Past-paper walk-through

Finding the Area Between Curves Expressed as Functions of x ▪ 8.4

eXercise

Questions in this set

- 2026 Q2
- 2025 Q2
- 2024 Q6
- 2022 Q2
- 2019 Q5
- 2015 Q2
- 2014 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2026 ▪ Q2

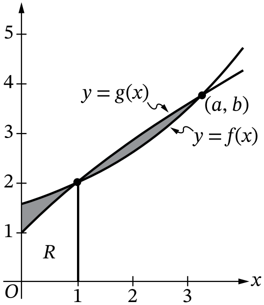
The function f is defined by $f(x) = 1.43^x + 0.57$, and the function g is defined by $g(x) = \dfrac{14x+12}{x+12}$. The graphs of f and g intersect at the points $(1, 2)$ and (a, b), as shown in Figure 1.

[Figure 1: A graph with x-axis from 0 to 3+ and y-axis from 0 to 5+. The curve $y = g(x)$ and the curve $y = f(x)$ both start near $(0, 1)$–$(0, 1.5)$, rise, and intersect at the labeled point $(1, 2)$ and again at the labeled point (a, b) (near $x = 3$, $y \approx 3.8$), with f above g just after the second intersection. The region R is shaded/bounded between the curve $y = g(x)$, the x-axis, the y-axis, and the vertical line $x = 1$ (region labeled R near the origin below g). The shaded region between the two curves from $x = 0$ to $x = a$ is shown shaded gray between $(1, 2)$ and (a, b).]

Question 2

For $x \geq 0$, the equation $y = g(x)$ can be rewritten as $x = h(y) = \dfrac{12y - 12}{14 - y}$, where h is a function of y. The graph of $x = h(y)$ and the horizontal line $y = 3.5$ are shown in Figure 2.
[Figure 2: A graph with x-axis from 0 to 3+ and y-axis from 0 to 5+. The curve $x = h(y)$ starts near $(0, 1)$ and increases, crossing the horizontal line $y = 3.5$ near $x = 3$. The region T is the region bounded by the curve $x = h(y)$, the y-axis, and the line $y = 3.5$, labeled T to the left of the curve.]
END OF PART A
(a) Let R be the region bounded by the graph of g, the x-axis, the y-axis, and the vertical line $x = 1$, as shown in Figure 1. Find the area of region R. Show the setup for your calculations.

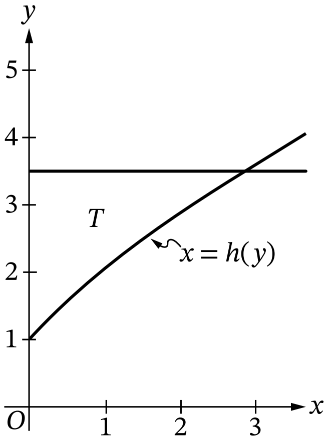
Question 2



For $x \geq 0$, the equation $y = g(x)$ can be rewritten as $x = h(y) = \dfrac{12y - 12}{14 - y}$, where h is a function of y. The graph of $x = h(y)$ and the horizontal line $y = 3.5$ are shown in Figure 2.

Figure 2

Question 2



Question 2

2026 ▪ Q2

The function f is defined by $f(x) = 1.43^x + 0.57$, and the function g is defined by $g(x) = \dfrac{14x + 12}{x + 12}$. The graphs of f and g intersect at the points $(1, 2)$ and (a, b), as shown in Figure 1.

[Figure 1: A graph with x-axis from 0 to 3+ and y-axis from 0 to 5+. The curve $y = g(x)$ and the curve $y = f(x)$ both start near $(0, 1)$–$(0, 1.5)$, rise, and intersect at the labeled point $(1, 2)$ and again at the labeled point (a, b) (near $x = 3$, $y \approx 3.8$), with f above g just after the second intersection. The region R is shaded/bounded between the curve $y = g(x)$, the x-axis, the y-axis, and the vertical line $x = 1$ (region labeled R near the origin below g). The shaded region between the two curves from $x = 0$ to $x = a$ is shown shaded gray between $(1, 2)$ and (a, b).]

