$, as shown in Figure 1. Find the area of region R . Show the setup for your calculations.

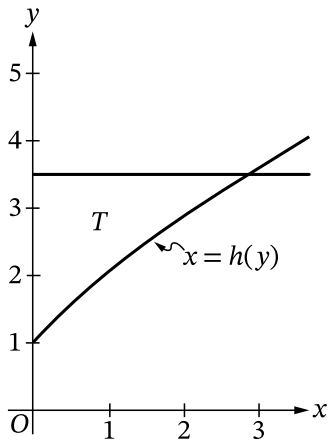
Question 2



For $x \geq 0$, the equation $y = g(x)$ can be rewritten as $x = h(y) = \frac{12y - 12}{14 - y}$, where h is a function of y . The graph of $x = h(y)$ and the horizontal line $y = 3.5$ are shown in Figure 2.

Figure 2

Question 2



Question 2

2026 ■ Q2

The function f is defined by $f(x) = 1.43^x + 0.57$, and the function g is defined by

$g(x) = \frac{14x + 12}{x + 12}$. The graphs of f and g intersect at the points $(1, 2)$ and (a, b) , as shown in Figure 1.

[Figure 1: A graph with x -axis from 0 to $3+$ and y -axis from 0 to $5+$. The curve $y = g(x)$ and the curve $y = f(x)$ both start near $(0, 1)-(0, 1.5)$, rise, and intersect at the labeled point $(1, 2)$ and again at the labeled point (a, b) (near $x = 3$, $y \approx 3.8$), with f above g just after the second intersection. The region R is shaded/bounded between the curve $y = g(x)$, the x -axis, the y -axis, and the vertical line $x = 1$ (region labeled R near the origin below g). The shaded region between the two curves from $x = 0$ to $x = a$ is shown shaded gray between $(1, 2)$ and (a, b) .]

For $x \geq 0$, the equation $y = g(x)$ can be rewritten as $x = h(y) = \frac{12y - 12}{14 - y}$, where h is a function of y . The graph of $x = h(y)$ and the horizontal line $y = 3.5$ are shown in Figure 2.

Question 2

[Figure 2: A graph with x -axis from 0 to $3+$ and y -axis from 0 to $5+$. The curve $x = h(y)$ starts near $(0, 1)$ and increases, crossing the horizontal line $y = 3.5$ near $x = 3$. The region T is the region bounded by the curve $x = h(y)$, the y -axis, and the line $y = 3.5$, labeled T to the left of the curve.]

END OF PART A

(b) Region R , described in part A, is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a rectangle whose height is $\frac{1}{3}$ times the length of its base in region R . Write, but do not evaluate, an integral expression that gives the volume of the solid.

Question 2

2026 ■ Q2

The function f is defined by $f(x) = 1.43^x + 0.57$, and the function g is defined by

$g(x) = \frac{14x + 12}{x + 12}$. The graphs of f and g intersect at the points $(1, 2)$ and (a, b) , as shown in Figure 1.

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For $x \geq 0$, the equation $y = g(x)$ can be rewritten as $x = h(y) = \frac{12y - 12}{14 - y}$, where h is a function of y . The graph of $x = h(y)$ and the horizontal line $y = 3.5$ are shown in Figure 2.

Question 2

[Figure 2: A graph with x-axis from 0 to 3+ and y-axis from 0 to 5+. The curve $x = h(y)$ starts near $(0, 1)$ and increases, crossing the horizontal line $y = 3.5$ near $x = 3$. The region T is the region bounded by the curve $x = h(y)$, the y-axis, and the line $y = 3.5$, labeled T to the left of the curve.]

END OF PART A

(c) The shaded region in Figure 1 is bounded by the graphs of f and g on the interval from $x = 0$ to $x = a$. Find the area of the shaded region. Show the setup for your calculations.

Question 2

2026 ■ Q2

The function f is defined by $f(x) = 1.43^x + 0.57$, and the function g is defined by

$g(x) = \frac{14x + 12}{x + 12}$. The graphs of f and g intersect at the points $(1, 2)$ and (a, b) , as shown in Figure 1.

