Past-paper walk-through

Using the Candidates Test to Determine Absolute (Global) Extrema ▪ 5.5

eXercise

Questions in this set

- 2026 Q4
- 2025 Q1
- 2025 Q4
- 2023 Q4
- 2022 Q1
- 2022 Q3
- 2019 Q1
- 2019 Q3
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- 2017 Q3

Questions in this set

- 2016 Q3
- 2015 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

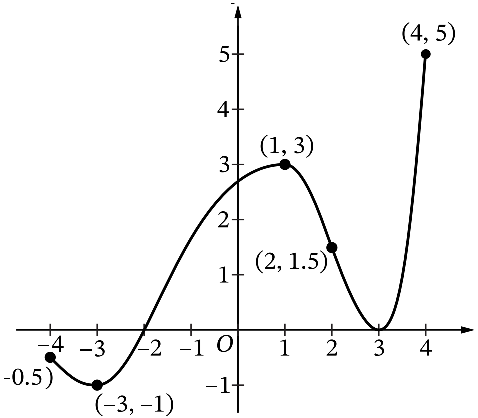
2026 ▪ Q4

Let f be a twice-differentiable function on the closed interval $[-4, 4]$ with $f(2) = 3$. The graph of f', the derivative of f, is shown.
[Graph of f': x-axis from -4 to 4, y-axis from -1 to 5. The curve passes through labeled points $(-4, -0.5)$, a local minimum at $(-3, -1)$, rises through the x-axis between $x = -2$ and $x = -1$, continues rising to a local maximum at $(1, 3)$, then decreases through $(2, 1.5)$, crosses the x-axis at $x = 3$ (touching down to a minimum of 0 at $x = 3$), then rises sharply to $(4, 5)$.]
(a) For $x > 0$, the function g is defined by $g(x) = f(x) - \ln x$. Find $g'(2)$. Show the work that leads to your answer.
(b) Find all values of x on the open interval $0 < x < 3$ at which the graph of f has a point of inflection. Give a reason for your answer.

Question 4

(c) For $-4 \leq x \leq 4$, on what open intervals, if any, is the graph of f both increasing and concave down? Give a reason for your answer.
(d) For $-4 \leq x \leq 4$, find the value of x at which f has an absolute minimum and the value of x at which f has an absolute maximum. Give reasons for your answers.

Question 4



Graph of f'

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(a) Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(b) Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \leq t \leq 4$. Show the setup for your calculations.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$. (Note: Your calculator should be in radian mode.)

(c) Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(d) At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)\,dx$, models the number of acres affected by the species over the time interval $4 \le t \le 36$. At what time t, for $4 \le t \le 36$, does A attain its maximum value? Justify your answer.

Solution 1

(a) Mark scheme

- Average value $= \dfrac{1}{4-0}\int_0^4 C(t)\,dt = \dfrac{1}{4}(11.112896) = 2.778224$, so the average number of acres affected from $t=0$ to $t=4$ is about 2.778 acres. Point 1: correct integral $\dfrac{1}{4-0}\int_0^4 C(t)\,dt$, with or without dt, with evidence of division by 4. Point 2: correct answer, accurate to 3 decimal places (2.778 or 2.778224), with or without supporting work.

Solution 1

(b) Mark scheme

- Average rate of change of C over $[0,4]$: $\dfrac{C(4)-C(0)}{4-0} = 1.282008$. Set $C'(t) = \dfrac{38}{25+t^2}$ equal to this: $\dfrac{38}{25+t^2} = 1.282008 \Rightarrow t = 2.154298 \approx 2.154$ weeks. Point 3: presents the average rate of change expression or value (e.g. $\dfrac{C(4)-C(0)}{4-0}$, $\dfrac{C(4)}{4}$, $\dfrac{5.128}{4}$, or 1.282). Point 4: correct answer $t \approx 2.154$, supported by the equation $C'(t) =$ average rate of change.

Solution 1

(c)

Mark scheme

- End behavior: $\lim_{t\to\infty} C'(t) = \lim_{t\to\infty} \dfrac{38}{25+t^2} = 0$. Point 5: correct limit expression, either $\lim_{t\to\infty} C'(t)$ or $\lim_{t\to\infty} C(t)$. Point 6: correct value 0 (only earnable if the limit expression used C', not C).

Solution 1

(d)

Mark scheme

- $A'(t) = C'(t) - 0.1 \ln t$. Set $A'(t) = 0$: $C'(t) = 0.1 \ln t \Rightarrow t = 11.441700$. Candidates test: $A(4) = 5.128031$, $A(11.4417) = 7.316978$, $A(36) = 1.743056$. The maximum is at $t = 11.442$ (or 11.441) weeks. Point 7: considers $A'(t) = 0$ (i.e., $C'(t) - 0.1 \ln t = 0$). Point 8: justification via a correct global (candidates-test) argument evaluating A at $t = 4$, $t = 11.4417$, and $t = 36$ (or an equivalent sign-analysis argument that $A'(t) > 0$ then $A'(t) < 0$). Point 9: correct answer $t \approx 11.442$ weeks, supported by work.

