Past-paper walk-through

Connecting Position, Velocity, and Acceleration of Functions Using Integrals ▪ 8.2

eXercise

Questions in this set

- 2026 Q5
- 2025 Q5
- 2024 Q2
- 2023 Q2
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- 2016 Q2
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Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 5

2026 ▪ Q5

A remote-controlled toy car moves back and forth along a straight path so that its velocity at time t is given by the function v, where $v(t)$ is measured in feet per second and t is measured in seconds.

$$v(t) = \begin{cases} t^4 - 8t^3 + 16t^2 & \text{for } 0 \le t \le 4 \\ 0 & \text{for } 4 < t < 6 \\ 10\cos\left(\frac{\pi}{3}t\right) - 10 & \text{for } 6 \le t \le 12 \end{cases}$$

The graph of $v(t)$ is shown.
[Graph of $v(t)$: t-axis from 0 to 12 in increments of 2 (labels at 2,4,6,8,10,12); v-axis unlabeled (vertical). The curve starts at the origin $(0,0)$, rises to a local maximum

Question 5

somewhere between $t = 2$ and $t = 4$, returns to and touches the t-axis at $t = 4$, stays at $v = 0$ (flat on the t-axis) from $t = 4$ to $t = 6$, then dips below the axis into a trough with minimum between $t = 8$ and $t = 10$, and rises back up to touch the t-axis at $t = 12$.]

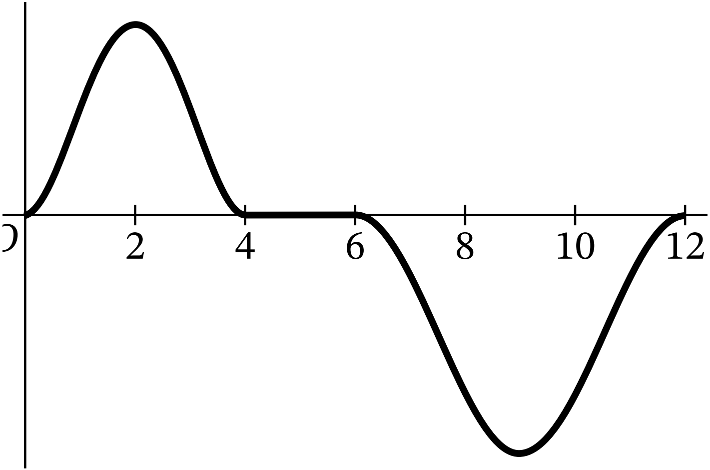
(a) Find the acceleration of the car at time $t = 1$ second. Show the work that leads to your answer.

(b) Is the car speeding up or slowing down at time $t = 1$ second? Give a reason for your answer.

(c) Find the distance, in feet, that the car traveled over the time interval $0 \leq t \leq 4$ seconds. Show the work that leads to your answer.

(d) Find the average velocity of the car over the time interval $6 \leq t \leq 12$ seconds. Show the work that leads to your answer.

Question 5



Question 5

2025 ▪ Q5

Two particles, H and J, are moving along the x-axis. For $0 \leq t \leq 5$, the position of particle H at time t is given by $x_H(t) = e^{t^2-4t}$ and the velocity of particle J at time t is given by $v_J(t) = 2t(t^2-1)^3$.

(a) Find the velocity of particle H at time $t = 1$. Show the work that leads to your answer.

(b) During what open intervals of time t, for $0 < t < 5$, are particles H and J moving in opposite directions? Give a reason for your answer.

(c) It can be shown that $v_J'(2) > 0$. Is the speed of particle J increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

(d) Particle J is at position $x = 7$ at time $t = 0$. Find the position of particle J at time $t = 2$. Show the work that leads to your answer.

Solution 5

(a)

Mark scheme

- $x_H'(t) = v_H(t) = (2t-4)e^{t^2-4t}$. At $t = 1$: $x_H'(1) = (2-4)e^{1-4} = -2e^{-3}$. Point 1: considers/presents $x_H'(t)$ (or $v_H(t)$, $x'(t)$, $x'(1)$, $(2t-4)e^{t^2-4t}$, or the unsimplified value at $t = 1$, e.g. $(2 \cdot 1 - 4)e^{1^2-4\cdot 1}$). Point 2: correct answer $-2e^{-3}$ (an unsupported answer of $-2e^{-3}$ alone earns this point but not Point 1).