[Figure 1: A graph with x -axis from 0 to $3+$ and y -axis from 0 to $5+$. The curve $y = g(x)$ and the curve $y = f(x)$ both start near $(0, 1)-(0, 1.5)$, rise, and intersect at the labeled point $(1, 2)$ and again at the labeled point (a, b) (near $x = 3$, $y \approx 3.8$), with f above g just after the second intersection. The region R is shaded/bounded between the curve $y = g(x)$, the x -axis, the y -axis, and the vertical line $x = 1$ (region labeled R near the origin below g). The shaded region between the two curves from $x = 0$ to $x = a$ is shown shaded gray between $(1, 2)$ and (a, b) .]

For $x \geq 0$, the equation $y = g(x)$ can be rewritten as $x = h(y) = \frac{12y - 12}{14 - y}$, where h is a function of y . The graph of $x = h(y)$ and the horizontal line $y = 3.5$ are shown in Figure 2.

Question 2

[Figure 2: A graph with x -axis from 0 to $3+$ and y -axis from 0 to $5+$. The curve $x = h(y)$ starts near $(0, 1)$ and increases, crossing the horizontal line $y = 3.5$ near $x = 3$. The region T is the region bounded by the curve $x = h(y)$, the y -axis, and the line $y = 3.5$, labeled T to the left of the curve.]

END OF PART A

(d) Let T be the region bounded by the graph of $x = h(y)$, the y -axis, and the horizontal line $y = 3.5$, as shown in Figure 2. Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region T is revolved about the y -axis.

Question 2

2025 ■ Q2

The shaded region R is bounded by the graphs of the functions f and g , where $f(x) = x^2 - 2x$ and $g(x) = x + \sin(\pi x)$, as shown in the figure.

[Figure: In the xy -plane, the graph of $y = g(x)$ and the graph of $y = f(x)$ are shown.

The shaded region R , labeled R , lies between the two curves. The point $(3, 3)$ is marked where the curves meet. The y -axis is labeled with values $-2, -1, 1, 2, 3, 4$ and the x -axis is labeled with values $1, 2, 3$, with origin O .]

(Note: Your calculator should be in radian mode.)

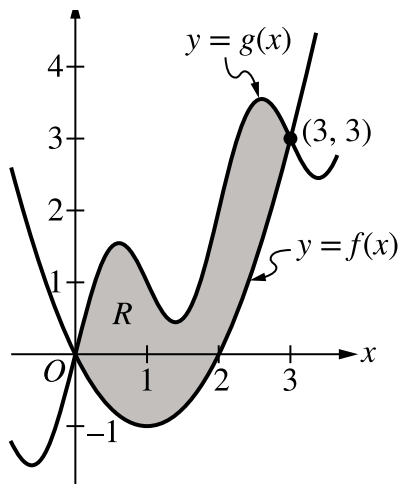
(a) Find the area of R . Show the setup for your calculations.

(b) Region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x -axis is a rectangle with height x and base in region R . Find the volume of the solid. Show the setup for your calculations.

Question 2

- (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when the region R is rotated about the horizontal line $y = -2$.
- (d) It can be shown that $g'(x) = 1 + \pi \cos(\pi x)$. Find the value of x , for $0 < x < 1$, at which the line tangent to the graph of f is parallel to the line tangent to the graph of g .

Question 2



Solution 2

(a)

Mark scheme

- Area = $\int_0^3 (g(x) - f(x)) dx = 5.136620 \approx 5.137$ (exact value $\frac{4 + 9\pi}{2\pi}$). Point 1: presents an integral of $g(x) - f(x)$, $|g(x) - f(x)|$, $f(x) - g(x)$, or $|f(x) - g(x)|$ over $[0, 3]$ (or an equivalent difference of definite integrals of g and f). Point 2: correct answer 5.137 (or 5.136), with or without supporting work.

Solution 2

(b)

Mark scheme

■ Volume = $\int_0^3 x(g(x) - f(x)) dx = 7.704930 \approx 7.705$ (exact value $\frac{12 + 27\pi}{4\pi}$).