Question 4

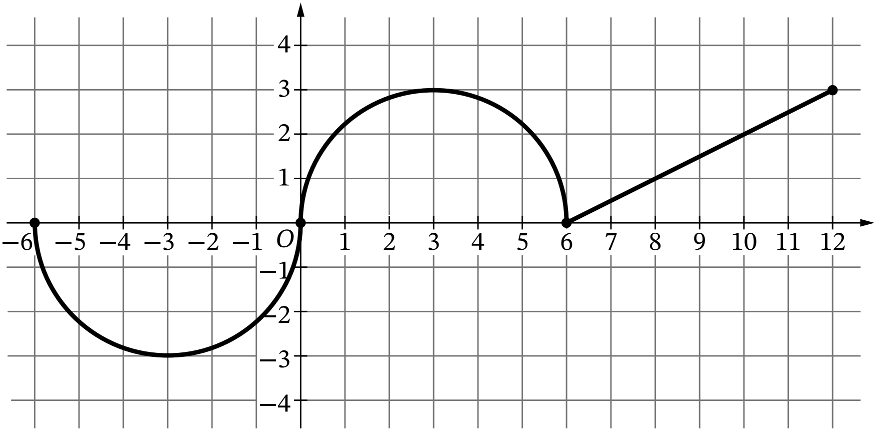
2025 ▪ Q4

The continuous function f is defined on the closed interval $-6 \leq x \leq 12$. The graph of f, consisting of two semicircles and one line segment, is shown in the figure.
[Graph of f: On the interval $-6 \leq x \leq 0$, the graph is a semicircle below the x-axis with endpoints at $(-6, 0)$ and $(0, 0)$, reaching a minimum near $(-3, -3)$. On the interval $0 \leq x \leq 6$, the graph is a semicircle above the x-axis with endpoints at $(0, 0)$ and $(6, 0)$, reaching a maximum near $(3, 3)$. On the interval $6 \leq x \leq 12$, the graph is a line segment from $(6, 0)$ to $(12, 3)$. The horizontal axis is labeled from -6 to 12 and the vertical axis from -4 to 4.]
Graph of f
Let g be the function defined by $g(x) = \int_6^x f(t)\, dt$.
(a) Find $g'(8)$. Give a reason for your answer.

Question 4

(b) Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.
(c) Find $g(12)$ and $g(0)$. Label your answers.
(d) Find the value of x at which g attains an absolute minimum on the closed interval $-6 \leq x \leq 12$. Justify your answer.

Question 4



Graph of f

Solution 4

(a)

Mark scheme

- By the Fundamental Theorem of Calculus, $g'(x) = f(x)$, so $g'(8) = f(8) = 1$ (read from the graph of f at $x = 8$, which is on the line segment). Point 1: considers $g'(x) = f(x)$ (or $g' = f$, or $g'(8) = f(8)$). Point 2: correct answer $g'(8) = 1$ (an implied application of the Fundamental Theorem of Calculus, e.g. just "$g'(8) = 1$" or "$f(8) = 1$", is also sufficient for this point alone).

Solution 4

(b) Mark scheme

- The graph of g has a point of inflection where $g'' = f'$ changes sign, i.e. where f changes from decreasing to increasing or vice versa (equivalently, where f attains a relative extremum, or where the slope of f changes sign). From the graph, this happens at $x = -3$, $x = 3$, and $x = 6$. Point 3: correct answer of exactly $x = -3$, $x = 3$, and $x = 6$ (if any other/additional values are declared, neither this point nor the reason point is earned; consideration of $x = -6$ or $x = 12$ does not impact scoring). Point 4: reason correctly tied to the given graph of f (e.g. "f changes from increasing to decreasing or decreasing to increasing there," or "f attains relative extrema there"; a reason relying on an ambiguous term like "the function" or "the graph" does not earn this point).

Solution 4

(c) Mark scheme

- $g(12) = \int_6^{12} f(t)\,dt = \frac{1}{2}(6)(3) = 9$ (area of the triangular line-segment region). $g(0) = \int_6^0 f(x)\,dx = -\int_0^6 f(x)\,dx = -\frac{\pi}{2}(3)^2 = -\frac{9\pi}{2}$ (negative of the semicircle area, since integrating from 6 down to 0). Point 5: $g(12) = 9$, with or without supporting work, correctly labeled as $g(12)$ (an unlabeled value does not earn the point). Point 6: $g(0) = -\frac{9\pi}{2}$, with or without supporting work, correctly labeled as $g(0)$.