Solution 5

(b) Mark scheme

- $x_H'(t) = (2t-4)e^{t^2-4t} = 0 \Rightarrow t = 2$ (only critical point on $0 < t < 5$); $x_H'(t) < 0$ for $0 < t < 2$ and $x_H'(t) > 0$ for $2 < t < 5$, so H moves left then right. $v_J(t) = 2t(t^2-1)^3 = 0$ for $0 < t < 5 \Rightarrow t = 1$; $v_J(t) < 0$ for $0 < t < 1$ and $v_J(t) > 0$ for $1 < t < 5$, so J moves left then right. H and J move in opposite directions exactly where their directions differ: for $1 < t < 2$ (H moving left, J moving right). Point 3: considers the sign of $x_H'(t)$ or $v_J(t)$ (sets it/the expression equal to 0, or correctly identifies its critical time $t = 2$ for H or $t = 1$ for J with no other values in $0 < t < 5$, or identifies the interval $1 < t < 2$). Point 4: correct direction-of-motion analysis on $0 < t < 5$ for one particle (H or J). Point 5:

Solution 5

correct answer $1 < t < 2$ with reasoning; requires correct analysis for BOTH particles on $0 < t < 5$.

Solution 5

(c)

Mark scheme

- $v_J(2) = 2(2)((2)^2 - 1)^3 = 108 > 0$, and $v'_J(2) > 0$ is given. Because $v_J(2)$ and $v'_J(2)$ have the same sign, the speed of particle J is increasing at $t = 2$. Point 6: correct answer (increasing) with correct reasoning that $v_J(2)$ and $v'_J(2)$ have the same sign (an explicit numeric evaluation of $v_J(2)$ is not required, but if given must be correct: $v_J(2) = 108$; the sign analysis for $v_J(t) > 0$ on $1 < t < 5$ from part B may be reused without restating $v_J(2) > 0$).

Solution 5

(d)

Mark scheme

- $x_J(2) = x_J(0) + \int_0^2 v_J(t)\,dt = 7 + \int_0^2 2t(t^2-1)^3\,dt = 7 + \left[\frac{1}{4}(t^2-1)^4\right]_0^2 = 7 + \frac{1}{4}\left((3)^4 - (-1)^4\right) = 7 + \frac{1}{4}(80) = 7 + 20 = 27$. Point 7: presents an integral (indefinite or definite) with integrand $v_J(t)$ or $2t(t^2-1)^3$ (with or without the differential dt). Point 8: correct antiderivative of the form $k(t^2-1)^4$ with $k = \frac{1}{4}$ for $k > 0$ (a different nonzero k does not earn Point 9). Point 9: correct answer $x_J(2) = 27$; requires Point 8.

Question 2

2024 ▪ Q2

A particle moves along the x-axis so that its velocity at time $t \geq 0$ is given by $v(t) = \ln\left(t^2 - 4t + 5\right) - 0.2t$.

(a) There is one time, $t = t_R$, in the interval $0 < t < 2$ when the particle is at rest (not moving). Find t_R. For $0 < t < t_R$, is the particle moving to the right or to the left? Give a reason for your answer.

(b) Find the acceleration of the particle at time $t = 1.5$. Show the setup for your calculations. Is the speed of the particle increasing or decreasing at time $t = 1.5$? Explain your reasoning.

(c) The position of the particle at time t is $x(t)$, and its position at time $t = 1$ is $x(1) = -3$. Find the position of the particle at time $t = 4$. Show the setup for your calculations.

Question 2

(d) Find the total distance traveled by the particle over the interval $1 \leq t \leq 4$. Show the setup for your calculations.

Solution 2

(a)

Mark scheme

- Solve $v(t) = \ln(t^2 - 4t + 5) - 0.2t = 0$ on $0 < t < 2$: $t_R = 1.425610 \approx 1.426$ (or 1.425). For $0 < t < t_R$, $v(t) > 0$, so the particle is moving to the right. 1 point for the correct value of t_R (particle at rest requires $v(t) = 0$), 1 point for the direction (right) with a reason based on the sign of $v(t)$ on that interval.

Solution 2

(b)

Mark scheme

- $a(1.5) = v'(1.5) = -1$ (or -0.999), using $a = v'$. $v(1.5) = -0.076856 < 0$; because $a(1.5)$ and $v(1.5)$ have the same sign, the speed of the particle is increasing at $t = 1.5$. 1 point for declaring the acceleration value via $v'(1.5)$, 1 point for the correct increasing/decreasing conclusion with reasoning consistent with the signs of $v(1.5)$ and $a(1.5)$.

