Past-paper walk-through

The Quotient Rule ▪ 2.9

eXercise

Questions in this set

- 2016 Q6
- 2014 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 6

2016 ▪ Q6

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

Question 6

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\displaystyle\int_1^3 f'(2x)\,dx$.

Question 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

Solution 6

(a)

Mark scheme

- k(x)=f(g(x)). k(3)=f(g(3))=f(6)=4. By the chain rule, k'(x)=f'(g(x))$\cdot g'(x), sok'(3) = f'(g(3)) \cdot g'(3) = f'(6) \cdot g'(3) = 5 \cdot 2 = 10. Tangentline : y = 10(x-3) + 4. Points : 2forcorrectlyfindingtheslopek'(3) = 10(chainruleusingtablevalues), 1forthecorrecttangentlineequation.$

Solution 6

(b) Mark scheme

- h(x)=g(x)/f(x). By the quotient rule: h'(x) $= \dfrac{f(x)g'(x) - g(x)f'(x)}{[f(x)]^2}.h'(1) = \dfrac{f(1)g'(1) - g(1)f'(1)}{[f(1)]^2} = \dfrac{(-6)(8) - (2)(3)}{(-6)^2} = \dfrac{-48-6}{36} = \dfrac{-54}{36} = -\dfrac{3}{2}.Points: 2 forthecorrectexpressionforh'(1) usingthequotientruleandtablevalues, 1 forthecorrectfinalanswer($

Solution 6

(c) Mark scheme

- $\int_1^3 f'(2x)\,dx = \frac{1}{2}\left[f(2x)\right]_1^3 = \frac{1}{2}\left[f(6) - f(2)\right] = \frac{1}{2}\left[5 - (-2)\right] = \frac{1}{2}(7) = \frac{7}{2}. Points: 2 for the correct antiderivative via u-substitution (\frac{1}{2}f(2x)), 1 for the correct final answer (7/2).$

Question 3

2014 ▪ Q3

[Graph of f on the closed interval $[-5, 4]$, consisting of three line segments: the graph starts at the open point $(-5, 2)$ and descends to a point at approximately $(-2, 0)$ on the x-axis; from there it rises steeply (steeper slope) through the origin area up to a peak near $x = 0$ (peak value not labeled, appears above $y = 3$); from the peak it descends steeply down to the point $(4, -4)$. Axis tick marks shown at 1 on both the x-axis and y-axis. Labelled "Graph of f".]

The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by

$$g(x) = \int_{-3}^{x} f(t)\,dt.$$

(a) Find $g(3)$.

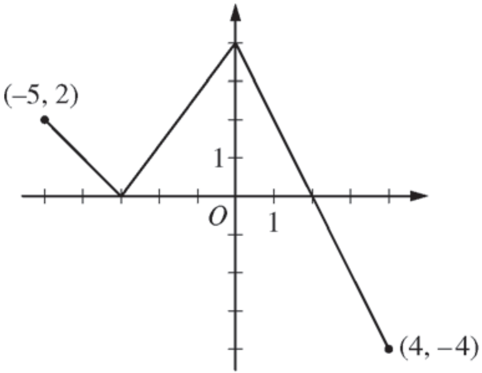
Question 3

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

(c) The function h is defined by $h(x) = \dfrac{g(x)}{5x}$. Find $h'(3)$.

(d) The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

Question 3



Graph of f

Solution 3

(a)

Mark scheme

- g(3) $= \int_{-3}^{3} f(t)\, dt = 6+4-1 = 9, computedassignedareasunderthethreelinesegmentsoffbetwe$
  $-3 and x = 3. Points : 1 forthecorrectanswer g(3) = 9.$

Solution 3

(b)

Mark scheme

- $g'(x) = f(x)$. The graph of g is increasing and concave down on the intervals $-5 < x < -3$ and $0 < x < 2$, because $g' = f$ is positive and decreasing on those intervals. Points: 1 for the correct intervals, 1 for a correct reason referencing $g' = f$ being both positive and decreasing on those intervals.

Solution 3

(c) Mark scheme

- h(x) = g(x)/(5x). By the quotient rule, h'(x) = 5x g'(x) - g(x)$\cdot 5\,\frac{}{(5x)^2=\frac{5x\,g'(x)-5g(x)}{25x^2}.h'(3)=\frac{(5)(3)g'(3)-5g(3)}{25\cdot 9}=\frac{15(-2)-5(9)}{225}=\frac{-75}{225}=-\frac{1}{3},\mathit{usingg}'(3)=\mathit{f}(3)=-2(\mathit{slopeoff}'\mathit{ssegmentat}}$

Solution 3

(d)

Mark scheme

- $p(x) = f(x^2 - x). Bythechainrule, p'(x) = f'(x^2 - x) \cdot (2x - 1). p'(-1) = f'(2) \cdot (-3) = (-2)(-3) = 6, usingf'(2) = -2(slopeoff'srightmostlinesegment). Points : 2foracorrectexpressionforp'(x)(chainruleappliedcorrectly), 1forthecorrectfinalanswerp'(-1) = 6.$