For $x \geq 0$, the equation $y = g(x)$ can be rewritten as $x = h(y) = \dfrac{12y - 12}{14 - y}$, where h is a function of y. The graph of $x = h(y)$ and the horizontal line $y = 3.5$ are shown in Figure 2.

Question 2

[Figure 2: A graph with x-axis from 0 to 3+ and y-axis from 0 to 5+. The curve $x = h(y)$ starts near $(0, 1)$ and increases, crossing the horizontal line $y = 3.5$ near $x = 3$. The region T is the region bounded by the curve $x = h(y)$, the y-axis, and the line $y = 3.5$, labeled T to the left of the curve.]

END OF PART A

(b) Region R, described in part A, is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is $\frac{1}{3}$ times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

Question 2

2026 ▪ Q2

The function f is defined by $f(x) = 1.43^x + 0.57$, and the function g is defined by $g(x) = \dfrac{14x + 12}{x + 12}$. The graphs of f and g intersect at the points $(1, 2)$ and (a, b), as shown in Figure 1.

[Figure 1: A graph with x-axis from 0 to 3+ and y-axis from 0 to 5+. The curve $y = g(x)$ and the curve $y = f(x)$ both start near $(0, 1)$–$(0, 1.5)$, rise, and intersect at the labeled point $(1, 2)$ and again at the labeled point (a, b) (near $x = 3$, $y \approx 3.8$), with f above g just after the second intersection. The region R is shaded/bounded between the curve $y = g(x)$, the x-axis, the y-axis, and the vertical line $x = 1$ (region labeled R near the origin below g). The shaded region between the two curves from $x = 0$ to $x = a$ is shown shaded gray between $(1, 2)$ and (a, b).]

For $x \geq 0$, the equation $y = g(x)$ can be rewritten as $x = h(y) = \dfrac{12y - 12}{14 - y}$, where h is a function of y. The graph of $x = h(y)$ and the horizontal line $y = 3.5$ are shown in Figure 2.

Question 2

[Figure 2: A graph with x-axis from 0 to 3+ and y-axis from 0 to 5+. The curve $x = h(y)$ starts near $(0, 1)$ and increases, crossing the horizontal line $y = 3.5$ near $x = 3$. The region T is the region bounded by the curve $x = h(y)$, the y-axis, and the line $y = 3.5$, labeled T to the left of the curve.]

END OF PART A

(c) The shaded region in Figure 1 is bounded by the graphs of f and g on the interval from $x = 0$ to $x = a$. Find the area of the shaded region. Show the setup for your calculations.

Question 2

2026 ▪ Q2

The function f is defined by $f(x) = 1.43^x + 0.57$, and the function g is defined by $g(x) = \dfrac{14x + 12}{x + 12}$. The graphs of f and g intersect at the points $(1, 2)$ and (a, b), as shown in Figure 1.

[Figure 1: A graph with x-axis from 0 to 3+ and y-axis from 0 to 5+. The curve $y = g(x)$ and the curve $y = f(x)$ both start near $(0, 1)$–$(0, 1.5)$, rise, and intersect at the labeled point $(1, 2)$ and again at the labeled point (a, b) (near $x = 3$, $y \approx 3.8$), with f above g just after the second intersection. The region R is shaded/bounded between the curve $y = g(x)$, the x-axis, the y-axis, and the vertical line $x = 1$ (region labeled R near the origin below g). The shaded region between the two curves from $x = 0$ to $x = a$ is shown shaded gray between $(1, 2)$ and (a, b).]

For $x \geq 0$, the equation $y = g(x)$ can be rewritten as $x = h(y) = \dfrac{12y - 12}{14 - y}$, where h is a function of y. The graph of $x = h(y)$ and the horizontal line $y = 3.5$ are shown in Figure 2.

Question 2

[Figure 2: A graph with x-axis from 0 to 3+ and y-axis from 0 to 5+. The curve $x = h(y)$ starts near $(0, 1)$ and increases, crossing the horizontal line $y = 3.5$ near $x = 3$. The region T is the region bounded by the curve $x = h(y)$, the y-axis, and the line $y = 3.5$, labeled T to the left of the curve.]

END OF PART A

(d) Let T be the region bounded by the graph of $x = h(y)$, the y-axis, and the horizontal line $y = 3.5$, as shown in Figure 2. Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region T is revolved about the y-axis.

