Past-paper walk-through

Modeling Situations with Differential Equations ▪ 7.1

eXercise

Questions in this set

- 2026 Q3
- 2023 Q3
- 2021 Q6

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 3

2026 ▪ Q3

A pie is taken from a hot oven and put on a table. The internal temperature of the pie at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

(a) Explain why the following could not be a slope field for the differential equation $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$.

[Slope field diagram: H-axis (vertical) from 30 to 70$+$ in increments of 10, t-axis (horizontal) from 0 to 25$+$ in increments of 5. At every grid point across the full range of t shown, the line segments have the same positive slope for a given horizontal row and the slopes appear to depend only on t (segments in each row are parallel and tilt

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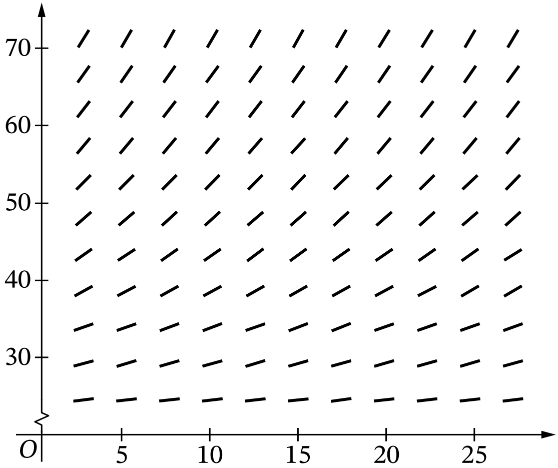
more steeply positive at greater height H, but they do not appear to flatten toward zero as $H \to 20$ nor do the segments vary with t within a row as required) — segments are drawn as short parallel dashes tilting up-and-to-the-right throughout, uniformly across each horizontal band of H values, without change as t increases.]

Question 3



Question 3

2026 ▪ Q3

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(b) Find the slope of the line tangent to the graph of H at time $t = 0$. Show the work that leads to your answer.

Question 3

2026 ▪ Q3

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(c) It can be shown that $\dfrac{d^2H}{dt^2} = \dfrac{1}{225}(H - 20)$. The line tangent to the graph of H at time $t = 0$ is used to approximate $H(5)$, the internal temperature of the pie at time $t = 5$. Is this approximation an overestimate or an underestimate for the actual value of $H(5)$? Give a reason for your answer.

Question 3

2026 ▪ Q3

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(d) Use separation of variables to find an expression for $H(t)$, the particular solution to the given differential equation with initial condition $H(0) = 75$.

Question 3

2023 ▪ Q3

A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius (°C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\dfrac{dM}{dt} = \dfrac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(a) A slope field for the differential equation $\dfrac{dM}{dt} = \dfrac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.
[Slope field diagram: axes labeled $M(t)$ (vertical, with gridlines at 5 and 70 marked) and t (horizontal, with a gridline at 15 marked), origin O. Short line segments are

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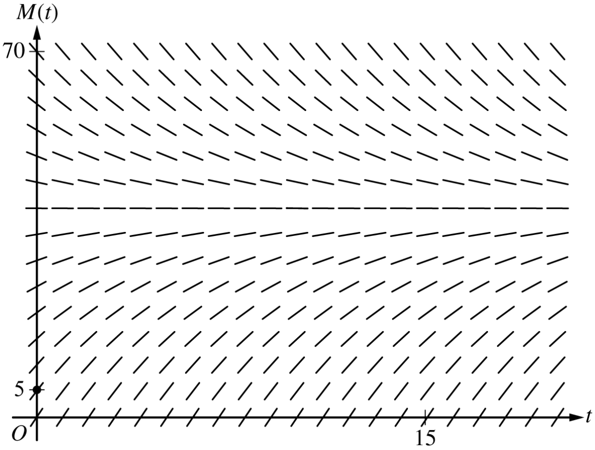
drawn at grid points across the region $0 \leq t \leq 15$-ish, $0 \leq M(t) \leq 70$-ish: segments are nearly horizontal (flat, slightly downward-sloping) near $M(t) = 70$, becoming steeper negative slope moving down toward $M(t) \approx 40$ (segments are horizontal/flat right at $M(t) = 40$), then the slopes become positive and increasingly steep as $M(t)$ decreases from 40 down toward $M(t) = 5$, with the steepest (near-vertical, slope ≈ 1) segments at the bottom of the field near $M(t) = 5$. The point $(0, 5)$ is marked with a star on the vertical axis.]