Solution 4

(d) Mark scheme

- g attains a minimum on $[-6, 12]$ either where $g'(x) = f(x) = 0$ or at an endpoint: candidate points are $x = -6, 0, 6, 12$. Candidates test: $g(-6) = 0$, $g(0) = -\frac{9\pi}{2} \approx -14.137$, $g(6) = 0$, $g(12) = 9$. The absolute minimum value is attained at $x = 0$. Point 7: considers $g'(x) = 0$ or $f(x) = 0$ (not earned by just presenting $x = 0$ and $x = 6$ with no justification). Point 8: justification via a correct global (candidates-test) argument evaluating g at all four candidate points $-6, 0, 6, 12$ (values for $g(0)$ and $g(12)$ may be imported from part C); alternatively, an argument that $g'(x) \leq 0$ (or $f(x) \leq 0$) for $-6 \leq x < 0$ and $g'(x) \geq 0$ (or $f(x) \geq 0$) for $0 < x \leq 12$. Point 9: correct answer $x = 0$; requires

Solution 4

Point 8 (a local-argument response, e.g. First/Second Derivative Test, or an incorrect global argument, does not earn Point 8 but remains eligible for Point 9 with the correct answer).

Question 4

2023 ▪ Q4

The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.
[Graph of f': x-axis from -2 to 8 with gridlines at every integer, y-axis from -2 to 2 with gridlines at every integer. The graph consists of: a line segment from $(-2, 2)$ down to $(0, -2)$; a line segment from $(0, -2)$ up to $(4, 2)$; and a semicircle (bulging downward, below the x-axis) connecting $(4, 2)$ down through the x-axis and back up to $(8, 2)$, dipping to a minimum of about 0 at $x = 6$. Solid dots are marked at $(-2, 2)$, $(0, -2)$, $(4, 2)$, and $(8, 2)$.]
(a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

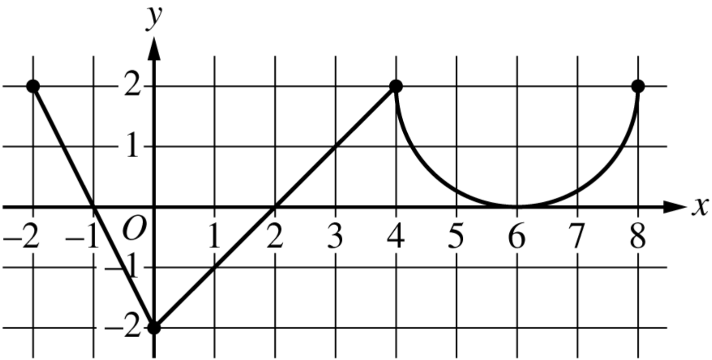
Question 4

(b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

(c) Find the value of $\lim_{x \to 2} \dfrac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

(d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

Question 4



Graph of f'

Solution 4

(a)

Mark scheme

- $f'(x) > 0$ on (2,6) and $f'(x) > 0$ on (6,8), so f'(x) does not change sign at x=6; therefore f has neither a relative maximum nor a relative minimum at x=6 (1 pt: correct answer — 'neither' — with the reason that f'(x) does not change sign at x=6).

Solution 4

(b)

Mark scheme

- The graph of f is concave down on (-2,0) and (4,6) because f' is decreasing on these intervals (1 pt: correct intervals (-2,0) and (4,6) — with or without endpoints; 1 pt: reason, must correctly discuss the behavior/slopes of f').

Solution 4

(c) Mark scheme

- Because f is differentiable, hence continuous, at x=2, $\lim_{x\to 2} f(x) = f(2) = 1$; so $\lim_{x\to 2}(6f(x) - 3x) = 6(1) - 3(2) = 0$ and $\lim_{x\to 2}(x^2 - 5x + 6) = 0$ (1 pt: both limits of numerator and denominator shown to equal 0; a limit presented as literally equal to 0/0 does not earn this point). Since the limit is of indeterminate form 0/0, L'Hospital's Rule applies (1 pt: uses L'Hospital's Rule — at least one correct derivative shown in the limit of a ratio of derivatives). $\lim_{x\to 2} \dfrac{6f(x) - 3x}{x^2 - 5x + 6} = \lim_{x\to 2} \dfrac{6f'(x) - 3}{2x - 5} = \dfrac{6(0) - 3}{2(2) - 5} = \dfrac{-3}{-1} = 3$ (1 pt: correct answer 3 with supporting work).

Solution 4

(d)

Mark scheme

- $f'(x) = 0 \Rightarrow x = -1, 2, 6$ (1 pt: considers f'(x)=0 — merely listing the zeros of f' without stating f'=0 is not sufficient). Since f is continuous on [-2,8], the candidates for the location of an absolute minimum are x=-2,-1,2,6,8, with $f(-2) = 3$, $f(-1) = 4$, $f(2) = 1$, $f(6) = 7 - \pi$, $f(8) = 11 - 2\pi$ (1 pt: justification — evaluate f at all critical points and both endpoints, or validly eliminate some by argument). The absolute minimum value of f is $f(2) = 1$ (1 pt: answer — must state the minimum VALUE 1, not merely its location x=2).