Point 3: presents a definite integral with integrand a product of two nonconstant factors, one factor being x , $g(x) - f(x)$, or $f(x) - g(x)$. Point 4: correct answer 7.705 (or 7.704), with or without supporting work.

Solution 2

(c)

Mark scheme

- Volume = $\pi \int_0^3 [(g(x) - (-2))^2 - (f(x) - (-2))^2] dx =$
 $\pi \int_0^3 [(g(x) + 2)^2 - (f(x) + 2)^2] dx$ (expression only, not evaluated). Point 5:
 integrand of the form $R^2 - r^2$ (or $|R^2 - r^2|$) where one of R, r is g or f (or a
 difference involving a nonzero constant) and the other is correct or a
 difference between f/g and a nonzero constant. Point 6: correct integrand
 $(g(x) + 2)^2 - (f(x) + 2)^2$ (or equivalent, e.g. reversed with a resolved sign).
 Point 7: correct limits of integration (0 to 3), the constant π , and the
 differential dx all present and correctly combined; requires Point 5.

Solution 2

(d)

Mark scheme

- Set $f'(x) = g'(x)$: $2x - 2 = 1 + \pi \cos(\pi x) \Rightarrow x = 0.675819 \approx 0.676$ (or 0.675).
Point 8: general statement $f'(x) = g'(x)$, or any correct equation substituting $2x - 2$ for $f'(x)$ and/or $1 + \pi \cos(\pi x)$ for $g'(x)$. Point 9: correct answer $x \approx 0.676$, with or without supporting work.

Question 6

2024 ■ Q6

[Graph of f and g ; x -axis labeled 0 to 5, y -axis labeled 5 to 25; curve $y = f(x)$ is an upward-opening parabola-like curve passing through about $(0, 2)$ and rising steeply to about $(5, 27)$; curve $y = g(x)$ passes through the origin area, dips slightly below the x -axis between $x = 0$ and $x = 2$, then rises through about $(5, 15)$; the shaded region R lies between the two curves from $x = 0$ to $x = 2$; the shaded region S lies between the graph of g and the x -axis from $x = 2$ to $x = 5$.]

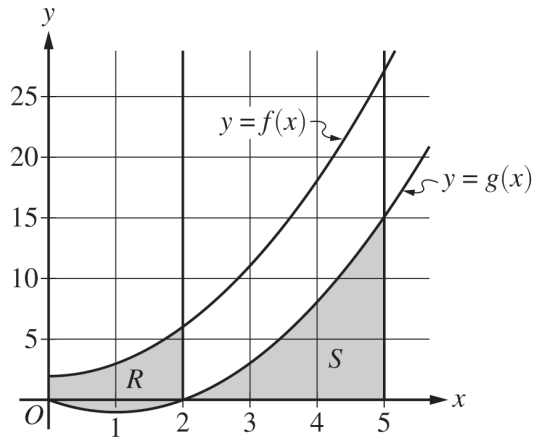
The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

(a) Let R be the region bounded by the graphs of f and g , from $x = 0$ to $x = 2$, as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region R .

Question 6

- (b)** Let S be the region bounded by the graph of g and the x -axis, from $x = 2$ to $x = 5$, as shown in the graph. Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x -axis is a rectangle with height equal to half its base in region S . Find the volume of the solid. Show the work that leads to your answer.
- (c)** Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region S , as described in part (b), is rotated about the horizontal line $y = 20$.

Question 6



Solution 6

(a)

Mark scheme

- With $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, the area of region R (bounded by f , g , from $x = 0$ to $x = 2$, where $f \geq g$) is given by the (unevaluated) integral $\text{Area} = \int_0^2 (f(x) - g(x)) \, dx$. 1 point for an integrand equivalent to $f(x) - g(x)$ (or $|f(x) - g(x)|$, or a difference of two separate integrals with integrands f and g), 1 point for the fully correct expression $\int_0^2 (f(x) - g(x)) \, dx$ (equivalent forms accepted, e.g. $\int_0^2 f(x) \, dx - \int_0^2 g(x) \, dx$). Do not evaluate.

