Past-paper walk-through

Using the Second Derivative Test to Determine Extrema ▪ 5.7

eXercise

Questions in this set

- 2015 Q4

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

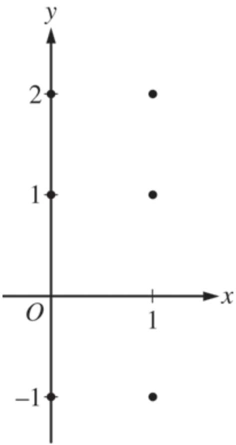
2015 ▪ Q4

Consider the differential equation $\dfrac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

[Slope-field axes labeled y (vertical) and x (horizontal), origin O. Six points are marked with dots, at coordinates $(1, 2)$, $(1, 1)$, $(1, -1)$, and their positions relative to axis tick marks at $y = 2$, $y = 1$, $y = -1$ on the y-axis and $x = 1$ on the x-axis. (The six indicated points are at $x = 1$ paired with $y = 2, 1, -1$, and at $x = 2$ paired with $y = 2, 1, -1$, based on the two columns of dots shown above $y = 2$, $y = 1$, and $y = -1$.)]

Question 4



Question 4

2015 ▪ Q4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

Question 4

2015 ▪ Q4

Consider the differential equation $\dfrac{dy}{dx} = 2x - y$.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

Question 4

2015 ▪ Q4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

Solution 4

(a)

Mark scheme

- Slope field at the six indicated points using $dy/dx = 2x - y$. At $x = 0$: (0,2) slope $= -2$, (0,1) slope $= -1$, (0,−1) slope $= 1$. At $x = 1$: (1,2) slope $= 0$, (1,1) slope $= 1$, (1,−1) slope $= 3$. Points: 1 for correct slope segments drawn at the three points where $x = 0$, 1 for correct slope segments drawn at the three points where $x = 1$. (This is a drawing task — grade by checking the drawn segment directions/steepness at each of the six given points match these computed slopes, not a numeric or algebraic answer.)

Solution 4

(b)

Mark scheme

- $d^2y/dx^2 = 2 - dy/dx = 2 - (2x - y) = 2 - 2x + y$. In Quadrant II, $x < 0$ and $y > 0$, so $2 - 2x + y > 0$ (each term is positive). Therefore all solution curves are concave up in Quadrant II. Points: 1 for the correct expression $d^2y/dx^2 = 2 - 2x + y$, 1 for the correct conclusion (concave up) with a valid reason (sign argument in Quadrant II).

Solution 4

(c)

Mark scheme

- dy/dx at $(x, y) = (2, 3)$ is $2(2) - 3 = 1 \neq 0$. Therefore f has neither a relative minimum nor a relative maximum at $x = 2$, since the derivative is nonzero there. Points: 1 for considering dy/dx at $(2, 3)$, 1 for the correct conclusion (neither min nor max) with justification ($dy/dx \neq 0$ at that point).

Solution 4

(d)

Mark scheme

- If $y = mx + b$, then $dy/dx = d/dx(mx + b) = m$. Substituting into $2x - y = m$ gives $2x - (mx + b) = m$, i.e. $(2 - m)x - (m + b) = 0$ for all x, so $2 - m = 0 \quad m = 2$, and $-(m + b) = 0 \quad b = -m = -2$. Therefore $m = 2$ and $b = -2$. Points: 1 for $d/dx(mx + b) = m$, 1 for the equation $2x - y = m$ (substituting $y = mx + b$ into the differential equation, or equivalent), 1 for the correct answer $m = 2$, $b = -2$.