Past-paper walk-through

Exploring Accumulations of Change ▪ 6.1

eXercise

Questions in this set

- 2019 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2019 ▪ Q1

Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

(a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)? Give your answer to the nearest whole number.

(b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)?

(c) At what time t, for $0 \leq t \leq 8$, is the greatest number of fish in the lake? Justify your answer.

Question 1

(d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$)? Explain your reasoning.

Solution 1

(a)

Mark scheme

- $\int_0^5 E(t)\,dt = 153.457690$ (calculator value). To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M. Points: 1 for the integral expression, 1 for the correct answer 153.

Solution 1

(b)

Mark scheme

- Average rate $= \dfrac{1}{5-0}\int_0^5 L(t)\,dt = 6.059038$. The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.
  Points: 1 for the integral expression, 1 for the correct answer 6.059.

Solution 1

(c) Mark scheme

- The rate of change in the number of fish in the lake at time t is $E(t) - L(t)$. Setting $E(t) - L(t) = 0$ gives $t = 6.20356$. Since $E(t) - L(t) > 0$ for $0 \leq t < 6.20356$ and $E(t) - L(t) < 0$ for $6.20356 < t \leq 8$, the number of fish increases then decreases, so the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203). [Alternate: let $A(t) = \int_0^t (E(s) - L(s))\, ds$ be the change in fish since midnight; $A'(t) = E(t) - L(t) = 0$ at $t = C = 6.20356$; compare $A(0) = 0$, $A(C) = 135.01492$, $A(8) = 80.91998$ — the maximum is at $t = C$.] Points: 1 for setting $E(t) - L(t) = 0$ (or the equivalent candidates-test setup), 1 for the answer $t = 6.204$ (or 6.203), 1 for justification (sign analysis or candidates test).

Solution 1

(d)

Mark scheme

- $E'(5) - L'(5) = -10.7228 < 0$. Since the rate of change in the number of fish is $E(t) - L(t)$, its derivative at $t = 5$ is $E'(5) - L'(5)$, which is negative, so the rate of change in the number of fish in the lake is DECREASING at $t = 5$ A.M. Points: 1 for considering $E'(5)$ and $L'(5)$, 1 for the correct answer (decreasing) with explanation.