Solution 6

(b) Mark scheme

- Cross sections perpendicular to the x -axis over region S (g and the x -axis, $x = 2$ to $x = 5$) are rectangles with height $= \frac{1}{2} \cdot \text{base}$, so area $= \frac{1}{2}(g(x))^2$:

$$V = \int_2^5 \frac{1}{2}(g(x))^2 dx = \int_2^5 \frac{1}{2}(x^2 - 2x)^2 dx = \frac{1}{2} \int_2^5 (x^4 - 4x^3 + 4x^2) dx =$$

$$\frac{1}{2} \left[\frac{x^5}{5} - x^4 + \frac{4x^3}{3} \right]_2^5 = \frac{1}{2} \left(\frac{500}{3} - \frac{16}{15} \right) = \frac{414}{5} = 82.8. \quad \text{1 point for the integrand}$$

of the form $k(g(x))^2$, 1 point for the correct limits $x = 2$ to $x = 5$ with an integrand of the form $a(x) \cdot g(x)$, 1 point for a correct antiderivative, 1 point for the final numeric answer $414/5$ (or 82.8).

Solution 6

(c)

Mark scheme

- Rotating region S about $y = 20$ gives washers with outer radius $20 - 0 = 20$ (from $y = 20$ down to the x -axis, $y = 0$) and inner radius $20 - g(x)$:
$$V = \pi \int_2^5 [20^2 - (20 - g(x))^2] dx = \pi \int_2^5 [400 - (20 - g(x))^2] dx =$$
$$\pi \int_2^5 [400 - (20 - (x^2 - 2x))^2] dx \text{ (do not evaluate).}$$
 1 point for the correct form of the integrand $20^2 - (20 - g(x))^2$ (or an equivalent difference-of-squares form), 1 point for substituting $g(x) = x^2 - 2x$ correctly into that integrand, 1 point for the correct limits $x = 2$ to $x = 5$ together with the constant π .

Question 2

2022 ■ Q2

[Graph: a coordinate plane with x-axis from -2 to 1 and y-axis from -1 to 1 (tick marks labeled -1 on y-axis, 1 on y-axis; -2 , -1 , O , 1 on x-axis). A curve labeled implicitly by the two functions below starts at $(-2, 0)$, dips down to a minimum below the x-axis between $x = -1$ and $x = 0$, rises back up crossing near the origin, and continues upward, ending with a marked point just above and to the right of $(1, 1)$.]

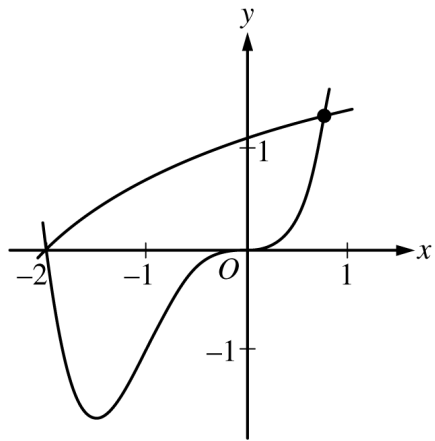
Let f and g be the functions defined by $f(x) = \ln(x + 3)$ and $g(x) = x^4 + 2x^3$. The graphs of f and g , shown in the figure above, intersect at $x = -2$ and $x = B$, where $B > 0$.

- (a) Find the area of the region enclosed by the graphs of f and g .
- (b) For $-2 \leq x \leq B$, let $h(x)$ be the vertical distance between the graphs of f and g . Is h increasing or decreasing at $x = -0.5$? Give a reason for your answer.

Question 2

- (c) The region enclosed by the graphs of f and g is the base of a solid. Cross sections of the solid taken perpendicular to the x -axis are squares. Find the volume of the solid.
- (d) A vertical line in the xy -plane travels from left to right along the base of the solid described in part (c). The vertical line is moving at a constant rate of 7 units per second. Find the rate of change of the area of the cross section above the vertical line with respect to time when the vertical line is at position $x = -0.5$.

