Exercise walk-through

Reasoning Using Slope Fields ▪ 7.4

eXercise

You must be able to

- sketch the **particular solution** through a given point on a slope field
- describe **long-run behavior** (where solutions level off or grow)
- identify **equilibrium solutions** 平衡解 where $\frac{dy}{dx} = 0$

Method — Equilibrium solutions

An **equilibrium** is a constant solution where $\frac{dy}{dx} = 0$ everywhere. For $\frac{dy}{dx} = y - 2$, setting $y - 2 = 0$ gives the equilibrium $y = 2$: a horizontal line the other solutions approach or leave.

Method — Long-run behavior

For $\frac{dy}{dx} = y - 2$ with a start **below** $y = 2$: there $\frac{dy}{dx} < 0$, so the solution moves **away** downward. Starting above, it moves up. So $y = 2$ is an **unstable** equilibrium.

Method — Sketching a particular solution

To sketch the solution through $(0, 1)$: from that point, follow the segments — the curve is always tangent to the local slope, curving as the field directs. Do not cross an equilibrium line.

Method — Reading a maximum from the field

Where a solution curve has a **horizontal** tangent ($\frac{dy}{dx} = 0$) and the slope changes $+$ to $-$ around it, the solution has a local maximum there.

Practice 2.1 ▪ 1 mark

Find the equilibrium solution of $\frac{dy}{dx} = y - 5$.

Solution 2.1

Worked steps

$y = 5$ B1.

Practice 2.2 ▪ 1 mark

For $\frac{dy}{dx} = 3 - y$, state the equilibrium value of y.

Solution 2.2

Worked steps

$y = 3$ B1.

Practice 2.3 ▪ 1 mark

For $\frac{dy}{dx} = y - 5$, if a solution starts at $y = 6$, does it move toward or away from $y = 5$?

Solution 2.3

Method: Long-run behavior

Worked steps

Away — at $y = 6$, $\dfrac{dy}{dx} = 1 > 0$, so y increases, moving away from 5 B1.

Exam-style 3.1 ▪ 1 mark

An equilibrium solution of a differential equation is a value of y where

A $\frac{dy}{dx} = 0$

B $y = 0$

C $x = 0$

D $\frac{dy}{dx}$ is undefined

Solution 3.1

Method: Equilibrium solutions

A $\dfrac{dy}{dx} = 0$ ✓

B $y = 0$

C $x = 0$

D $\dfrac{dy}{dx}$ is undefined

Worked steps

A — an equilibrium has $\dfrac{dy}{dx} = 0$.

Exam-style 3.2 ▪ 1 mark

Consider $\frac{dy}{dx} = 2 - y$.

(a) Find the equilibrium solution.

(b) For a solution starting at $y = 5$, state whether y increases or decreases, with a reason.

(c) State $\lim_{x \to \infty} y$ for that solution.

Solution 3.2

Method: Long-run behavior

Worked steps

(a) $y = 2$ **B1**. (b) Decreases — at $y = 5$, $\frac{dy}{dx} = 2 - 5 = -3 < 0$ **M1** **A1**. (c) $y \to 2$ **B1**.

Exam-style 3.3 ▪ 3 marks

A slope field for $\dfrac{dy}{dx} = y(4 - y)$ has two equilibrium lines. State both, and say which one solutions starting near $y = 1$ approach.

Solution 3.3

Method: Equilibrium solutions

Worked steps

Equilibria $y = 0$ and $y = 4$ B1 B1; a solution starting at $y = 1$ has $\dfrac{dy}{dx} = 1(3) > 0$, so it rises toward $y = 4$ B1.