Question 3



Question 3

2023 ▪ Q3

A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius ($^\circ$C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

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2023 ▪ Q3

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(c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

Question 3

2023 ▪ Q3

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(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\dfrac{dM}{dt} = \dfrac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

Solution 3

(a)

Mark scheme

- The solution curve through (0,5) starts at (0,5), increases and is concave down, and approaches (curving up toward, but always staying below) the flat slope segments at M=40 as t increases across the shown window; it must not cross any drawn slope-field segments (1 pt: curve passes through (0,5), extends reasonably close to the left and right edges of the given rectangle, has no obvious conflicts with the slope lines, and lies entirely below M=40).

Solution 3

(b) Mark scheme

- $\left.\frac{dM}{dt}\right|_{t=0} = \frac{1}{4}(40-5) = \frac{35}{4}$ (1 pt: value of the derivative at t=0). Tangent line: $y = 5 + \frac{35}{4}(t-0)$, so $M(2) \approx 5 + \frac{35}{4} \cdot 2 = 22.5$; the temperature of the milk at t=2 minutes is approximately 22.5°C (1 pt: approximation using a tangent line through (0,5) with slope 35/4, correctly simplified).

Solution 3

(c) Mark scheme

- $\dfrac{d^2M}{dt^2} = -\dfrac{1}{4}\dfrac{dM}{dt} = -\dfrac{1}{4}\left(\dfrac{1}{4}(40 - M)\right) = -\dfrac{1}{16}(40 - M)$ (1 pt: correct second-derivative expression in terms of M, no subsequent simplification error). Since $M(t) < 40$, $\dfrac{d^2M}{dt^2} < 0$, so the graph of M is concave down; therefore the tangent-line approximation from part (b) is an overestimate (1 pt: overestimate conclusion with a reason based on concavity/decreasing dM/dt).

Solution 3

(d) Mark scheme

- Separate variables: $\dfrac{dM}{40-M} = \dfrac{1}{4}\,dt$ (1 pt: separates variables). Integrate: $\displaystyle\int \frac{dM}{40-M} = \int \frac{1}{4}\,dt \Rightarrow -\ln|40-M| = \frac{1}{4}t + C$ (1 pt: finds antiderivatives). Apply the initial condition M(0)=5: $-\ln|40-5| = 0 + C \Rightarrow C = -\ln 35$; since M(t)<40, $|40-M| = 40-M$, giving $-\ln(40-M) = \dfrac{1}{4}t - \ln 35$, i.e. $\ln(40-M) = -\dfrac{1}{4}t + \ln 35$ (1 pt: constant of integration correctly found and initial condition used). Solve for M: $40 - M = 35e^{-t/4} \Rightarrow M(t) = 40 - 35e^{-t/4}$ (1 pt: solves for M — the final answer must be exactly this closed form, or equivalent).

Question 6

2021 ▪ Q6

A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

(a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.

[Slope field diagram, t-axis horizontal from 0 rightward, y-axis vertical; short line segments drawn at grid points showing slope direction: segments near the top of the grid slope downward (negative slope, since $y > 12$ there), a horizontal row of flat dashes at $y = 12$ (zero slope), and segments below slope upward (positive slope, since

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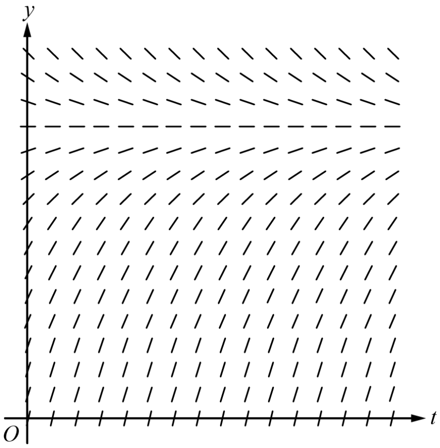
$y < 12$), with segments closer to $y = 0$ sloping steeply upward and flattening as y approaches 12 across each column.]

