

Past-paper walk-through

Using the Mean Value Theorem ■ 5.1

eXercise

Questions in this set

- 2025 Q1
- 2023 Q1
- 2021 Q4
- 2019 Q2
- 2018 Q4
- 2017 Q6
- 2014 Q5

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- **//** separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 1

2025 ■ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6 \arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be

shown that $C'(t) = \frac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(a) Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

Question 1

2025 ■ Q1

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(b) Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \leq t \leq 4$. Show the setup for your calculations.

Question 1

2025 ■ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6 \arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \frac{38}{25 + t^2}$. (Note: Your calculator should be in radian mode.)

(c) Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

Question 1

2025 ■ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6 \arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \frac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(d) At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A , defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x) \, dx$, models the number of acres affected by the species over the time interval $4 \leq t \leq 36$. At what time t , for $4 \leq t \leq 36$, does A attain its maximum value? Justify your answer.

Solution 1

(a) Mark scheme

- Average value = $\frac{1}{4-0} \int_0^4 C(t) dt = \frac{1}{4}(11.112896) = 2.778224$, so the average number of acres affected from $t = 0$ to $t = 4$ is about 2.778 acres. Point 1: correct integral $\frac{1}{4-0} \int_0^4 C(t) dt$, with or without dt , with evidence of division by 4. Point 2: correct answer, accurate to 3 decimal places (2.778 or 2.778224), with or without supporting work.

Solution 1

(b) Mark scheme

- Average rate of change of C over $[0, 4]$: $\frac{C(4) - C(0)}{4 - 0} = 1.282008$. Set $C'(t) = \frac{38}{25 + t^2}$ equal to this: $\frac{38}{25 + t^2} = 1.282008 \Rightarrow t = 2.154298 \approx 2.154$ weeks. Point 3: presents the average rate of change expression or value (e.g. $\frac{C(4) - C(0)}{4 - 0}$, $\frac{C(4)}{4}$, $\frac{5.128}{4}$, or 1.282). Point 4: correct answer $t \approx 2.154$, supported by the equation $C'(t) = \text{average rate of change}$.

Solution 1

(c)

Mark scheme

- End behavior: $\lim_{t \rightarrow \infty} C'(t) = \lim_{t \rightarrow \infty} \frac{38}{25 + t^2} = 0$. Point 5: correct limit expression, either $\lim_{t \rightarrow \infty} C'(t)$ or $\lim_{t \rightarrow \infty} C(t)$. Point 6: correct value 0 (only earnable if the limit expression used C' , not C).

Solution 1

(d)

Mark scheme

- $A'(t) = C'(t) - 0.1 \ln t$. Set $A'(t) = 0$: $C'(t) = 0.1 \ln t \Rightarrow t = 11.441700$. Candidates test: $A(4) = 5.128031$, $A(11.4417) = 7.316978$, $A(36) = 1.743056$. The maximum is at $t = 11.442$ (or 11.441) weeks. Point 7: considers $A'(t) = 0$ (i.e., $C'(t) - 0.1 \ln t = 0$). Point 8: justification via a correct global (candidates-test) argument evaluating A at $t = 4$, $t = 11.4417$, and $t = 36$ (or an equivalent sign-analysis argument that $A'(t) > 0$ then $A'(t) < 0$). Point 9: correct answer $t \approx 11.442$ weeks, supported by work.

Question 1

2023 ■ Q1

A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f , where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

Question 1

(a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t) dt$ in the context of the problem. Use a right Riemann sum with the three subintervals $[60, 90]$, $[90, 120]$, and $[120, 135]$ to approximate the value of $\int_{60}^{135} f(t) dt$.

Question 1

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