Past-paper walk-through

Connecting a Function, Its First Derivative, and Its Second Derivative ▪ 5.9

eXercise

Questions in this set

- 2025 Q4
- 2018 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

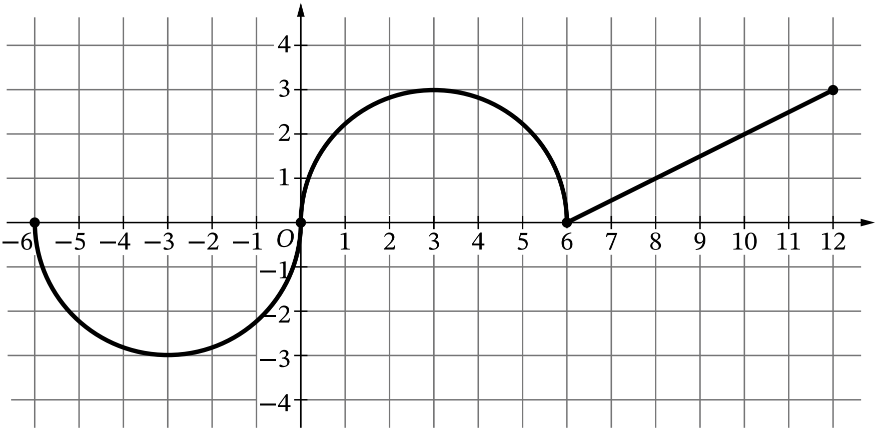
2025 ▪ Q4

The continuous function f is defined on the closed interval $-6 \leq x \leq 12$. The graph of f, consisting of two semicircles and one line segment, is shown in the figure.
[Graph of f: On the interval $-6 \leq x \leq 0$, the graph is a semicircle below the x-axis with endpoints at $(-6, 0)$ and $(0, 0)$, reaching a minimum near $(-3, -3)$. On the interval $0 \leq x \leq 6$, the graph is a semicircle above the x-axis with endpoints at $(0, 0)$ and $(6, 0)$, reaching a maximum near $(3, 3)$. On the interval $6 \leq x \leq 12$, the graph is a line segment from $(6, 0)$ to $(12, 3)$. The horizontal axis is labeled from -6 to 12 and the vertical axis from -4 to 4.]
Graph of f
Let g be the function defined by $g(x) = \int_6^x f(t)\,dt$.
(a) Find $g'(8)$. Give a reason for your answer.

Question 4

(b) Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.
(c) Find $g(12)$ and $g(0)$. Label your answers.
(d) Find the value of x at which g attains an absolute minimum on the closed interval $-6 \leq x \leq 12$. Justify your answer.

Question 4



Graph of f

Solution 4

(a)

Mark scheme

- By the Fundamental Theorem of Calculus, $g'(x) = f(x)$, so $g'(8) = f(8) = 1$ (read from the graph of f at $x = 8$, which is on the line segment). Point 1: considers $g'(x) = f(x)$ (or $g' = f$, or $g'(8) = f(8)$). Point 2: correct answer $g'(8) = 1$ (an implied application of the Fundamental Theorem of Calculus, e.g. just "$g'(8) = 1$" or "$f(8) = 1$", is also sufficient for this point alone).

Solution 4

(b) Mark scheme

- The graph of g has a point of inflection where $g'' = f'$ changes sign, i.e. where f changes from decreasing to increasing or vice versa (equivalently, where f attains a relative extremum, or where the slope of f changes sign). From the graph, this happens at $x = -3$, $x = 3$, and $x = 6$. Point 3: correct answer of exactly $x = -3$, $x = 3$, and $x = 6$ (if any other/additional values are declared, neither this point nor the reason point is earned; consideration of $x = -6$ or $x = 12$ does not impact scoring). Point 4: reason correctly tied to the given graph of f (e.g. "f changes from increasing to decreasing or decreasing to increasing there," or "f attains relative extrema there"; a reason relying on an ambiguous term like "the function" or "the graph" does not earn this point).