Question 6



Question 6

2021 ▪ Q6

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(b) Using correct units, interpret the statement $\lim_{t \to \infty} A(t) = 12$ in the context of this problem.

Question 6

2021 ▪ Q6

A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12-y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

(c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation $\frac{dy}{dt} = \frac{12-y}{3}$ with initial condition $A(0) = 0$.

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2021 ▪ Q6

A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

(d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\frac{dy}{dt} = 3 - \frac{y}{t+2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

Solution 6

(a) Mark scheme

- Sketch the solution curve to $\dfrac{dy}{dt} = \dfrac{12 - y}{3}$ through $(0, 0)$: the curve must start at $(0, 0)$, be generally increasing, be concave down throughout, and approach the horizontal asymptote $y = 12$ from below as t increases (following the slope-field segments, which are positive below $y = 12$, flat at $y = 12$, and negative above $y = 12$). Point 1: earned only if a single such curve is drawn passing through $(0, 0)$ with the correct increasing/concave-down/asymptotic shape; not earned if two or more solution curves are presented.

Solution 6

(b)

Mark scheme

- Over time, the amount of medication in the patient approaches 12 milligrams. Point 1: the interpretation must include ”medication in the patient,” ”approaches 12,” and units (milligrams), or their equivalents.

Solution 6

(c) Mark scheme

- Separate variables: $\dfrac{dy}{dt} = \dfrac{12-y}{3} \Rightarrow \dfrac{dy}{12-y} = \dfrac{dt}{3}$. Integrate: $\displaystyle\int \frac{dy}{12-y} = \int \frac{dt}{3} \Rightarrow -\ln|12-y| = \frac{t}{3} + C$. Solve for y: $\ln|12-y| = -\dfrac{t}{3} - C \Rightarrow |12-y| = e^{-t/3-C} \Rightarrow y = 12 + Ke^{-t/3}$. Apply $A(0) = 0$: $0 = 12 + K \Rightarrow K = -12$. Final: $y = A(t) = 12 - 12e^{-t/3}$. Point 1: correct separation of variables ($dy/(12-y) = dt/3$; a "bad separation" like $dy/(12-y) = 3\,dt$ does not earn this point but the response stays eligible for points 2-3). Point 2: correct antiderivatives on both sides (absolute value bars not required). Point 3: correctly introduces and solves for the constant of integration

Solution 6

using the initial condition $A(0) = 0$ (earned consecutively after point 2). Point 4: solves explicitly for $y = A(t) = 12 - 12e^{-t/3}$ (needs points 1-3 in the standard path).

Solution 6

(d) Mark scheme

- $\frac{dy}{dt} = 3 - \frac{y}{t+2} \Rightarrow \frac{d^2y}{dt^2} = -\frac{\frac{dy}{dt}(t+2) - y}{(t+2)^2}$ (quotient rule). At $t = 1$: $B'(1) = 3 - \frac{B(1)}{3} = 3 - \frac{2.5}{3} = \frac{6.5}{3}$. Then $B''(1) = -\frac{B'(1) \cdot 3 - B(1)}{3^2} = -\frac{6.5 - 2.5}{9} = -\frac{4}{9} < 0$. The rate of change of the amount of medication in the second patient is DECREASING at $t = 1$ because $B''(1) < 0$ and d^2y/dt^2 is continuous on an interval containing $t = 1$. Point 1: correctly applies the quotient rule to $y/(t+2)$ (or product rule to $y(t+2)^{-1}$); errors differentiating the constant 3 or mishandling the sign forfeit this point. Point 2: states $B''(1) < 0$ with a correct value/sign — cannot be earned unless

Solution 6

the second derivative itself is correct. Point 3: consistent conclusion (decreasing) based on the declared value/sign of $B''(1)$; only needs an attempt at the quotient/product rule to be eligible.