Question 1

2022 ▪ Q1

From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

(a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

(b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

(c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

(d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t, for $a \leq t \leq 4$, is given by

Question 1

$$N(t) = \int_a^t (A(x) - 400)\, dx,$$

where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \le t \le 4$. Justify your answer.

Solution 1

(a)

Mark scheme

- The total number of vehicles arriving from 6 A.M. to 10 A.M. is given by the definite integral $\int_1^5 A(t)\,dt$. Full credit requires a definite integral with the correct lower limit $t = 1$ and upper limit $t = 5$; the integrand should be $A(t)$ (equivalently $|A(t)|$, since $A(t) \geq 0$ on this interval). Do not evaluate the integral.

Solution 1

(b) Mark scheme

- Average value $= \dfrac{1}{5-1}\int_1^5 A(t)\,dt = \dfrac{1}{4}(1502.147865) = 375.536966.$ The average rate at which vehicles arrive at the toll plaza from 6 A.M. to 10 A.M. is about 375.537 (or 375.536) vehicles per hour. One point for using the correct average value formula $\frac{1}{b-a}\int_a^b A(t)\,dt$ with $a = 1$, $b = 5$; one point for the correct answer, accurate to three decimal places.

Solution 1

(c)

Mark scheme

- $A'(1) = 148.947272 > 0$, so the rate at which vehicles arrive at the toll plaza is increasing at $t = 1$ (6 A.M.). One point for considering $A'(1)$; one point for the correct answer (increasing) together with the reason $A'(1) > 0$.

Solution 1

(d)

Mark scheme

- $N'(t) = A(t) - 400 = 0 \Rightarrow A(t) = 400 \Rightarrow t = a = 1.469372$ or $t = b = 3.597713$. Evaluating $N(t) = \int_a^t (A(x) - 400)\,dx$ at the critical points and the right endpoint: $N(a) = 0$, $N(b) = 71.254129$, $N(4) = 62.338346$. The greatest number of vehicles in line is 71 (attained at $t = b$). Points: (1) considers $N'(t) = 0$; (2) correctly finds both $t = a$ and $t = b$; (3) correct answer of 71 (or 71.254...); (4) justification via a candidates-test table comparing $N(a)$, $N(b)$, and $N(4)$.

Question 3

2022 ▪ Q3

[Graph titled "Graph of f'": a coordinate plane with x-axis from 0 to 7 (integer gridlines at $1, 2, 3, 4, 5, 6, 7$) and y-axis from -2 to 2 (gridlines at $-2, -1, 1, 2$). The graph consists of a semicircle below the x-axis from $x = 0$ to $x = 4$, dipping down to a minimum of about -2 near $x = 2$, returning to 0 at $x = 4$. From $x = 4$ to $x = 6$ a line segment rises from $(4, 0)$ to the marked point $(6, 2)$. From $x = 6$ to $x = 7$ a line segment falls from $(6, 2)$ to the marked point $(7, 1)$.]

Let f be a differentiable function with $f(4) = 3$. On the interval $0 \leq x \leq 7$, the graph of f', the derivative of f, consists of a semicircle and two line segments, as shown in the figure above.

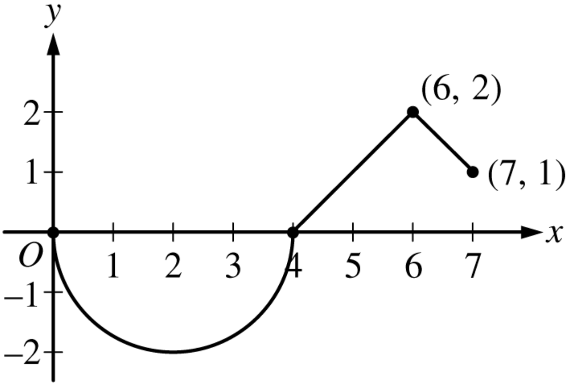
(a) Find $f(0)$ and $f(5)$.

(b) Find the x-coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

Question 3

(c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \leq x \leq 7$? Show the analysis that leads to your answer.
(d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \leq x \leq 7$. Justify your answer.

Question 3



Graph of f'

Solution 3

(a)

Mark scheme

- $f(0) = f(4) + \int_4^0 f'(x)\,dx = 3 - \int_0^4 f'(x)\,dx = 3 + 2\pi$ (the semicircle over $[0, 4]$ has radius 2 and lies below the axis, so $\int_0^4 f'(x)\,dx = -2\pi$). $f(5) = f(4) + \int_4^5 f'(x)\,dx = 3 + \frac{1}{2} = \frac{7}{2}$. Points: (1) area of either region ($\int_0^4 f'$ or $\int_4^5 f'$); (2) correct $f(0) = 3 + 2\pi$; (3) correct $f(5) = 7/2$.

