Past-paper walk-through

Riemann Sums, Summation Notation, and Definite Integral Notation ▪ 6.3

eXercise

Questions in this set

- 2022 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2022 ▪ Q1

From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

(a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

(b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

(c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

(d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t, for $a \leq t \leq 4$, is given by

Question 1

$$N(t) = \int_a^t (A(x) - 400)\, dx,$$

where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

Solution 1

(a)

Mark scheme

- The total number of vehicles arriving from 6 A.M. to 10 A.M. is given by the definite integral $\int_1^5 A(t)\,dt$. Full credit requires a definite integral with the correct lower limit $t = 1$ and upper limit $t = 5$; the integrand should be $A(t)$ (equivalently $|A(t)|$, since $A(t) \geq 0$ on this interval). Do not evaluate the integral.

Solution 1

(b) Mark scheme

- Average value $= \dfrac{1}{5-1}\int_1^5 A(t)\,dt = \dfrac{1}{4}(1502.147865) = 375.536966$. The average rate at which vehicles arrive at the toll plaza from 6 A.M. to 10 A.M. is about 375.537 (or 375.536) vehicles per hour. One point for using the correct average value formula $\frac{1}{b-a}\int_a^b A(t)\,dt$ with $a = 1$, $b = 5$; one point for the correct answer, accurate to three decimal places.

Solution 1

(c)

Mark scheme

- $A'(1) = 148.947272 > 0$, so the rate at which vehicles arrive at the toll plaza is increasing at $t = 1$ (6 A.M.). One point for considering $A'(1)$; one point for the correct answer (increasing) together with the reason $A'(1) > 0$.

Solution 1

(d)

Mark scheme

- $N'(t) = A(t) - 400 = 0 \Rightarrow A(t) = 400 \Rightarrow t = a = 1.469372$ or $t = b = 3.597713$. Evaluating $N(t) = \int_a^t (A(x) - 400)\,dx$ at the critical points and the right endpoint: $N(a) = 0$, $N(b) = 71.254129$, $N(4) = 62.338346$. The greatest number of vehicles in line is 71 (attained at $t = b$). Points: (1) considers $N'(t) = 0$; (2) correctly finds both $t = a$ and $t = b$; (3) correct answer of 71 (or 71.254...); (4) justification via a candidates-test table comparing $N(a)$, $N(b)$, and $N(4)$.