Solution 2

(c)

Mark scheme

- $x(4) = x(1) + \int_1^4 v(t)\,dt = -3 + 0.197117 = -2.802883 \approx -2.803$ (or -2.802). 1 point for the definite integral of $v(t)$ with correct limits 1 to 4, 1 point for adding the initial condition $x(1) = -3$, 1 point for the final numeric answer with supporting work shown (an answer alone with no work earns no points).

Solution 2

(d)

Mark scheme

- Total distance $= \int_1^4 |v(t)|\, dt = 0.9581 \approx 0.958$ (0.958, 0.959, or 0.96 all accepted due to calculator variation). 1 point for setting up the integral of $|v(t)|$ (equivalently, splitting into signed pieces at the sign changes of v near $t \approx 1.425$ and $t \approx 2.883$), 1 point for the correct numeric answer.

Question 2

2023 ▪ Q2

Stephen swims back and forth along a straight path in a 50-meter-long pool for 90 seconds. Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t}\sin\left(\frac{\pi}{56}t\right)$, where t is measured in seconds and $v(t)$ is measured in meters per second.

(a) Find all times t in the interval $0 < t < 90$ at which Stephen changes direction. Give a reason for your answer.

(b) Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.

(c) Find the distance between Stephen's position at time $t = 20$ seconds and his position at time $t = 80$ seconds. Show the setup for your calculations.

Question 2

(d) Find the total distance Stephen swims over the time interval $0 \leq t \leq 90$ seconds. Show the setup for your calculations.

Solution 2

(a)

Mark scheme

- For $0 < t < 90$, $v(t) = 0 \Rightarrow t = 56$ (1 pt: considers the sign of v(t)). Stephen changes direction when his velocity changes sign; this occurs at t=56 seconds (1 pt: answer with reason — must include t=56, unsupported t=56 alone earns nothing).

Solution 2

(b)

Mark scheme

- $v'(60) = a(60) \approx -0.0360162 \approx -0.036$ (1 pt: a(60) with setup, e.g. explicitly showing v'(60)=a(60)); the units of acceleration are meters per second per second (1 pt: units). Since $v(60) \approx -0.1595124 < 0$, Stephen is speeding up at t=60 seconds because his velocity and acceleration are both negative at that time (1 pt: speeding-up conclusion with reason — the presented sign of a(60) must be negative and consistent with a negative v(60)).

Solution 2

(c)

Mark scheme

- Distance $= \displaystyle\int_{20}^{80} v(t)\,dt$ (1 pt: integral setup). Evaluating, $\int_{20}^{80} v(t)\,dt = 23.383997 \approx 23.384$ (or 23.383) meters (1 pt: answer).

Solution 2

(d)

Mark scheme

- Total distance $= \displaystyle\int_0^{90} |v(t)|\,dt$ (1 pt: integral setup, or equivalently $\int_0^{56} v(t)\,dt - \int_{56}^{90} v(t)\,dt$). Evaluating, $\int_0^{90} |v(t)|\,dt = 62.164216 \approx 62.164$ meters (1 pt: answer).

Question 3

2022 ▪ Q3

[Graph titled "Graph of f'": a coordinate plane with x-axis from 0 to 7 (integer gridlines at $1, 2, 3, 4, 5, 6, 7$) and y-axis from -2 to 2 (gridlines at $-2, -1, 1, 2$). The graph consists of a semicircle below the x-axis from $x = 0$ to $x = 4$, dipping down to a minimum of about -2 near $x = 2$, returning to 0 at $x = 4$. From $x = 4$ to $x = 6$ a line segment rises from $(4, 0)$ to the marked point $(6, 2)$. From $x = 6$ to $x = 7$ a line segment falls from $(6, 2)$ to the marked point $(7, 1)$.]

Let f be a differentiable function with $f(4) = 3$. On the interval $0 \leq x \leq 7$, the graph of f', the derivative of f, consists of a semicircle and two line segments, as shown in the figure above.

(a) Find $f(0)$ and $f(5)$.