Solution 3

(b)

Mark scheme

- The graph of f has points of inflection at $x = 2$ and $x = 6$, because f' changes from decreasing to increasing at $x = 2$ (the bottom of the semicircle) and from increasing to decreasing at $x = 6$ (the corner/peak of the graph of f' at $(6, 2)$). Points: (1) states both $x = 2$ and $x = 6$; (2) correct justification tied to the graph of f' (e.g., discussing where the slope/behavior of f' changes, or citing $x = 2, 6$ as locations of local extrema on the graph of f').

Solution 3

(c)

Mark scheme

- $g(x) = f(x) - x \Rightarrow g'(x) = f'(x) - 1$. Since $g'(x) \leq 0 \iff f'(x) \leq 1$, and from the graph $f'(x) \leq 1$ for $0 \leq x \leq 5$, the graph of g is decreasing on the interval $0 \leq x \leq 5$ (because $g'(x) \leq 0$ there). Points: (1) correct $g'(x) = f'(x) - 1$ (or equivalent); (2) correct interval $[0, 5]$ with reason.

Solution 3

(d) Mark scheme

- g is continuous, $g'(x) < 0$ for $0 < x < 5$, and $g'(x) > 0$ for $5 < x < 7$, so the absolute minimum occurs at $x = 5$: $g(5) = f(5) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$. The absolute minimum value of g on $[0, 7]$ is $-\frac{3}{2}$. (Alternative: a Candidates Test comparing $g(0) = 3 + 2\pi$, $g(5) = -3/2$, $g(7) = -1/2$ gives the same result.) Points: (1) considers $g'(x) = 0$, isolating $x = 5$ as the only critical number on $(0, 7)$; (2) correct answer $-3/2$ with justification.

Question 1

2019 ▪ Q1

Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

(a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)? Give your answer to the nearest whole number.

(b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)?

(c) At what time t, for $0 \leq t \leq 8$, is the greatest number of fish in the lake? Justify your answer.

Question 1

(d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$)? Explain your reasoning.

Solution 1

(a)

Mark scheme

- $\int_0^5 E(t)\,dt = 153.457690$ (calculator value). To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M. Points: 1 for the integral expression, 1 for the correct answer 153.

Solution 1

(b)

Mark scheme

- Average rate $= \dfrac{1}{5-0}\int_0^5 L(t)\,dt = 6.059038$. The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour. Points: 1 for the integral expression, 1 for the correct answer 6.059.

Solution 1

(c) Mark scheme

- The rate of change in the number of fish in the lake at time t is $E(t) - L(t)$. Setting $E(t) - L(t) = 0$ gives $t = 6.20356$. Since $E(t) - L(t) > 0$ for $0 \leq t < 6.20356$ and $E(t) - L(t) < 0$ for $6.20356 < t \leq 8$, the number of fish increases then decreases, so the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203). [Alternate: let $A(t) = \int_0^t (E(s) - L(s))\, ds$ be the change in fish since midnight; $A'(t) = E(t) - L(t) = 0$ at $t = C = 6.20356$; compare $A(0) = 0$, $A(C) = 135.01492$, $A(8) = 80.91998$ — the maximum is at $t = C$.] Points: 1 for setting $E(t) - L(t) = 0$ (or the equivalent candidates-test setup), 1 for the answer $t = 6.204$ (or 6.203), 1 for justification (sign analysis or candidates test).

Solution 1

(d)

Mark scheme

- $E'(5) - L'(5) = -10.7228 < 0$. Since the rate of change in the number of fish is $E(t) - L(t)$, its derivative at $t = 5$ is $E'(5) - L'(5)$, which is negative, so the rate of change in the number of fish in the lake is DECREASING at $t = 5$ A.M. Points: 1 for considering $E'(5)$ and $L'(5)$, 1 for the correct answer (decreasing) with explanation.

Question 3

2019 ▪ Q3

The continuous function f is defined on the closed interval $-6 \leq x \leq 5$. The figure below shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3,\ 3-\sqrt{5})$ is on the graph of f.
[Graph of f; x-axis from -2 to 5 (gridlines shown from -2 to 5), y-axis from -1 to 3. A line segment goes from a closed point at $(-2, 1)$ down to a closed point at the origin $(0, 0)$. From $(0, 0)$ a line segment rises to a closed point at $(2, 3)$. From $(2, 3)$ a curve (quarter circle centered at $(5, 3)$) decreases and flattens out, passing through $(3, 3-\sqrt{5})$, ending at a closed point at $(5, 0)$ on the x-axis. Labeled "Graph of f".]
(a) If $\displaystyle\int_{-6}^{5} f(x)\,dx = 7$, find the value of $\displaystyle\int_{-6}^{-2} f(x)\,dx$. Show the work that leads to your answer.