Question 2

2025 ▪ Q2

The shaded region R is bounded by the graphs of the functions f and g, where $f(x) = x^2 - 2x$ and $g(x) = x + \sin(\pi x)$, as shown in the figure.
[Figure: In the xy-plane, the graph of $y = g(x)$ and the graph of $y = f(x)$ are shown. The shaded region R, labeled R, lies between the two curves. The point $(3, 3)$ is marked where the curves meet. The y-axis is labeled with values $-2, -1, 1, 2, 3, 4$ and the x-axis is labeled with values $1, 2, 3$, with origin O.]
(Note: Your calculator should be in radian mode.)

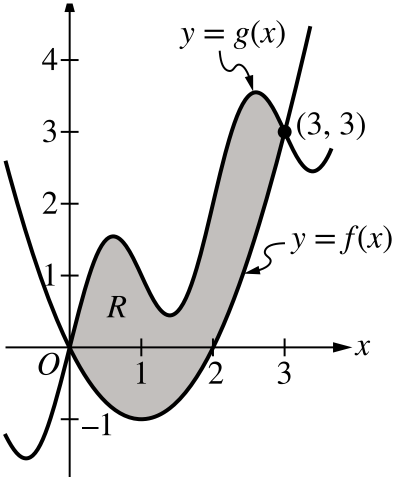
(a) Find the area of R. Show the setup for your calculations.

(b) Region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height x and base in region R. Find the volume of the solid. Show the setup for your calculations.

Question 2

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when the region R is rotated about the horizontal line $y = -2$.
(d) It can be shown that $g'(x) = 1 + \pi \cos(\pi x)$. Find the value of x, for $0 < x < 1$, at which the line tangent to the graph of f is parallel to the line tangent to the graph of g.

Question 2



Solution 2

(a)

Mark scheme

- Area $= \int_0^3 (g(x) - f(x))\, dx = 5.136620 \approx 5.137$ (exact value $\dfrac{4+9\pi}{2\pi}$). Point 1: presents an integral of $g(x) - f(x)$, $|g(x) - f(x)|$, $f(x) - g(x)$, or $|f(x) - g(x)|$ over $[0, 3]$ (or an equivalent difference of definite integrals of g and f). Point 2: correct answer 5.137 (or 5.136), with or without supporting work.

Solution 2

(b)

Mark scheme

- Volume $= \int_0^3 x(g(x) - f(x))\,dx = 7.704930 \approx 7.705$ (exact value $\dfrac{12 + 27\pi}{4\pi}$).
  Point 3: presents a definite integral with integrand a product of two nonconstant factors, one factor being x, $g(x) - f(x)$, or $f(x) - g(x)$. Point 4: correct answer 7.705 (or 7.704), with or without supporting work.

Solution 2

(c)

Mark scheme

- Volume $= \pi \int_0^3 \left[(g(x) - (-2))^2 - (f(x) - (-2))^2\right] dx = \pi \int_0^3 \left[(g(x) + 2)^2 - (f(x) + 2)^2\right] dx$ (expression only, not evaluated). Point 5: integrand of the form $R^2 - r^2$ (or $|R^2 - r^2|$) where one of R, r is g or f (or a difference involving a nonzero constant) and the other is correct or a difference between f/g and a nonzero constant. Point 6: correct integrand $(g(x) + 2)^2 - (f(x) + 2)^2$ (or equivalent, e.g. reversed with a resolved sign). Point 7: correct limits of integration (0 to 3), the constant π, and the differential dx all present and correctly combined; requires Point 5.

Solution 2

(d)

Mark scheme

- Set $f'(x) = g'(x)$: $2x - 2 = 1 + \pi\cos(\pi x) \Rightarrow x = 0.675819 \approx 0.676$ (or 0.675). Point 8: general statement $f'(x) = g'(x)$, or any correct equation substituting $2x - 2$ for $f'(x)$ and/or $1 + \pi\cos(\pi x)$ for $g'(x)$. Point 9: correct answer $x \approx 0.676$, with or without supporting work.

