

Learn these by heart before the exam.

Limits and Continuity

$$\lim_{x \rightarrow c} f(x) = L \iff \lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = L$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1, \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

Continuous at c needs all three: $f(c)$ exists, $\lim_{x \rightarrow c} f(x)$ exists, and they are equal.

Indeterminate $\frac{0}{0}$: factor, rationalise, or use L'Hopital. Never just substitute and stop.

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)} \quad \text{only for } \frac{0}{0} \text{ or } \frac{\infty}{\infty}; \text{ verify the form FIRST}$$

IVT: f continuous on $[a, b]$ means it takes every value between $f(a)$ and $f(b)$. Cite continuity explicitly.

Differentiation

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$(x^n)' = nx^{n-1}, \quad (e^x)' = e^x, \quad (\ln x)' = \frac{1}{x}, \quad (a^x)' = a^x \ln a$$

$$(\sin x)' = \cos x, \quad (\cos x)' = -\sin x, \quad (\tan x)' = \sec^2 x$$

$$(uv)' = u'v + uv', \quad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, \quad (\arctan x)' = \frac{1}{1+x^2}$$

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Implicit: differentiate both sides in x , attach $\frac{dy}{dx}$ by the chain rule to every y , then solve.

Contextual Applications

Motion: $v(t) = x'(t)$, $a(t) = v'(t)$. Speed is $|v|$; speed rises when v and a share a sign.

$$\text{total distance} = \int_a^b |v(t)| dt, \quad \text{displacement} = \int_a^b v(t) dt$$

Related rates: write the relation, differentiate in t , THEN substitute the instant's values. Never substitute first.

$L(x) = f(a) + f'(a)(x-a)$ over-estimates when the curve is concave down

Analytical Applications

$f' > 0$ increasing, $f' < 0$ decreasing. $f'' > 0$ concave up, $f'' < 0$ concave down.

Critical point: $f' = 0$ or undefined. It is a

MAXIMUM if f' changes $+$ to $-$; say that in words.

Inflection point: f'' CHANGES SIGN. $f'' = 0$ alone is not enough.

$f'(c) = \frac{f(b) - f(a)}{b-a}$ needs continuous on $[a, b]$ and differentiable on (a, b) ; state both

Candidates test for a closed interval: check every critical point AND both endpoints.

Integration and Accumulation

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1), \quad \int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C, \quad \int \sin x dx = -\cos x + C, \quad \int \sec^2 x dx = \tan x + C$$

$$\frac{d}{dx} \int_a^{g(x)} f(t) dt = f(g(x)) g'(x) \quad (1)$$

$$\int_a^b f'(x) dx = f(b) - f(a) \quad (1)$$

$$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx$$

u -substitution: change the LIMITS too, or convert back to x before evaluating. Do not mix.

Riemann sums: left under-estimates an increasing function, right over-estimates it. Trapezoid over-estimates when concave up.

Differential Equations

Separable: get all y with dy and all x with dx , integrate both sides, add ONE $+C$, then use the initial condition.

$$\frac{dy}{dt} = ky \Rightarrow y = y_0 e^{kt}$$

Slope field: at each point the segment has slope $\frac{dy}{dx}$. Follow the segments to sketch a solution curve.

Applications of Integration

$$A = \int_a^b [f(x) - g(x)] dx \quad \text{top minus bottom; split where they cross}$$

$$V = \pi \int_a^b [R(x)^2 - r(x)^2] dx \quad /$$

$$V = \int_a^b A(x) dx \quad \text{square cross-section } A = s^2; \text{ semicircle } A = \frac{\pi}{2} r^2$$

Accumulation: $f(b) = f(a) + \int_a^b f'(x) dx$. Read this as 'start plus the change'.

Exam technique

Justify in words, with the theorem named: 'by the MVT, since f is continuous on $[a, b]$ and differentiable on (a, b) '.

Calculator sections: show the SETUP integral or equation. A bare number earns almost nothing.

Keep 3 decimal places and units. $\frac{dy}{dx}$ answers usually need units of y per unit of x .