Past-paper walk-through

Integrating Using Substitution ▪ 6.9

eXercise

Questions in this set

- 2016 Q6
- 2014 Q5

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 6

2016 ▪ Q6

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

Question 6

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\displaystyle\int_1^3 f'(2x)\,dx$.

Question 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

Solution 6

(a)

Mark scheme

- k(x)=f(g(x)). k(3)=f(g(3))=f(6)=4. By the chain rule, k'(x)=f'(g(x))·$g'(x), sok'(3) = f'(g(3)) \cdot g'(3) = f'(6) \cdot g'(3) = 5 \cdot 2 = 10. Tangentline : y = 10(x-3)+4. Points : 2forcorrectlyfindingtheslopek'(3) = 10(chainruleusingtablevalues), 1forthecorrecttangentlineequation.$

Solution 6

(b) Mark scheme

- h(x)=g(x)/f(x). By the quotient rule: h'(x) $= \dfrac{f(x)g'(x) - g(x)f'(x)}{[f(x)]^2}.h'(1) = \dfrac{f(1)g'(1) - g(1)f'(1)}{[f(1)]^2} = \dfrac{(-6)(8) - (2)(3)}{(-6)^2} = \dfrac{-48 - 6}{36} = \dfrac{-54}{36} = -\dfrac{3}{2}.Points : 2 for the correct expression for h'(1) using the quotient rule and table values, 1 for the correct final answer($

Solution 6

(c) Mark scheme

- $\int_1^3 f'(2x)\,dx = \frac{1}{2}\left[f(2x)\right]_1^3 = \frac{1}{2}\left[f(6) - f(2)\right] = \frac{1}{2}\left[5 - (-2)\right] = \frac{1}{2}(7) = \frac{7}{2}. Points: 2 for the correct antiderivative via u-substitution (\frac{1}{2}f(2x)), 1 for the correct final answer (7/2).$

Question 5

2014 ▪ Q5

The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

Question 5

(a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.
(b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.
(c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.
(d) Evaluate $\displaystyle\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

Question 5

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

Solution 5

(a)

Mark scheme

- $x = 1$ is the only critical point at which f' changes sign from negative to positive (f' is negative on $-1<x<1$ and positive on $1<x<3$, so f' changes sign at $x=1$). Therefore, f has a relative minimum at $x = 1$. Points: 1 for the correct x-value ($x=1$) together with a valid justification (first-derivative sign change from negative to positive at $x=1$).

Solution 5

(b)

Mark scheme

- f' is differentiable, so f' is continuous on [-1,1]. f'(1)-f'(-1) $\overline{1-(-1)=\frac{0-0}{2}=0.\mathit{Therefore,bytheMeanValueTheoremappliedtof'on}[-1,1],\mathit{thereisatleastonevaluec},-1<c<1,\mathit{suchtha}}$

Solution 5

(c) Mark scheme

- h(x) = $\ln(f(x)). Bythechainrule, h'(x) = \frac{1}{f(x)} \cdot f'(x). h'(3) = \frac{1}{f(3)} \cdot f'(3) = \frac{1}{7} \cdot \frac{1}{2} = \frac{1}{14}. Points: 2 foracorrectexpressionforh'(x)(chainruleforln(f(x))), 1 forthecorrectfinalanswerh'(3) = 1/14.$

Solution 5

(d)

Mark scheme

- $\int_{-2}^{3} f'(g(x))g'(x)\,dx = \left[f(g(x))\right]_{x=-2}^{x=3} = f(g(3)) - f(g(-2)) = f(1) - f(-1) = 2 - 8 = -6, usingtheFundamentalTheoremofCalculus(antiderivativef(g(x))viathechainrule/u-substitutionu = g(x))withtablevaluesg(3) = 1, g(-2) = -1, f(1) = 2, f(-1) = 8. Points : 2forcorrectlyapplyingtheFundamentalTheoremofCalculus(recognizingf(g(x))astheantiderivat$
  $6.$