Exercise walk-through

Estimating Limit Values from Graphs ▪ 1.3

eXercise

You must be able to

- read **one-sided limits 单侧极限** from a graph
- decide whether the two-sided limit exists

Method — Limits from graphs

Read the *y*-value the graph approaches from the left ($\lim_{x\to a^-}$) and from the right ($\lim_{x\to a^+}$).
The two-sided limit $\lim_{x\to a} f(x)$ exists only if **both** one-sided limits equal the same L — even if there is a **hole 空心点** at a.

Practice 2.1 ▪ 1 mark

State when a two-sided limit exists.

Solution 2.1

Method: Limits from graphs

Worked steps

when the left-hand and right-hand limits are equal B1.

Practice 2.2 ▪ 1 mark

State the notation for a left-hand limit.

Solution 2.2

Method: Limits from graphs

Worked steps

$\lim_{x \to a^-} f(x)$ B1.

Practice 2.3 ▪ 2 marks

If the limit from the left is 4 and from the right is 4, state the two-sided limit.

Solution 2.3

Method: Limits from graphs

Worked steps

the two-sided limit is 4 B1 B1.

Exam-style 3.1 ▪ 1 mark

A two-sided limit exists if and only if

A $f(a)$ is defined

B the left and right limits are equal

C the graph is a straight line

D the limit is 0

Solution 3.1

Method: Limits from graphs

A $f(a)$ is defined

B the left and right limits are equal ✓

C the graph is a straight line

D the limit is 0

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

A hole in the graph at $x = a$ means

A the limit cannot exist

B the limit may still exist

C f is linear

D $a = 0$

Solution 3.2

Method: Limits from graphs

A the limit cannot exist

B the limit may still exist ✓

C f is linear

D $a = 0$

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

A graph approaches 5 from the left and 3 from the right at $x = 2$.
(a) State the left-hand limit.
(b) State the right-hand limit.
(c) State whether the two-sided limit exists.

Solution 3.3

Method: Limits from graphs

Worked steps

(a) 5 B1. (b) 3 B1. (c) no, the one-sided limits differ B1.