Past-paper walk-through

Applying the Power Rule ▪ 2.5

eXercise

Questions in this set

- 2025 Q2
- 2018 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2025 ▪ Q2

The shaded region R is bounded by the graphs of the functions f and g, where $f(x) = x^2 - 2x$ and $g(x) = x + \sin(\pi x)$, as shown in the figure.
[Figure: In the xy-plane, the graph of $y = g(x)$ and the graph of $y = f(x)$ are shown. The shaded region R, labeled R, lies between the two curves. The point $(3, 3)$ is marked where the curves meet. The y-axis is labeled with values $-2, -1, 1, 2, 3, 4$ and the x-axis is labeled with values $1, 2, 3$, with origin O.]
(Note: Your calculator should be in radian mode.)

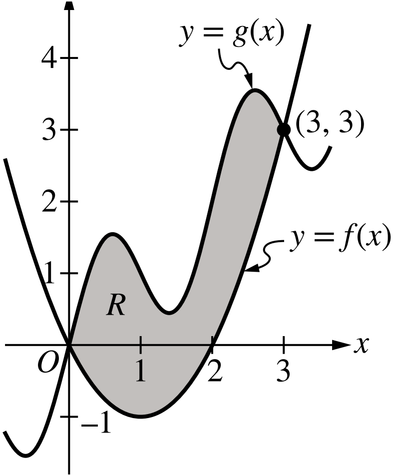
(a) Find the area of R. Show the setup for your calculations.

(b) Region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height x and base in region R. Find the volume of the solid. Show the setup for your calculations.

Question 2

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when the region R is rotated about the horizontal line $y = -2$.
(d) It can be shown that $g'(x) = 1 + \pi \cos(\pi x)$. Find the value of x, for $0 < x < 1$, at which the line tangent to the graph of f is parallel to the line tangent to the graph of g.

Question 2



Solution 2

(a)

Mark scheme

- Area $= \int_0^3 (g(x) - f(x))\,dx = 5.136620 \approx 5.137$ (exact value $\frac{4+9\pi}{2\pi}$). Point 1: presents an integral of $g(x) - f(x)$, $|g(x) - f(x)|$, $f(x) - g(x)$, or $|f(x) - g(x)|$ over $[0, 3]$ (or an equivalent difference of definite integrals of g and f). Point 2: correct answer 5.137 (or 5.136), with or without supporting work.

Solution 2

(b)

Mark scheme

- Volume $= \displaystyle\int_0^3 x(g(x) - f(x))\,dx = 7.704930 \approx 7.705$ (exact value $\dfrac{12+27\pi}{4\pi}$).
  Point 3: presents a definite integral with integrand a product of two nonconstant factors, one factor being x, $g(x) - f(x)$, or $f(x) - g(x)$. Point 4: correct answer 7.705 (or 7.704), with or without supporting work.

Solution 2

(c)

Mark scheme

- Volume $= \pi \int_0^3 \left[(g(x) - (-2))^2 - (f(x) - (-2))^2\right] dx =$ $\pi \int_0^3 \left[(g(x) + 2)^2 - (f(x) + 2)^2\right] dx$ (expression only, not evaluated). Point 5: integrand of the form $R^2 - r^2$ (or $|R^2 - r^2|$) where one of R, r is g or f (or a difference involving a nonzero constant) and the other is correct or a difference between f/g and a nonzero constant. Point 6: correct integrand $(g(x) + 2)^2 - (f(x) + 2)^2$ (or equivalent, e.g. reversed with a resolved sign). Point 7: correct limits of integration (0 to 3), the constant π, and the differential dx all present and correctly combined; requires Point 5.

Solution 2

(d)

Mark scheme

- Set $f'(x) = g'(x)$: $2x - 2 = 1 + \pi\cos(\pi x) \Rightarrow x = 0.675819 \approx 0.676$ (or 0.675). Point 8: general statement $f'(x) = g'(x)$, or any correct equation substituting $2x - 2$ for $f'(x)$ and/or $1 + \pi\cos(\pi x)$ for $g'(x)$. Point 9: correct answer $x \approx 0.676$, with or without supporting work.

Question 2

2018 ▪ Q2

A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) Evaluate $\displaystyle\int_0^{3.5} v(t)\,dt$, and evaluate $\displaystyle\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

Solution 2

(a)

Mark scheme

- $v'(3) = -2.118$. The acceleration of the particle at time $t = 3$ is -2.118. (1 pt: the answer.)

Solution 2

(b)

Mark scheme

- $x(3) = x(0) + \int_0^3 v(t)\,dt = -5 + \int_0^3 v(t)\,dt = -1.760213$. The position of the particle at $t = 3$ is -1.760. (1 pt: sets up $\int_0^3 v(t)\,dt$; 1 pt: uses the initial condition $x(0) = -5$; 1 pt: the answer -1.760.)

Solution 2

(c)

Mark scheme

- $\int_0^{3.5} v(t)\,dt = 2.844$ (or 2.843), and $\int_0^{3.5} |v(t)|\,dt = 3.737$. The integral $\int_0^{3.5} v(t)\,dt$ is the displacement of the particle over $0 \leq t \leq 3.5$. The integral $\int_0^{3.5} |v(t)|\,dt$ is the total distance traveled by the particle over $0 \leq t \leq 3.5$. (1 pt: both numeric answers; 2 pts: correct interpretations, 1 each for $\int v\,dt =$ displacement and $\int |v|\,dt =$ total distance.)

Solution 2

(d)

Mark scheme

- Set $v(t) = x_2'(t)$. Since $x_2(t) = t^2 - t$, $x_2'(t) = 2t - 1$, so $v(t) = 2t - 1 \Rightarrow t = 1.57054$. The two particles are moving with the same velocity at $t = 1.571$ (or 1.570). (1 pt: sets $v(t) = x_2'(t)$; 1 pt: the answer.)