

Exercise walk-through

Introduction to Related Rates ■ 4.4

eXercise

You must be able to

- differentiate a relation with respect to **time** 时间 to link two rates

Method — Related rates

When two quantities are related, their rates are related: differentiate the relation with respect to t (using the chain rule).

Method — Example

$$A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}.$$

Practice 2.1 ■ 1 mark

State the first step in a related-rates problem.

Solution 2.1

Method: Related rates

Worked steps

write the relation connecting the quantities **B1**.

Practice 2.2 ■ 2 marks

Differentiate $A = \pi r^2$ with respect to t .

Solution 2.2

Method: Related rates

Worked steps

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} \quad \text{M1} \quad \text{A1}.$$

Practice 2.3 ■ 1 mark

State what links the two rates.

Solution 2.3

Worked steps

the derivative of the relation with respect to t **B1**.

Exam-style 3.1 ■ 1 mark

In related rates, you differentiate the relation with respect to

A x

B time t

C r

D nothing

Solution 3.1

Method: Related rates

A x

B time t



C r

D nothing

Worked steps

B.

Exam-style 3.2 ■ 1 mark

Differentiating $A = \pi r^2$ with respect to t gives

A $2\pi r$

B $2\pi r \frac{dr}{dt}$

C πr^2

D $2r$

Solution 3.2

A $2\pi r$

B $2\pi r \frac{dr}{dt}$



C πr^2

D $2r$

Worked steps

B.

Exam-style 3.3 ■ 1 mark

$V = s^3$ (a cube).

(a) Differentiate with respect to t .

(b) State the chain-rule factor.

(c) Write $\frac{dV}{dt}$.

Solution 3.3

Method: Related rates

Worked steps

$$(a) \frac{dV}{dt} = 3s^2 \frac{ds}{dt} \quad \text{B1}. \quad (b) \frac{ds}{dt} \quad \text{B1}. \quad (c) \frac{dV}{dt} = 3s^2 \frac{ds}{dt} \quad \text{B1}.$$