Question 6

2024 ▪ Q6

[Graph of f and g; x-axis labeled 0 to 5, y-axis labeled 5 to 25; curve $y = f(x)$ is a upward-opening parabola-like curve passing through about $(0, 2)$ and rising steeply to about $(5, 27)$; curve $y = g(x)$ passes through the origin area, dips slightly below the x-axis between $x = 0$ and $x = 2$, then rises through about $(5, 15)$; the shaded region R lies between the two curves from $x = 0$ to $x = 2$; the shaded region S lies between the graph of g and the x-axis from $x = 2$ to $x = 5$.]

The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

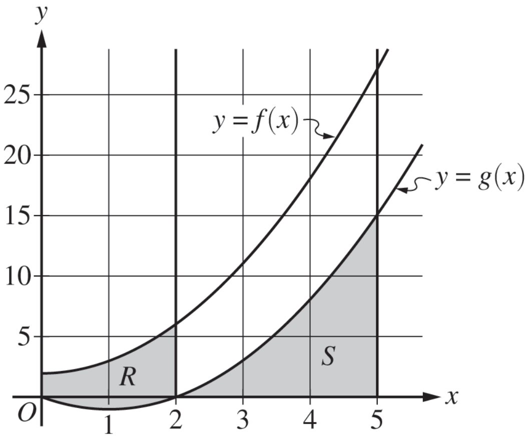
(a) Let R be the region bounded by the graphs of f and g, from $x = 0$ to $x = 2$, as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region R.

Question 6

(b) Let S be the region bounded by the graph of g and the x-axis, from $x = 2$ to $x = 5$, as shown in the graph. Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height equal to half its base in region S. Find the volume of the solid. Show the work that leads to your answer.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region S, as described in part (b), is rotated about the horizontal line $y = 20$.

Question 6



Solution 6

(a)

Mark scheme

- With $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, the area of region R (bounded by f, g, from $x = 0$ to $x = 2$, where $f \geq g$) is given by the (unevaluated) integral Area $= \int_0^2 (f(x) - g(x))\,dx$. 1 point for an integrand equivalent to $f(x) - g(x)$ (or $|f(x) - g(x)|$, or a difference of two separate integrals with integrands f and g), 1 point for the fully correct expression $\int_0^2 (f(x) - g(x))\,dx$ (equivalent forms accepted, e.g. $\int_0^2 f(x)\,dx - \int_0^2 g(x)\,dx$). Do not evaluate.

Solution 6

(b) Mark scheme

- Cross sections perpendicular to the x-axis over region S (g and the x-axis, $x = 2$ to $x = 5$) are rectangles with height $= \frac{1}{2}\cdot$base, so area $= \frac{1}{2}(g(x))^2$: $V = \int_2^5 \frac{1}{2}(g(x))^2\,dx = \int_2^5 \frac{1}{2}(x^2 - 2x)^2\,dx = \frac{1}{2}\int_2^5 (x^4 - 4x^3 + 4x^2)\,dx = \frac{1}{2}\left[\frac{x^5}{5} - x^4 + \frac{4x^3}{3}\right]_2^5 = \frac{1}{2}\left(\frac{500}{3} - \frac{16}{15}\right) = \frac{414}{5} = 82.8$. 1 point for the integrand of the form $k(g(x))^2$, 1 point for the correct limits $x = 2$ to $x = 5$ with an integrand of the form $a(x) \cdot g(x)$, 1 point for a correct antiderivative, 1 point for the final numeric answer $414/5$ (or 82.8).

Solution 6

(c)

Mark scheme

- Rotating region S about $y = 20$ gives washers with outer radius $20 - 0 = 20$ (from $y = 20$ down to the x-axis, $y = 0$) and inner radius $20 - g(x)$: $V = \pi \int_2^5 \left[20^2 - (20 - g(x))^2\right] dx = \pi \int_2^5 \left[400 - (20 - g(x))^2\right] dx = \pi \int_2^5 \left[400 - \left(20 - (x^2 - 2x)\right)^2\right] dx$ (do not evaluate). 1 point for the correct form of the integrand $20^2 - (20 - g(x))^2$ (or an equivalent difference-of-squares form), 1 point for substituting $g(x) = x^2 - 2x$ correctly into that integrand, 1 point for the correct limits $x = 2$ to $x = 5$ together with the constant π.