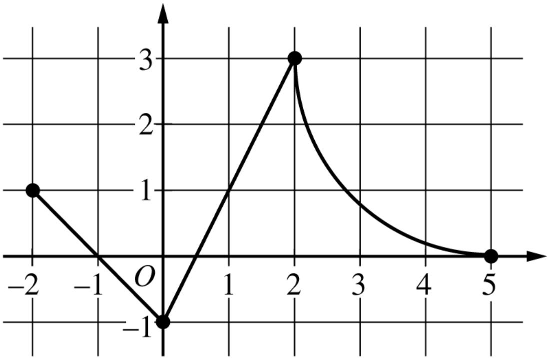
Question 3

(b) Evaluate $\int_3^5 \left(2f'(x) + 4\right) dx$.

(c) The function g is given by $g(x) = \int_{-2}^{x} f(t)\, dt$. Find the absolute maximum value of g on the interval $-2 \leq x \leq 5$. Justify your answer.

(d) Find $\lim_{x \to 1} \dfrac{10^x - 3f'(x)}{f(x) - \arctan x}$.

Question 3



Graph of f

Solution 3

(a) Mark scheme

- $\int_{-6}^{5} f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx$. Given $\int_{-6}^{5} f(x)\,dx = 7$ and (from the given graph of f) $\int_{-2}^{5} f(x)\,dx = 2 + \left(9 - \frac{9\pi}{4}\right) = 11 - \frac{9\pi}{4}$, so $7 = \int_{-6}^{-2} f(x)\,dx + \left(11 - \frac{9\pi}{4}\right)$, giving $\int_{-6}^{-2} f(x)\,dx = 7 - \left(11 - \frac{9\pi}{4}\right) = \frac{9\pi}{4} - 4$. Points: 1 for splitting the integral correctly ($\int_{-6}^{5} = \int_{-6}^{-2} + \int_{-2}^{5}$), 1 for the correct value of $\int_{-2}^{5} f(x)\,dx$, 1 for the final answer $\frac{9\pi}{4} - 4$.

Solution 3

(b)

Mark scheme

- $\int_3^5 (2f'(x) + 4)\, dx = 2\int_3^5 f'(x)\, dx + \int_3^5 4\, dx = 2(f(5) - f(3)) + 4(5-3) = 2\left(0 - (3 - \sqrt{5})\right) + 8 = 2(-3 + \sqrt{5}) + 8 = 2 + 2\sqrt{5}$. Points: 1 for using the Fundamental Theorem of Calculus (i.e. $2(f(5) - f(3))$ term), 1 for the final answer $2 + 2\sqrt{5}$.

Solution 3

(c) Mark scheme

- $g(x) = \int_{-2}^{x} f(t)\,dt$, so $g'(x) = f(x)$. $g'(x) = 0$ at $x = -1,\ x = \frac{1}{2},\ x = 5$ (the critical points/endpoints from the given graph of f on $[-2, 5]$). Evaluating g at the candidates: $g(-2) = 0$, $g(-1) = \frac{1}{2}$, $g(\frac{1}{2}) = -\frac{1}{4}$, $g(5) = 11 - \frac{9\pi}{4}$. On $-2 \leq x \leq 5$, the absolute maximum value of g is $g(5) = 11 - \frac{9\pi}{4}$. Points: 1 for $g'(x) = f(x)$, 1 for identifying $x = -1$ as a candidate (local max, since f changes from positive to negative there), 1 for the correct answer $g(5) = 11 - \frac{9\pi}{4}$ with justification (candidates test/table).

Solution 3

(d) Mark scheme

- $\lim_{x\to 1} \dfrac{10^x - 3f'(x)}{f(x) - \arctan x} = \dfrac{10^1 - 3f'(1)}{f(1) - \arctan 1} = \dfrac{10 - 3\cdot 2}{1 - \arctan 1} = \dfrac{4}{1 - \frac{\pi}{4}}$. Points: 1 for the correct final answer, $\dfrac{4}{1-\pi/4}$.

Question 5

2018 ▪ Q5

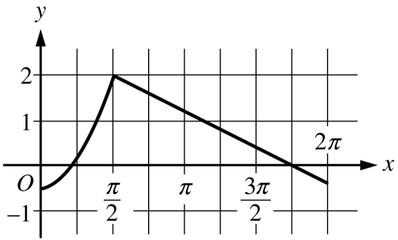
Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

Question 5

Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', th

below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify



Graph of g'

Question 5

2018 ▪ Q5

Let f be the function defined by $f(x) = e^x \cos x$.