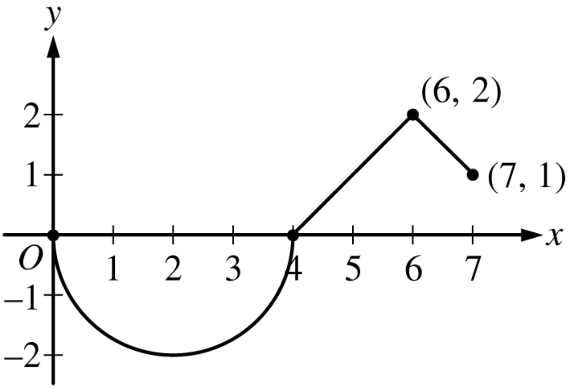
(b) Find the x-coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

Question 3

(c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \leq x \leq 7$? Show the analysis that leads to your answer.

(d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \leq x \leq 7$. Justify your answer.

Question 3



Graph of f'

Solution 3

(a)

Mark scheme

- $f(0) = f(4) + \int_4^0 f'(x)\,dx = 3 - \int_0^4 f'(x)\,dx = 3 + 2\pi$ (the semicircle over $[0,4]$ has radius 2 and lies below the axis, so $\int_0^4 f'(x)\,dx = -2\pi$). $f(5) = f(4) + \int_4^5 f'(x)\,dx = 3 + \frac{1}{2} = \frac{7}{2}$. Points: (1) area of either region ($\int_0^4 f'$ or $\int_4^5 f'$); (2) correct $f(0) = 3 + 2\pi$; (3) correct $f(5) = 7/2$.

Solution 3

(b)

Mark scheme

- The graph of f has points of inflection at $x = 2$ and $x = 6$, because f' changes from decreasing to increasing at $x = 2$ (the bottom of the semicircle) and from increasing to decreasing at $x = 6$ (the corner/peak of the graph of f' at $(6, 2)$). Points: (1) states both $x = 2$ and $x = 6$; (2) correct justification tied to the graph of f' (e.g., discussing where the slope/behavior of f' changes, or citing $x = 2, 6$ as locations of local extrema on the graph of f').

Solution 3

(c)

Mark scheme

- $g(x) = f(x) - x \Rightarrow g'(x) = f'(x) - 1$. Since $g'(x) \leq 0 \iff f'(x) \leq 1$, and from the graph $f'(x) \leq 1$ for $0 \leq x \leq 5$, the graph of g is decreasing on the interval $0 \leq x \leq 5$ (because $g'(x) \leq 0$ there). Points: (1) correct $g'(x) = f'(x) - 1$ (or equivalent); (2) correct interval $[0, 5]$ with reason.

Solution 3

(d) Mark scheme

- g is continuous, $g'(x) < 0$ for $0 < x < 5$, and $g'(x) > 0$ for $5 < x < 7$, so the absolute minimum occurs at $x = 5$: $g(5) = f(5) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$. The absolute minimum value of g on $[0, 7]$ is $-\frac{3}{2}$. (Alternative: a Candidates Test comparing $g(0) = 3 + 2\pi$, $g(5) = -3/2$, $g(7) = -1/2$ gives the same result.) Points: (1) considers $g'(x) = 0$, isolating $x = 5$ as the only critical number on $(0, 7)$; (2) correct answer $-3/2$ with justification.

Question 6

2022 ▪ Q6

Particle P moves along the x-axis such that, for time $t > 0$, its position is given by $x_P(t) = 6 - 4e^{-t}$.

Particle Q moves along the y-axis such that, for time $t > 0$, its velocity is given by $v_Q(t) = \frac{1}{t^2}$. At time $t = 1$, the position of particle Q is $y_Q(1) = 2$.

(a) Find $v_P(t)$, the velocity of particle P at time t.

(b) Find $a_Q(t)$, the acceleration of particle Q at time t. Find all times t, for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

(c) Find $y_Q(t)$, the position of particle Q at time t.

(d) As $t \to \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

Solution 6

(a)

Mark scheme

- $v_P(t) = x_P'(t) = 4e^{-t}$. 1 point for the correct answer (an unlabeled response still earns the point; a response that equates $x_P(t)$ with $v_P(t)$ does not).

Solution 6

(b) Mark scheme

- $a_Q(t) = v'_Q(t) = -\dfrac{2}{t^3}$. For $t > 0$, $a_Q(t) < 0$ and $v_Q(t) = \dfrac{1}{t^2} > 0$. Because the velocity and acceleration have opposite signs, the speed of particle Q is decreasing for all $t > 0$. Points: (1) correct expression $a_Q(t) = -2/t^3$; (2) considers/states the signs of $a_Q(t)$ and $v_Q(t)$ (consistent with the presented $a_Q(t)$); (3) correct answer (decreasing for all $t > 0$) with justification that velocity and acceleration have opposite signs.

