

Exercise walk-through

The Chain Rule ■ 3.1

eXercise

You must be able to

- differentiate a **composite function** 复合函数 with the **chain rule** 链式法则

Method — The chain rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

— derivative of the outer (evaluated at the inner) times the derivative of the inner.

Method — Example

$$\frac{d}{dx}[\sin(x^2)] = \cos(x^2) \cdot 2x = 2x \cos(x^2).$$

Practice 2.1 ■ 1 mark

State the chain rule.

Solution 2.1

Method: The chain rule

Worked steps

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x) \quad \text{B1}.$$

Practice 2.2 ■ 2 marks

Differentiate $(x^2 + 1)^3$.

Solution 2.2

Worked steps

$$3(x^2 + 1)^2 \cdot 2x = 6x(x^2 + 1)^2 \quad \text{M1} \quad \text{A1}.$$

Practice 2.3 ■ 1 mark

State what the chain rule is for.

Solution 2.3

Method: The chain rule

Worked steps

differentiating a composite (a function of a function) **B1**.

Exam-style 3.1 ■ 1 mark

$\frac{d}{dx}[f(g(x))]$ equals

A $f'(x)g'(x)$

B $f'(g(x)) \cdot g'(x)$

C $f'(g(x))$

D $g'(f(x))$

Solution 3.1

A $f'(x)g'(x)$

B $f'(g(x)) \cdot g'(x)$



C $f'(g(x))$

D $g'(f(x))$

Worked steps

B.

Exam-style 3.2 ■ 1 mark

$\frac{d}{dx}[\sin(x^2)]$ equals

A $\cos(x^2)$

B $2x \cos(x^2)$

C $2x \sin(x^2)$

D $\cos(2x)$

Solution 3.2

A $\cos(x^2)$

B $2x\cos(x^2)$



C $2x\sin(x^2)$

D $\cos(2x)$

Worked steps

B.

Exam-style 3.3 ■ 1 mark

$$y = (3x + 1)^4.$$

- (a) Identify the outer and inner functions.
- (b) Apply the chain rule.
- (c) State y' .

Solution 3.3

Method: The chain rule

Worked steps

(a) outer u^4 , inner $3x + 1$ **B1**. (b) $4(3x + 1)^3 \cdot 3$ **B1**. (c) $y' = 12(3x + 1)^3$ **B1**.