Question 2

2022 ▪ Q2

[Graph: a coordinate plane with x-axis from -2 to 1 and y-axis from -1 to 1 (tick marks labeled -1 on y-axis, 1 on y-axis; -2, -1, O, 1 on x-axis). A curve labeled implicitly by the two functions below starts at $(-2, 0)$, dips down to a minimum below the x-axis between $x = -1$ and $x = 0$, rises back up crossing near the origin, and continues upward, ending with a marked point just above and to the right of $(1, 1)$.]

Let f and g be the functions defined by $f(x) = \ln(x+3)$ and $g(x) = x^4 + 2x^3$. The graphs of f and g, shown in the figure above, intersect at $x = -2$ and $x = B$, where $B > 0$.

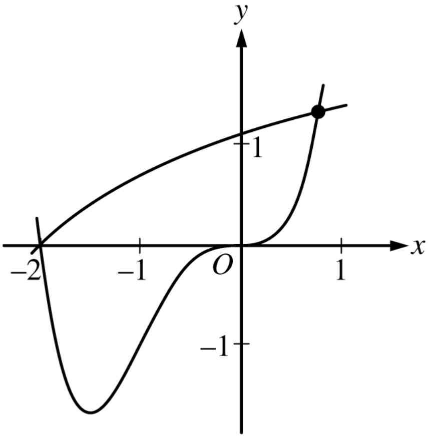
(a) Find the area of the region enclosed by the graphs of f and g.

(b) For $-2 \leq x \leq B$, let $h(x)$ be the vertical distance between the graphs of f and g. Is h increasing or decreasing at $x = -0.5$? Give a reason for your answer.

Question 2

(c) The region enclosed by the graphs of f and g is the base of a solid. Cross sections of the solid taken perpendicular to the x-axis are squares. Find the volume of the solid.
(d) A vertical line in the xy-plane travels from left to right along the base of the solid described in part (c). The vertical line is moving at a constant rate of 7 units per second. Find the rate of change of the area of the cross section above the vertical line with respect to time when the vertical line is at position $x = -0.5$.

Question 2



Solution 2

(a)

Mark scheme

- $\ln(x+3) = x^4 + 2x^3 \Rightarrow x = -2,\ x = B = 0.781975$. Area $= \int_{-2}^{B} (f(x) - g(x))\, dx = \int_{-2}^{B} \left(\ln(x+3) - (x^4 + 2x^3)\right) dx = 3.603548$. The area of the region is 3.604 (or 3.603). Points: (1) correct integrand $f(x) - g(x)$ (or an equivalent form such as $|f(x) - g(x)|$ or $g(x) - f(x)$); (2) correct limits of integration, -2 to B; (3) correct answer.

Solution 2

(b)

Mark scheme

- $h(x) = f(x) - g(x)$, so $h'(x) = f'(x) - g'(x)$.
  $h'(-0.5) = f'(-0.5) - g'(-0.5) = -0.6$ (or -0.599). Since $h'(-0.5) < 0$, h is decreasing at $x = -0.5$. Points: (1) considers $h'(-0.5)$ (or equivalently $f'(-0.5) - g'(-0.5)$); (2) correct answer (decreasing) with reason.

Solution 2

(c)

Mark scheme

- Volume $= \int_{-2}^{B} (f(x) - g(x))^2 \, dx = 5.340102$. The volume of the solid is 5.340. Points: (1) correct integrand $k(f(x) - g(x))^2$ with $k = 1$ in a definite integral; (2) correct answer.

Solution 2

(d) Mark scheme

- The cross section has area $A(x) = (f(x) - g(x))^2$. By the chain rule, $\dfrac{d}{dt}[A(x)] = \dfrac{dA}{dx} \cdot \dfrac{dx}{dt}$. At $x = -0.5$: $\dfrac{d}{dt}[A(x)]\Big|_{x=-0.5} = A'(-0.5) \cdot 7 = -9.271842$. At $x = -0.5$, the area of the cross section is changing at a rate of about -9.272 (or -9.271) square units per second. Points: (1) correct chain-rule expression $\frac{dA}{dx} \cdot \frac{dx}{dt}$ (equivalently $A'(x) \cdot dx/dt$); (2) correct answer.