Question 2



Solution 2

(a)

Mark scheme

- $\ln(x+3) = x^4 + 2x^3 \Rightarrow x = -2, x = B = 0.781975$. Area
 $= \int_{-2}^B (f(x) - g(x)) dx = \int_{-2}^B (\ln(x+3) - (x^4 + 2x^3)) dx = 3.603548$. The area of the region is 3.604 (or 3.603). Points: (1) correct integrand $f(x) - g(x)$ (or an equivalent form such as $|f(x) - g(x)|$ or $g(x) - f(x)$); (2) correct limits of integration, -2 to B ; (3) correct answer.

Solution 2

(b)

Mark scheme

- $h(x) = f(x) - g(x)$, so $h'(x) = f'(x) - g'(x)$.
 $h'(-0.5) = f'(-0.5) - g'(-0.5) = -0.6$ (or -0.599). Since $h'(-0.5) < 0$, h is decreasing at $x = -0.5$. Points: (1) considers $h'(-0.5)$ (or equivalently $f'(-0.5) - g'(-0.5)$); (2) correct answer (decreasing) with reason.

Solution 2

(c)

Mark scheme

- Volume = $\int_{-2}^B (f(x) - g(x))^2 dx = 5.340102$. The volume of the solid is 5.340.
Points: (1) correct integrand $k(f(x) - g(x))^2$ with $k = 1$ in a definite integral;
(2) correct answer.

Solution 2

(d) Mark scheme

- The cross section has area $A(x) = (f(x) - g(x))^2$. By the chain rule,

$$\frac{d}{dt}[A(x)] = \frac{dA}{dx} \cdot \frac{dx}{dt}. \text{ At } x = -0.5: \left. \frac{d}{dt}[A(x)] \right|_{x=-0.5} = A'(-0.5) \cdot 7 = -9.271842.$$

At $x = -0.5$, the area of the cross section is changing at a rate of about -9.272 (or -9.271) square units per second. Points: (1) correct chain-rule expression $\frac{dA}{dx} \cdot \frac{dx}{dt}$ (equivalently $A'(x) \cdot dx/dt$); (2) correct answer.

Question 5

2019 ■ Q5

Let R be the region enclosed by the graphs of $g(x) = -2 + 3 \cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x - 1)^2$, the y -axis, and the vertical line $x = 2$, as shown in the figure below.

[Graph showing region R shaded gray; x -axis from O to past 2 (gridlines at 1 and 2), y -axis with gridline at 1. The upper boundary curve $h(x)$ starts above the y -axis, rises to a peak between $x = 0$ and $x = 1$, then descends, meeting the vertical line $x = 2$ near $y \approx 1.5$ -2. The lower boundary curve $g(x)$ starts near $(0, 1)$, dips down crossing the x -axis around $x = 1$, continues down to a minimum, then rises back up to meet $h(x)$ at the line $x = 2$. Region R is the shaded area between the two curves from $x = 0$ to $x = 2$, bounded on the left by the y -axis and on the right by the line $x = 2$.]

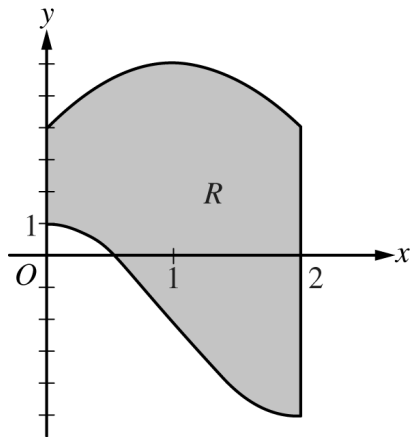
(a) Find the area of R .

Question 5

(b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x -axis has area $A(x) = \frac{1}{x+3}$. Find the volume of the solid.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

Question 5



Solution 5

(a) Mark scheme

- Area of $R = \int_0^2 (h(x) - g(x)) dx = \int_0^2 \left[(6 - 2(x-1)^2) - (-2 + 3 \cos(\frac{\pi}{2}x)) \right] dx = \left[(6x - \frac{2}{3}(x-1)^3) - (-2x + \frac{6}{\pi} \sin(\frac{\pi}{2}x)) \right]_{x=0}^{x=2} = \left((12 - \frac{2}{3}) - (-4 + 0) \right) - \left((0 + \frac{2}{3}) - (0 + 0) \right) = 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}$. The area of R is $\frac{44}{3}$. Points: 1 for the integrand (correctly set up as $h(x) - g(x)$), 1 for the antiderivative of $3 \cos(\frac{\pi}{2}x)$, 1 for the antiderivative of the remaining terms, 1 for the final answer $\frac{44}{3}$.