(b) What is the slope of the line tangent to the graph of f at $x = \dfrac{3\pi}{2}$?

Question 5

2018 ▪ Q5

Let f be the function defined by $f(x) = e^x \cos x$.

(c) Find the absolute minimum value of f on the interval $0 \leq x \leq 2\pi$. Justify your answer.

Question 5

2018 ▪ Q5

Let f be the function defined by $f(x) = e^x \cos x$.

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.

[Graph of g': axes with x-axis labeled at O, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, 2π, and y-axis labeled -1, 1, 2. The curve begins slightly below 0 near $x = 0$, curves upward crossing the x-axis before $x = \frac{\pi}{2}$, rises sharply to a peak at $\left(\frac{\pi}{2}, 2\right)$, then decreases linearly, crossing the x-axis near $x = \frac{3\pi}{2}$, and continues down to a value slightly below 0 at $x = 2\pi$.]

Graph of g'

Solution 5

(a)

Mark scheme

- The average rate of change of f on $0 \leq x \leq \pi$ is $\dfrac{f(\pi) - f(0)}{\pi - 0} = \dfrac{-e^{\pi} - 1}{\pi}$. (1 pt: the answer.)

Solution 5

(b)

Mark scheme

- $f'(x) = e^x \cos x - e^x \sin x$. $f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$. The slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$ is $e^{3\pi/2}$. (1 pt: correct $f'(x)$; 1 pt: the slope $e^{3\pi/2}$.)

Solution 5

(c) Mark scheme

- $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$. Candidates test: $f(0) = 1$, $f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}e^{\pi/4}$, $f\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}e^{5\pi/4}$, $f(2\pi) = e^{2\pi}$. The absolute minimum value of f on $0 \leq x \leq 2\pi$ is $-\frac{1}{\sqrt{2}}e^{5\pi/4}$. (1 pt: sets $f'(x) = 0$; 1 pt: identifies $x = \frac{\pi}{4}, \frac{5\pi}{4}$ as candidates; 1 pt: the answer with justification via the candidates test.)

Solution 5

(d) Mark scheme

- $\lim_{x\to\pi/2} f(x) = 0$. Because g is differentiable, g is continuous, so $\lim_{x\to\pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$. By L'Hospital's Rule, $\lim_{x\to\pi/2} \dfrac{f(x)}{g(x)} = \lim_{x\to\pi/2} \dfrac{f'(x)}{g'(x)} = \dfrac{-e^{\pi/2}}{2}$. (1 pt: g continuous at $x = \frac{\pi}{2}$ and both limits equal 0; 1 pt: applies L'Hospital's Rule; 1 pt: the answer $\frac{-e^{\pi/2}}{2}$. Note: max $1/3$ if no limit notation is attached to a ratio of derivatives.)

Question 3

2017 ▪ Q3

[Graph of f', labeled "Graph of f'"; consists of a semicircle and three line segments. x-axis x, y-axis y. A line segment starts at the labeled point $(-6, 2)$ and decreases linearly to the point $(-2, 0)$ on the x-axis. From $(-2, 0)$ to $(0, 0)$ the graph is a semicircle dipping below the x-axis (minimum around $(-1, -1)$) and returning to the x-axis at $(0, 0)$. From $(0, 0)$ the graph rises linearly to the labeled point $(3, 2)$. From $(3, 2)$ the graph decreases linearly to the point $(5, 0)$ on the x-axis. Gridlines/tick marks are shown at integer x-values and at $y = 1$.]

The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of $f(-6)$ and $f(5)$.

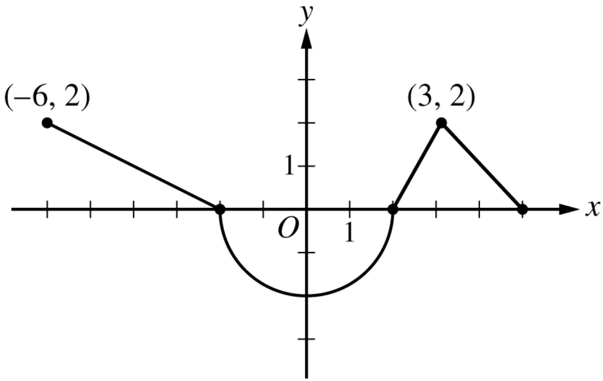
(b) On what intervals is f increasing? Justify your answer.

