Exercise walk-through

Extreme Value Theorem, Global Versus Local Extrema, and Critical Points ▪ 5.2

eXercise

You must be able to

- state the **Extreme Value Theorem** 极值定理
- find the **critical points** 临界点 of a function

Method — EVT and critical points

Extreme Value Theorem: a function continuous on a closed interval $[a, b]$ attains a global maximum and minimum.
Critical points are where $f'(x) = 0$ or f' does not exist — the candidates for local extrema.

Method — Example

$f(x) = x^2 - 4x$: $f'(x) = 2x - 4 = 0$ gives the critical point $x = 2$.

Practice 2.1 ▪ 1 mark

State the Extreme Value Theorem.

Solution 2.1

Method: EVT and critical points

Worked steps

a function continuous on a closed interval attains a global max and min B1.

Practice 2.2 ▪ 1 mark

State what a critical point is.

Solution 2.2

Method: Example

Worked steps

a point where $f' = 0$ or f' does not exist **B1**.

Practice 2.3 ▪ 2 marks

Find the critical point of $f(x) = x^2 - 4x$.

Solution 2.3

Method: Example

Worked steps

$f'(x) = 2x - 4 = 0$, so $x = 2$ M1 A1.

Exam-style 3.1 ▪ 1 mark

A critical point occurs where

A $f = 0$

B $f' = 0$ or f' does not exist

C $f'' = 0$

D $x = 0$

Solution 3.1

A $f = 0$

B $f' = 0$ or f' does not exist

C $f'' = 0$

D $x = 0$

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

The EVT guarantees extrema when f is continuous on

A an open interval

B a closed interval

C all real numbers

D no interval

Solution 3.2

Method: EVT and critical points

A an open interval

B a closed interval ✓

C all real numbers

D no interval

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$f(x) = x^2 - 6x$.
(a) Find $f'(x)$.
(b) Set $f' = 0$.
(c) State the critical point.

Solution 3.3

Method: Example

Worked steps

(a) $2x - 6$ B1. (b) $2x - 6 = 0$ B1. (c) $x = 3$ B1.