Solution 4

(c) Mark scheme

- $g(12) = \int_6^{12} f(t)\,dt = \frac{1}{2}(6)(3) = 9$ (area of the triangular line-segment region). $g(0) = \int_6^0 f(x)\,dx = -\int_0^6 f(x)\,dx = -\frac{\pi}{2}(3)^2 = -\frac{9\pi}{2}$ (negative of the semicircle area, since integrating from 6 down to 0). Point 5: $g(12) = 9$, with or without supporting work, correctly labeled as $g(12)$ (an unlabeled value does not earn the point). Point 6: $g(0) = -\frac{9\pi}{2}$, with or without supporting work, correctly labeled as $g(0)$.

Solution 4

(d) Mark scheme

- g attains a minimum on $[-6, 12]$ either where $g'(x) = f(x) = 0$ or at an endpoint: candidate points are $x = -6, 0, 6, 12$. Candidates test: $g(-6) = 0$, $g(0) = -\dfrac{9\pi}{2} \approx -14.137$, $g(6) = 0$, $g(12) = 9$. The absolute minimum value is attained at $x = 0$. Point 7: considers $g'(x) = 0$ or $f(x) = 0$ (not earned by just presenting $x = 0$ and $x = 6$ with no justification). Point 8: justification via a correct global (candidates-test) argument evaluating g at all four candidate points $-6, 0, 6, 12$ (values for $g(0)$ and $g(12)$ may be imported from part C); alternatively, an argument that $g'(x) \leq 0$ (or $f(x) \leq 0$) for $-6 \leq x < 0$ and $g'(x) \geq 0$ (or $f(x) \geq 0$) for $0 < x \leq 12$. Point 9: correct answer $x = 0$; requires

Solution 4

Point 8 (a local-argument response, e.g. First/Second Derivative Test, or an incorrect global argument, does not earn Point 8 but remains eligible for Point 9 with the correct answer).

Question 3

2018 ▪ Q3

[Graph of g: a piecewise-defined curve on axes with x from -5 to 6 (gridlines every 1 unit) and y from -3 to 8 (gridlines every 1 unit). The curve is constant at $y = -3$ for $-5 \leq x \leq -2$; rises linearly from $(-2, -3)$ to $(-1, 0)$; is constant at $y = 0$ for $-1 \leq x \leq 0$; rises linearly from $(0, 0)$ to $(1, 2)$; is constant at $y = 2$ for $1 \leq x \leq 3$; then follows a smooth curve for $3 \leq x \leq 6$ that dips down to a minimum value of 0 at $x = 4$ and rises back up, passing through approximately $(5, 1)$ and reaching $(6, 8)$.]

Graph of g

The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \leq x < 3$, and $g(x) = 2(x - 4)^2$ for $3 \leq x \leq 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

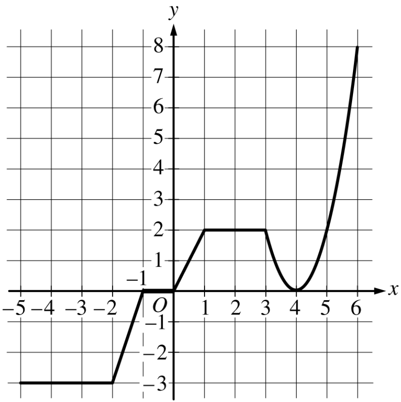
Question 3

(b) Evaluate $\int_1^6 g(x)\,dx$.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

Question 3



Graph of g

Solution 3

(a)

Mark scheme

- $f(-5) = f(1) + \int_1^{-5} g(x)\,dx = f(1) - \int_{-5}^{1} g(x)\,dx = 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$. (1 pt: the integral expression for $f(-5)$; 1 pt: the answer $\frac{25}{2}$.)

Solution 3

(b)

Mark scheme

- $\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx = \int_1^3 2\,dx + \int_3^6 2(x-4)^2\,dx =$ $4 + \left[\frac{2}{3}(x-4)^3\right]_3^6 = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10.$ (1 pt: splits the integral at $x = 3$; 1 pt: antiderivative $\frac{2}{3}(x-4)^3$ of $2(x-4)^2$; 1 pt: the answer 10.)

Solution 3

(c)

Mark scheme

- The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$, because $f'(x) = g(x) > 0$ and $f'(x) = g(x)$ is increasing (so $f''(x) = g'(x) > 0$) on those intervals. (1 pt: the correct intervals; 1 pt: the reason.)

Solution 3

(d)

Mark scheme

- The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$ (so f'' changes sign there). (1 pt: the answer $x = 4$; 1 pt: the reason.)