Question 5

2019 ▪ Q5

Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the y-axis, and the vertical line $x = 2$, as shown in the figure below.

[Graph showing region R shaded gray; x-axis from O to past 2 (gridlines at 1 and 2), y-axis with gridline at 1. The upper boundary curve $h(x)$ starts above the y-axis, rises to a peak between $x = 0$ and $x = 1$, then descends, meeting the vertical line $x = 2$ near $y \approx 1.5$-2. The lower boundary curve $g(x)$ starts near $(0, 1)$, dips down crossing the x-axis around $x = 1$, continues down to a minimum, then rises back up to meet $h(x)$ at the line $x = 2$. Region R is the shaded area between the two curves from $x = 0$ to $x = 2$, bounded on the left by the y-axis and on the right by the line $x = 2$.]

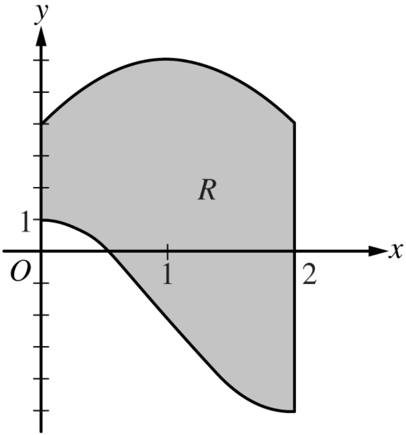
(a) Find the area of R.

Question 5

(b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \dfrac{1}{x+3}$. Find the volume of the solid.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

Question 5



Solution 5

(a) Mark scheme

- Area of $R = \int_0^2 (h(x) - g(x))\,dx = \int_0^2 \left[\left(6 - 2(x-1)^2\right) - \left(-2 + 3\cos\left(\frac{\pi}{2}x\right)\right)\right] dx = \left[\left(6x - \frac{2}{3}(x-1)^3\right) - \left(-2x + \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right)\right)\right]_{x=0}^{x=2} = \left(\left(12 - \frac{2}{3}\right) - (-4+0)\right) - \left(\left(0 + \frac{2}{3}\right) - (0+0)\right) = 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}$. The area of R is $\frac{44}{3}$. Points: 1 for the integrand (correctly set up as $h(x) - g(x)$), 1 for the antiderivative of $3\cos\left(\frac{\pi}{2}x\right)$, 1 for the antiderivative of the remaining terms, 1 for the final answer $\frac{44}{3}$.

Solution 5

(b)

Mark scheme

- Volume $= \int_0^2 A(x)\,dx = \int_0^2 \frac{1}{x+3}\,dx = \big[\ln(x+3)\big]_{x=0}^{x=2} = \ln 5 - \ln 3$. The volume of the solid is $\ln 5 - \ln 3$. Points: 1 for the correct integral, 1 for the final answer $\ln 5 - \ln 3$.

Solution 5

(c)

Mark scheme

- Volume when R is rotated about the line $y = 6$ (washer method), written but NOT evaluated: $\pi \int_0^2 \left[(6 - g(x))^2 - (6 - h(x))^2\right] dx$. Points: 1 for correct limits and constant (π, from 0 to 2), 1 for the correct form of the integrand (outer-radius-squared minus inner-radius-squared, using $6 - g(x)$ and $6 - h(x)$), 1 for the fully correct integrand overall.

Question 2

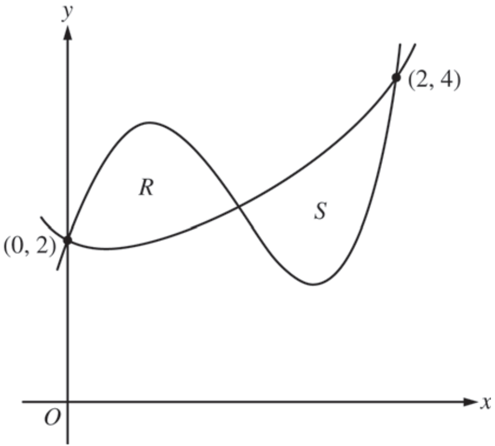
2015 ▪ Q2

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2 - 2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.
[Graph showing the curves f and g on axes labeled x and y. Both curves pass through the point $(0, 2)$ on the y-axis and meet again at the point $(2, 4)$. Between these intersection points, one curve rises to a local peak and the other dips to a local trough, crossing each other partway through; the region enclosed on the left (near the peak) is labeled R, and the region enclosed on the right (near the trough) is labeled S. The labeled points are $(0, 2)$ and $(2, 4)$.]
(a) Find the sum of the areas of regions R and S.
(b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

