Exercise walk-through

Connecting Infinite Limits and Vertical Asymptotes ▪ 1.14

eXercise

You must be able to

- relate an **infinite limit 无穷极限** at a to a **vertical asymptote 垂直渐近线** $x = a$

Method — Infinite limits

If $f(x) \to \pm\infty$ as $x \to a$, the line $x = a$ is a **vertical asymptote**. This happens where the denominator $\to 0$ but the numerator does not.

Method — Example

$f(x) = \dfrac{1}{x-2}$ has a vertical asymptote at $x = 2$; as $x \to 2^+$, $f(x) \to +\infty$.

Practice 2.1 ▪ 1 mark

State what an infinite limit at a indicates.

Solution 2.1

Worked steps

the graph has a vertical asymptote there B1.

Practice 2.2 ▪ 1 mark

For $f(x) = \frac{1}{x}$, state the vertical asymptote.

Solution 2.2

Method: Example

Worked steps

$x = 0$ B1.

Practice 2.3 ▪ 2 marks

State where a rational function has a vertical asymptote.

Solution 2.3

Method: Infinite limits

Worked steps

where the denominator is 0 but the numerator is not **B1** **B1**.

Exam-style 3.1 ▪ 1 mark

If $\lim_{x \to a} f(x) = \infty$, then $x = a$ is a

A horizontal asymptote

B vertical asymptote

C hole

D root

Solution 3.1

A horizontal asymptote

B vertical asymptote ✓

C hole

D root

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

$f(x) = \dfrac{1}{x-2}$ has a vertical asymptote at

A $x = 0$

B $x = 2$

C $y = 2$

D $x = -2$

Solution 3.2

Method: Example

A $x = 0$

B $x = 2$ ✓

C $y = 2$

D $x = -2$

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$f(x) = \dfrac{1}{x-5}$.

(a) Where is the denominator 0?

(b) State the vertical asymptote.

(c) State the behaviour of $\lim_{x \to 5^+} f(x)$.

Solution 3.3

Method: Infinite limits

Worked steps

(a) $x = 5$ B1. (b) $x = 5$ B1. (c) $+\infty$ B1.