Exercise walk-through

Volume with Washer Method: Revolving Around Other Axes ▪ 8.12

eXercise

You must be able to

- shift **both** radii to the distance from each curve to a line $y = k$ or $x = k$
- integrate $\pi\left(R_{\text{outer}}^2 - R_{\text{inner}}^2\right)$ with the shifted radii
- sketch the region and label the two radii

Method — Both radii shift

Revolving about a line $y = k$ moves **both** radii. Each becomes the distance from its curve to that line:

$$R_{\text{outer}} = |f_{\text{far}} - k|, \quad R_{\text{inner}} = |f_{\text{near}} - k|.$$

Method — A worked example

Revolve the region between $y = \sqrt{x}$ and $y = x$ on $[0, 1]$ about the line $y = -1$.
Distances to $y = -1$: outer (from $\sqrt{x}$) $= \sqrt{x} + 1$; inner (from x) $= x + 1$:

$$V = \pi \int_0^1 \left((\sqrt{x} + 1)^2 - (x + 1)^2 \right) dx.$$

Method — Keep the order

The outer curve (farther from the axis) still gives the larger radius. When the axis is **below** the region, adding the same amount to each keeps their order.

Method — Method summary

Sketch the region and the axis, mark R_{outer} and R_{inner} as distances to the axis, then integrate $\pi\left(R_{\text{outer}}^2 - R_{\text{inner}}^2\right)$.

Practice 2.1 ▪ 1 mark

A curve $y = f(x)$ is revolved about $y = -2$. Write its radius.

Solution 2.1

Worked steps

$R = f(x) + 2$ $\boxed{\mathbf{B1}}$.

Practice 2.2 ▪ 2 marks

For outer curve $y = 4$ and inner curve $y = 1$ revolved about $y = -1$, state both radii.

Solution 2.2

Method: Both radii shift

Worked steps

outer $= 4 - (-1) = 5$; inner $= 1 - (-1) = 2$ **B1** **B1**.

Practice 2.3 ▪ 1 mark

Expand $(x+1)^2 - (\sqrt{x}+1)^2$? No — first expand $(x+1)^2$.

Solution 2.3

Worked steps

$(x+1)^2 = x^2 + 2x + 1$ **B1**.

Exam-style 3.1 ▪ 1 mark

Revolving a region between two curves about $y = k$ uses radii

- **A** the curves themselves
- **B** each curve's distance to the line $y = k$
- **C** only the outer curve
- **D** k for both

Solution 3.1

A the curves themselves

B each curve's distance to the line $y = k$ ✓

C only the outer curve

D k for both

Worked steps

B — each radius is the curve's distance to the line $y = k$.

Exam-style 3.2 ▪ 2 marks

The region between $y = x^2$ (lower) and $y = 2x$ (upper) on $[0, 2]$ is revolved about the line $y = -1$.
(a) Write the outer and inner radii.
(b) Set up the volume integral.

Solution 3.2

Method: A worked example

Worked steps

(a) On $(0,2)$, $2x \geq x^2$, so outer (from $2x$) $= 2x+1$, inner (from x^2) $= x^2+1$ **M1** **A1**. (b) $V = \pi \int_0^2 \left((2x+1)^2 - (x^2+1)^2\right) dx$ **M1** **A1**.

Exam-style 3.3 ▪ 4 marks

The region between $y = \sqrt{x}$ and $y = x$ on $[0, 1]$ is revolved about the line $y = 2$. Write the outer and inner radii (as distances to $y = 2$), and set up the volume integral.

Solution 3.3

Method: A worked example

Worked steps

Distances to $y = 2$: outer (from $y = x$, the lower curve, farther below 2) $= 2 - x$; inner (from $y = \sqrt{x}$) $= 2 - \sqrt{x}$ M1 M1; $V = \pi \int_0^1 \left((2 - x)^2 - (2 - \sqrt{x})^2\right) dx$ M1 A1.