Question 2

(c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

Question 2



Solution 2

(a)

Mark scheme

- The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at (0, 2), (2, 4), and $(A, B) = (1.032832, 2.401108)$. Sum of areas $= \int$ ^A [g(x) − f(x)] dx + $\int$_A^2 [f(x) − g(x)] dx $= 0.997427 + 1.006919 = 2.004$. Points: 1 for correct limits (0, A, 2, using the intersection point $A \approx 1.033$), 2 for correct integrands ([g − f] on [0, A] and [f − g] on [A, 2]), 1 for the correct final numeric answer 2.004.

Solution 2

(b)

Mark scheme

- Region S is the base of a solid with square cross sections perpendicular to the x-axis, side length f(x) − g(x). Volume = ∫_A^2 $[f(x) - g(x)]^2$ dx = 1.283, using A ≈ 1.032832 as the lower limit (region S spans x = A to x = 2). Points: 2 for the correct integrand $[f(x) - g(x)]^2$ with correct limits of integration, 1 for the correct numeric answer 1.283.

Solution 2

(c)

Mark scheme

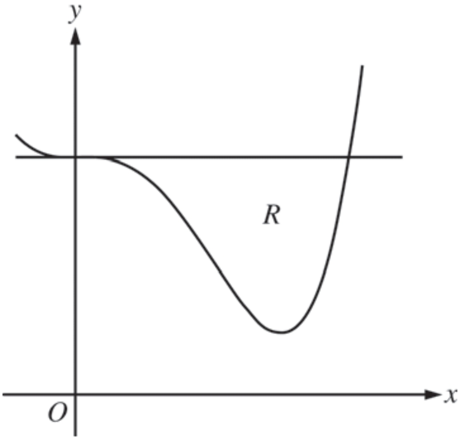
- $h(x) = f(x) - g(x)$, so $h'(x) = f'(x) - g'(x)$. $h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811). Points: 1 for considering $h'(x) = f'(x) - g'(x)$, 1 for the correct numeric answer ≈ -3.812.

Question 2

2014 ▪ Q2

[Graph of $y = f(x)$ for $x \geq 0$: a curve starting near the origin, dipping down below the x-axis to a minimum, then rising steeply back up and crossing above $y = 0$; the region R is labelled in the dip between the curve and the x-axis/line $y = 4$, roughly between the two crossing points of $f(x) = 4$.]
Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.
(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.
(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

Question 2



Solution 2

(a)

Mark scheme

- f(x) = 4 at x = 0 and x = 2.3 (the intersection points bounding R). Volume = $\pi \int_0^{2.3} \left[(4+2)^2 - (f(x)+2)^2\right] dx = 98.868(or 98.867), usingwasherswithouterradius 6(from y = -2 to y = 4) and inner radius f(x) + 2(from y = -2 to y = f(x)). Points : 2 for the correct integrand, 1 for correct limits of integration (0 to 2.3), 1 for the correct numerical answer$

Solution 2

(b)

Mark scheme

- Volume $= \int_0^{2.3} \frac{1}{2}(4-f(x))^2\,dx = 3.574(or 3.573), usingthatanisoscelesrighttrianglewithlegs = 4 - f(x)(theverticaldistanceinR)hasarea(1/2)s^2. Points : 2forthecorrectintegrand, 1forthecorrectnumericanswer.$

Solution 2

(c)

Mark scheme

- Set the area of the left piece equal to the area of the right piece: $\int_0^k (4 - f(x))\,dx = \int_k^{2.3} (4 - f(x))\,dx.(Equationonly - -donotsolvefork.)Points : 1foracorrectexpressionfortheareaofoneregion, 1foracorrectequationsettingthetworegions' areas$