Exercise walk-through

Connecting Limits at Infinity and Horizontal Asymptotes ▪ 1.15

eXercise

You must be able to

- relate a **limit at infinity** 无穷远处极限 to a **horizontal asymptote** 水平渐近线
- find it from the ratio of leading terms

Method — Limits at infinity

$\lim_{x\to\infty} f(x) = L$ means $y = L$ is a horizontal asymptote. For a rational function, compare leading terms:

- degree(top) $<$ degree(bottom) $\to$ $y = 0$;
- degrees equal $\to$ $y = \dfrac{\text{leading coeff of top}}{\text{leading coeff of bottom}}$;
- degree(top) $>$ degree(bottom) $\to$ no horizontal asymptote.

Method — Example

$\lim_{x\to\infty} \frac{1}{x} = 0$, so $y = 0$ is a horizontal asymptote.

Practice 2.1 ▪ 1 mark

State what $\lim_{x\to\infty} f(x) = L$ indicates.

Solution 2.1

Worked steps

the graph has a horizontal asymptote $y = L$ B1.

Practice 2.2 ▪ 1 mark

For $f(x) = \dfrac{1}{x}$, state $\lim_{x \to \infty} f(x)$.

Solution 2.2

Worked steps

0 B1.

Practice 2.3 ▪ 2 marks

For $f(x) = \dfrac{2x}{x+1}$, state the horizontal asymptote.

Solution 2.3

Worked steps

$y = 2$ (equal degrees, ratio $2/1$) M1 A1.

Exam-style 3.1 ▪ 1 mark

$\lim_{x\to\infty} f(x) = L$ means $y = L$ is a

A vertical asymptote

B horizontal asymptote

C hole

D root

Solution 3.1

Method: Limits at infinity

A vertical asymptote

B horizontal asymptote ✓

C hole

D root

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

$\lim_{x \to \infty} \frac{1}{x}$ equals

A 1

B 0

C ∞

D -1

Solution 3.2

A 1

B 0 ✓

C ∞

D -1

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$f(x) = \dfrac{3x^2}{x^2 + 1}$.

(a) Compare the degrees.

(b) Take the ratio of the leading coefficients.

(c) State the horizontal asymptote.

Solution 3.3

Method: Limits at infinity

Worked steps

(a) equal (both degree 2) B1. (b) $3/1 = 3$ B1. (c) $y = 3$ B1.