Exercise walk-through

Volume with Disc Method: Revolving Around Other Axes ▪ 8.10

eXercise

You must be able to

- adjust the **radius** to the distance from the curve to a non-coordinate axis
- write $R = |f(x) - k|$ for a horizontal axis $y = k$ (or $|x(y) - k|$ for $x = k$)
- integrate πR^2 as before

Method — Radius to a shifted axis

When the axis of revolution is $y = k$ (not the x-axis), the radius is the **distance from the curve to that line**:

$$R = |f(x) - k|, \qquad V = \pi \int_a^b \left(f(x) - k\right)^2 \, dx.$$

Method — A worked example

Revolve $y = \sqrt{x}$, $0 \le x \le 4$, about the line $y = -1$. The radius is $\sqrt{x} - (-1) = \sqrt{x} + 1$:

$$V = \pi \int_0^4 (\sqrt{x} + 1)^2 \, dx = \pi \int_0^4 (x + 2\sqrt{x} + 1) \, dx.$$

Method — Revolving about a vertical line

For an axis $x = k$, use $R = |x(y) - k|$ and integrate in y. Always draw a picture and measure the radius from the curve **to** the axis.

Method — Watch the direction

If the region lies **below** the axis line, the distance is $k - f(x)$; squaring makes the sign irrelevant, but set the radius as a genuine (positive) distance.

Practice 2.1 ▪ 1 mark

A curve $y = f(x)$ is revolved about $y = 3$. Write the radius.

Solution 2.1

Method: Revolving about a vertical line

Worked steps

$R = |f(x) - 3|$ **B1**.

Practice 2.2 ▪ 1 mark

Revolving $y = 2$ (constant) region about $y = -1$: state the radius.

Solution 2.2

Method: Revolving about a vertical line

Worked steps

$R = 2 - (-1) = 3$ **B1**.

Practice 2.3 ▪ 1 mark

Expand $(\sqrt{x}+1)^2$.

Solution 2.3

Worked steps

$x + 2\sqrt{x} + 1$ B1.

Exam-style 3.1 ▪ 1 mark

Revolving $y = f(x)$ about the line $y = k$ uses radius

A $f(x)$

B $f(x) - k$

C k

D $f(x) + k$ always

Solution 3.1

Method: Revolving about a vertical line

A $f(x)$

B $f(x) - k$ ✓

C k

D $f(x) + k$ always

Worked steps

B — the radius is the distance $f(x) - k$ to the line.

Exam-style 3.2 ▪ 1 mark

The region under $y = x^2$ from $x = 0$ to $x = 2$ is revolved about the line $y = -1$.
(a) Write the radius $R(x)$.
(b) Set up the volume integral.

Solution 3.2

Worked steps

(a) $R = x^2 - (-1) = x^2 + 1$ **B1**. (b) $\pi \int_0^2 (x^2+1)^2 \, dx$ **M1** **A1** *area, limits.*

Exam-style 3.3 ▪ 3 marks

The region under $y = \sqrt{x}$ on $[0, 4]$ is revolved about the line $y = 2$. Set up the volume integral for the solid (radius $= 2 - \sqrt{x}$).

Solution 3.3

Method: A worked example

Worked steps

$R = 2 - \sqrt{x}$ **M1**; $V = \pi \int_0^4 (2 - \sqrt{x})^2 \, dx$ **M1** **A1**.