Question 3

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

Question 3



Graph of f'

Question 3

2016 ▪ Q3

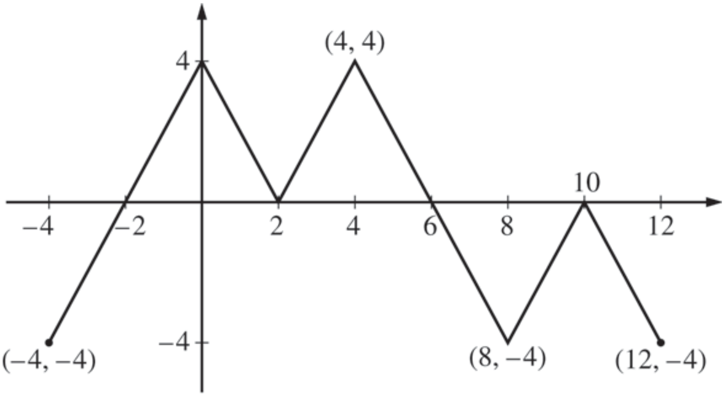
[Graph of the piecewise-linear function f, labeled "Graph of f"; x-axis from -4 to 12 with gridlines/labels at $-4, -2, 2, 4, 6, 8, 10, 12$; y-axis showing 4 and -4. The graph consists of straight line segments connecting the labeled points: $(-4, -4)$ up to $(0, 4)$, down to $(2, 0)$, up to $(4, 4)$, down to $(8, -4)$, up to $(10, 0)$, down to $(12, -4)$.]
The figure above shows the graph of the piecewise-linear function f. For $-4 \leq x \leq 12$, the function g is defined by $g(x) = \displaystyle\int_2^x f(t)\,dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.
(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.
(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.

Question 3

(d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.

Question 3



Graph of f

Solution 3

(a)

Mark scheme

- Since g(x)=$\int_2^x f(t)\, dt, g'(x) = f(x)[thisrecognitionpointmaybeearnedinpart(a), (b), (c), or(d)-10, sinceg'(x) = f(x) andf(x) \leq 0(doesnotchangesign)for8 \leq x \leq 12. Points : 1forrecognizingg'(x) = f(x)(awardhereifnotalreadycreditedinanotherpart), 1forthecorrectanswer(neither)withvalidjus$

Solution 3

(b)

Mark scheme

- The graph of g has a point of inflection at x=4, since g'(x)=f(x) is increasing for 2$\leq x \leq 4 anddecreasingfor 4 \leq x \leq 8 (sog' changesfromincreasingtodecreasingatx = 4, meaningg changesconcavitythere). Points : 1 forthecorrectanswer(yes) withvalidjustification.$

Solution 3

(c)

Mark scheme

- g'(x)=f(x) changes sign only at x=-2 and x=6. Evaluating g at the candidates and endpoints: g(-4)=-4, g(-2)=-8, g(6)=8, g(12)=-4. On -4$\leq$ x $\leq$ $12, theabsoluteminimumvalueisg(-2) = -8andtheabsolutemaximumvalueisg(6) = 8.Points : 1foridentifyingx = -2andx = 6ascandidates(whereg' changessign), 1foralsoconsideringtheendpointsx = -4andx = 12, 2forthecorrectabsoluteminimumandmaximumvalueswithjustification.$

Solution 3

(d)

Mark scheme

- g(x)$\leq 0 for -4 \leq x \leq 2 and 10 \leq x \leq 12, readfromthevalues/signofgestablishedinpart(c)togetherwiththegraphoff. Points : 2forthecorrectpairofintervals.$

Question 1

2015 ▪ Q1

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \leq t \leq 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \leq t \leq 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \leq t \leq 8$, is the amount of water in the pipe at a minimum? Justify your answer.

Question 1

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

Solution 1

(a)

Mark scheme

- $\int$ R(t) dt $=$ 76.570 cubic feet. Points: 1 for the integrand R(t) with correct limits of integration 0 to 8, 1 for the correct numeric answer 76.570.

Solution 1

(b)

Mark scheme

- $R(3) - D(3) = -0.313632 < 0$. Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at $t = 3$ hours. Points: 1 for considering (evaluating) both $R(3)$ and $D(3)$, 1 for the correct answer (decreasing) with the reason $R(3) < D(3)$.

Solution 1

(c)

Mark scheme

- The amount of water in the pipe at time t, $0 \leq t \leq 8$, is $W(t) = 30 + \int [R(x) - D(x)]\,dx$. Setting $R(t) - D(t) = 0$ gives $t = 0$ or $t = 3.271658$. Evaluating W at these candidates and the endpoint: $W(0) = 30$, $W(3.271658) = 27.964561$, $W(8) = 48.543686$. The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours. Points: 1 for considering $R(t) - D(t) = 0$, 1 for the correct answer $t \approx 3.271$ or 3.272 hours, 1 for justification (comparing W at the candidate point and endpoints, e.g. via a table, to show it is the minimum).

Solution 1

(d)

Mark scheme

- $30 + \int \hat{}w\ [R(t) - D(t)]\ dt = 50$ (equation only — do not solve). Points: 1 for the correct integral $\int \hat{}w\ [R(t) - D(t)]\ dt$, 1 for the complete equation (starting quantity 30 plus that integral set equal to 50).