Solution 6

(c)

Mark scheme

- $y_Q(t) = y_Q(1) + \int_1^t \frac{1}{s^2}\, ds = 2 - \left[\frac{1}{s}\right]_1^t = 2 - \frac{1}{t} + 1 = 3 - \frac{1}{t}$. Points: (1) correct integral expression; (2) uses the initial condition $y_Q(1) = 2$; (3) correct answer, $y_Q(t) = 3 - 1/t$.

Solution 6

(d)

Mark scheme

- For particle P: $\lim_{t\to\infty}(6 - 4e^{-t}) = 6$. For particle Q: $\lim_{t\to\infty}\left(3 - \frac{1}{t}\right) = 3$. Because $6 > 3$, particle P will eventually be farther from the origin. Points: (1) one correct limit; (2) correct answer with reason based on both limits.

Question 2

2021 ▪ Q2

A particle, P, is moving along the x-axis. The velocity of particle P at time t is given by $v_P(t) = \sin\left(t^{1.5}\right)$ for $0 \le t \le \pi$. At time $t = 0$, particle P is at position $x = 5$.
A second particle, Q, also moves along the x-axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \le t \le \pi$. At time $t = 0$, particle Q is at position $x = 10$.

(a) Find the positions of particles P and Q at time $t = 1$.

(b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.

(c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.

(d) Find the total distance traveled by particle P over the time interval $0 \le t \le \pi$.

Solution 2

(a)

Mark scheme

- $x_P(1) = 5 + \displaystyle\int_0^1 v_P(t)\,dt = 5.370660$, so at $t = 1$ particle P is at $x = 5.371$ (or 5.370). $x_Q(1) = 10 + \displaystyle\int_0^1 v_Q(t)\,dt = 8.564355$, so particle Q is at $x = 8.564$.

  Point 1: explicit presentation of at least one definite integral ($\int_0^1 v_P(t)\,dt$ or $\int_0^1 v_Q(t)\,dt$) — required to be eligible for points 2 and 3. Point 2: correct position for P (adding the initial condition and evaluating). Point 3: correct position for Q.

Solution 2

(b)

Mark scheme

- $v_P(1) = \sin(1^{1.5}) = 0.841471 > 0$, so particle P is moving to the right. $v_Q(1) = (1 - 1.8) \cdot 1.25^1 = -1 < 0$, so particle Q is moving to the left. Since $x_P(1) < x_Q(1)$ (particle P is to the left of particle Q), the particles are moving TOWARD each other at $t = 1$. Point 1: uses the sign of $v_P(1)$ or $v_Q(1)$ to determine direction of motion for one particle (must reference a sign, an explicit value is not required). Point 2: answer with explanation based on the signs of both velocities and the relative positions of P and Q at $t = 1$ (may use imported positions from part (a)).

Solution 2

(c)

Mark scheme

- $a_Q(1) = v'_Q(1) = 1.026856$ (or 1.027), so the acceleration of particle Q at $t = 1$ is about 1.027. Since $v_Q(1) = -1 < 0$ and $a_Q(1) > 0$ (opposite signs), the speed of particle Q is DECREASING at $t = 1$. Point 1: acceleration explicitly connected to $v'_Q(1)$ (presenting only the numeric value without showing $v'_Q = a_Q$ does not earn it). Point 2: correct conclusion (speed decreasing) with the opposite-signs reasoning; may be earned independent of point 1.

Solution 2

(d)

Mark scheme

- Total distance traveled by $P = \displaystyle\int_0^{\pi} |v_P(t)|\, dt = 1.93148 \approx 1.931$. Point 1: the definite integral $\int_0^{\pi} |v_P(t)|\, dt$ (a split sum/difference of definite integrals using a verified sign-change point is also acceptable). Point 2: the correct final answer 1.931 (an unsupported numeric answer alone earns no points; using v_Q instead of v_P earns only point 1).

Question 2

2019 ▪ Q2

The velocity of a particle, P, moving along the x-axis is given by the differentiable function v_P, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table below. Particle P is at the origin at time $t = 0$.

t **(hours)**	**0**	**0.3**	**1.7**	**2.8**	**4**
$v_P(t)$ (meters per hour)	0	55	-29	55	48

Question 2

(a) Justify why there must be at least one time t, for $0.3 \leq t \leq 2.8$, at which $v_P'(t)$, the acceleration of particle P, equals 0 meters per hour per hour.

Question 2

t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	–29	55	48