Past-paper walk-through

The Chain Rule ▪ 3.1

eXercise

Questions in this set

- 2026 Q6
- 2025 Q5
- 2023 Q1
- 2023 Q5
- 2017 Q6
- 2016 Q6
- 2014 Q3
- 2014 Q5

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 6

2026 ▪ Q6

The function f is twice differentiable. The table gives values of f and its derivative f' at selected values of x.

x	**0**	**2**	**3**	**6**
$f(x)$	-1	3	8	5
$f'(x)$	-5	4	9	-2

Question 6

(a) Find $\lim_{x\to 2} \frac{f(x)}{x}$, or state that the limit does not exist.
(b) Let $g(x) = f(f(x))$. Find $g'(2)$. Show the work that leads to your answer.
(c) Let h be a differentiable function such that $h(0) = 10$ and $h'(x) = f'(3x)$. Find $h(2)$. Show the work that leads to your answer.
(d)(i) Let k be the function defined by $k(x) = \int_0^x t^2 f(t)\,dt$. Find $k'(x)$.
(d)(ii) Find $k''(3)$. Show the work that leads to your answer.

Question 6

x	0	2	3	6
$f(x)$	-1	3	8	5
$f'(x)$	-5	4	9	-2

Question 5

2025 ▪ Q5

Two particles, H and J, are moving along the x-axis. For $0 \leq t \leq 5$, the position of particle H at time t is given by $x_H(t) = e^{t^2-4t}$ and the velocity of particle J at time t is given by $v_J(t) = 2t(t^2-1)^3$.

(a) Find the velocity of particle H at time $t = 1$. Show the work that leads to your answer.

(b) During what open intervals of time t, for $0 < t < 5$, are particles H and J moving in opposite directions? Give a reason for your answer.

(c) It can be shown that $v_J'(2) > 0$. Is the speed of particle J increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

(d) Particle J is at position $x = 7$ at time $t = 0$. Find the position of particle J at time $t = 2$. Show the work that leads to your answer.

Solution 5

(a)

Mark scheme

- $x_H'(t) = v_H(t) = (2t-4)e^{t^2-4t}$. At $t=1$: $x_H'(1) = (2-4)e^{1-4} = -2e^{-3}$. Point 1: considers/presents $x_H'(t)$ (or $v_H(t)$, $x'(t)$, $x'(1)$, $(2t-4)e^{t^2-4t}$, or the unsimplified value at $t=1$, e.g. $(2\cdot 1-4)e^{1^2-4\cdot 1}$). Point 2: correct answer $-2e^{-3}$ (an unsupported answer of $-2e^{-3}$ alone earns this point but not Point 1).

Solution 5

(b) Mark scheme

- $x_H'(t) = (2t-4)e^{t^2-4t} = 0 \Rightarrow t = 2$ (only critical point on $0 < t < 5$); $x_H'(t) < 0$ for $0 < t < 2$ and $x_H'(t) > 0$ for $2 < t < 5$, so H moves left then right. $v_J(t) = 2t(t^2-1)^3 = 0$ for $0 < t < 5 \Rightarrow t = 1$; $v_J(t) < 0$ for $0 < t < 1$ and $v_J(t) > 0$ for $1 < t < 5$, so J moves left then right. H and J move in opposite directions exactly where their directions differ: for $1 < t < 2$ (H moving left, J moving right). Point 3: considers the sign of $x_H'(t)$ or $v_J(t)$ (sets it/the expression equal to 0, or correctly identifies its critical time $t = 2$ for H or $t = 1$ for J with no other values in $0 < t < 5$, or identifies the interval $1 < t < 2$). Point 4: correct direction-of-motion analysis on $0 < t < 5$ for one particle (H or J). Point 5:

Solution 5

correct answer $1 < t < 2$ with reasoning; requires correct analysis for BOTH particles on $0 < t < 5$.

Solution 5

(c)

Mark scheme

- $v_J(2) = 2(2)((2)^2 - 1)^3 = 108 > 0$, and $v'_J(2) > 0$ is given. Because $v_J(2)$ and $v'_J(2)$ have the same sign, the speed of particle J is increasing at $t = 2$. Point 6: correct answer (increasing) with correct reasoning that $v_J(2)$ and $v'_J(2)$ have the same sign (an explicit numeric evaluation of $v_J(2)$ is not required, but if given must be correct: $v_J(2) = 108$; the sign analysis for $v_J(t) > 0$ on $1 < t < 5$ from part B may be reused without restating $v_J(2) > 0$).

Solution 5

(d)

Mark scheme

- $x_J(2) = x_J(0) + \int_0^2 v_J(t)\,dt = 7 + \int_0^2 2t(t^2-1)^3\,dt = 7 + \left[\frac{1}{4}(t^2-1)^4\right]_0^2 = 7 + \frac{1}{4}\left((3)^4 - (-1)^4\right) = 7 + \frac{1}{4}(80) = 7 + 20 = 27$. Point 7: presents an integral (indefinite or definite) with integrand $v_J(t)$ or $2t(t^2-1)^3$ (with or without the differential dt). Point 8: correct antiderivative of the form $k(t^2-1)^4$ with $k = \frac{1}{4}$ for $k > 0$ (a different nonzero k does not earn Point 9). Point 9: correct answer $x_J(2) = 27$; requires Point 8.

Question 1

2023 ▪ Q1

A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f, where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

t **(seconds)**	**0**	**60**	**90**	**120**	**135**	**150**
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

Question 1

(a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t)\,dt$ in the context of the problem. Use a right Riemann sum with the three subintervals $[60, 90]$, $[90, 120]$, and $[120, 135]$ to approximate the value of $\int_{60}^{135} f(t)\,dt$.

Question 1

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0