Solution 5

(b)

Mark scheme

- Volume = $\int_0^2 A(x) dx = \int_0^2 \frac{1}{x+3} dx = [\ln(x+3)]_{x=0}^{x=2} = \ln 5 - \ln 3$. The volume of the solid is $\ln 5 - \ln 3$. Points: 1 for the correct integral, 1 for the final answer $\ln 5 - \ln 3$.

Solution 5

(c)

Mark scheme

- Volume when R is rotated about the line $y = 6$ (washer method), written but NOT evaluated: $\pi \int_0^2 [(6 - g(x))^2 - (6 - h(x))^2] dx$. Points: 1 for correct limits and constant (π , from 0 to 2), 1 for the correct form of the integrand (outer-radius-squared minus inner-radius-squared, using $6 - g(x)$ and $6 - h(x)$), 1 for the fully correct integrand overall.

Question 2

2015 ■ Q2

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

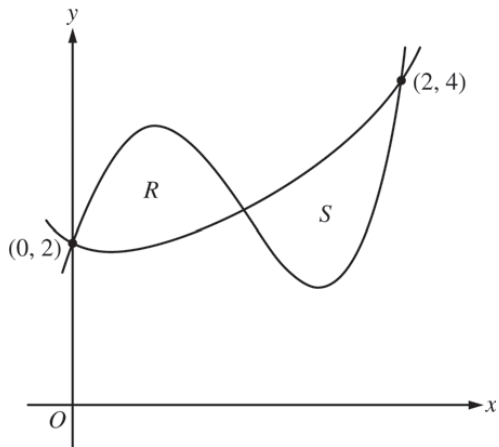
[Graph showing the curves f and g on axes labeled x and y . Both curves pass through the point $(0, 2)$ on the y -axis and meet again at the point $(2, 4)$. Between these intersection points, one curve rises to a local peak and the other dips to a local trough, crossing each other partway through; the region enclosed on the left (near the peak) is labeled R , and the region enclosed on the right (near the trough) is labeled S . The labeled points are $(0, 2)$ and $(2, 4)$.]

- (a) Find the sum of the areas of regions R and S .
- (b) Region S is the base of a solid whose cross sections perpendicular to the x -axis are squares. Find the volume of the solid.

Question 2

(c) Let h be the vertical distance between the graphs of f and g in region S . Find the rate at which h changes with respect to x when $x = 1.8$.

Question 2



Solution 2

(a)

Mark scheme

- The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at $(0, 2)$, $(2, 4)$, and $(A, B) = (1.032832, 2.401108)$. Sum of areas $= \int_0^A [g(x) - f(x)] dx + \int_A^2 [f(x) - g(x)] dx = 0.997427 + 1.006919 = 2.004$. Points: 1 for correct limits (0, A, 2, using the intersection point $A \approx 1.033$), 2 for correct integrands ($[g - f]$ on $[0, A]$ and $[f - g]$ on $[A, 2]$), 1 for the correct final numeric answer 2.004.

Solution 2

(b)

Mark scheme

- Region S is the base of a solid with square cross sections perpendicular to the x-axis, side length $f(x) - g(x)$. Volume = $\int_A^2 [f(x) - g(x)]^2 dx = 1.283$, using $A \approx 1.032832$ as the lower limit (region S spans $x = A$ to $x = 2$). Points: 2 for the correct integrand $[f(x) - g(x)]^2$ with correct limits of integration, 1 for the correct numeric answer 1.283.

Solution 2

(c)

Mark scheme

- $h(x) = f(x) - g(x)$, so $h'(x) = f'(x) - g'(x)$. $h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811). Points: 1 for considering $h'(x) = f'(x) - g'(x)$, 1 for the correct numeric answer ≈ -3.812 .