1.1 Introducing Calculus: Can Change Occur at an Instant?

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS change • average rate of change • interval • instantaneous rate of change • limit

Objective

Build the skills to answer exam questions on **whether change can occur at an instant**.

You must be able to:

- find an **average rate of change** over an interval
- describe the **instantaneous rate of change** as a limit of average rates

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ Average vs instantaneous rate

The **average rate of change** of f over $[a, b]$ is $\dfrac{f(b) - f(a)}{b - a}$.

The **instantaneous rate** at a point is the **limit** of these average rates as the interval shrinks to zero – this is the idea calculus makes precise.

■ Example

$f(x) = x^2$ on $[1, 3]$: average rate $= \dfrac{9 - 1}{3 - 1} = 4$.

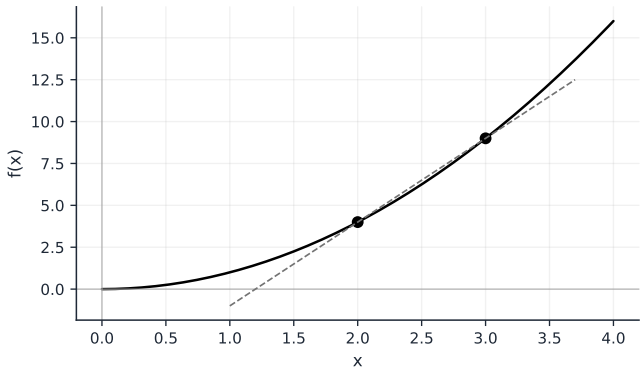
■ Key knowledge: Change over a shrinking interval

Average rate uses two different inputs. To describe an instant, simplify the quotient for $h \neq 0$ and then take its limit; never substitute $h = 0$ into the unsimplified quotient.

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■ **Worked example: Change over a shrinking interval**

A position is $s(t) = t^2$ metres. From $t = 2$ to $t = 3$, $v_{\text{avg}} = [s(3) - s(2)]/(3 - 2) = 5\,\text{m}\,\text{s}^{-1}$. Over $[2, 2{+}h]$, $v_{\text{avg}} = [(2{+}h)^2{-}4]/h = 4{+}h$. As $h \to 0$, the instantaneous rate approaches $4\,\text{m}\,\text{s}^{-1}$. The finite-interval rate and instantaneous rate answer different questions.



2 Practice

2.1 State the average rate of change of f on $[a, b]$. [1]

2.2 State what the instantaneous rate of change is the limit of. [1]

2.3 For $f(x) = x^2$, find the average rate of change on $[1, 3]$. [2]

3 Exam-style questions

3.1 The instantaneous rate of change is [1]

- **A** the average over $[a, b]$
- **B** the limit of average rates as the interval shrinks
- **C** always zero
- **D** the slope of a single secant line

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3.2 The average rate of change of f on $[a, b]$ is [1]

- **A** $f(b) \cdot f(a)$
- **B** $\dfrac{f(b) - f(a)}{b - a}$
- **C** $f(b) + f(a)$
- **D** $b - a$

3.3 $f(x) = x^2$.

(a) Find $f(1)$ and $f(3)$. [1]

(b) Find the average rate of change on $[1, 3]$. [1]

(c) State what a shrinking interval approaches. [1]

3.4 A position is $s(t) = t^2 + 2t$ metres. (a) Find the average velocity on $[1, 2]$. [2]

(b) Simplify the average-velocity quotient on $[1, 1+h]$ and use it to find the instantaneous velocity at $t = 1$. [2]

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Solutions

Each answer shows the steps, then the **result**.

2.1

- $\dfrac{f(b)-f(a)}{b-a}$

2.2

- the average rates of change over intervals shrinking to zero

2.3

- $\dfrac{f(3)-f(1)}{3-1}=\dfrac{9-1}{2}=4$

3.1 B

- the limit of average rates as the interval shrinks.

3.2 B

- $\dfrac{f(b)-f(a)}{b-a}$.

3.3

(a)

- $f(1)=1,\ f(3)=9$

(b)

- $\dfrac{9-1}{2}=4$

(c)

- the instantaneous rate of change

3.4

(a) $v_{\text{avg}}=[s(2)-s(1)]/(2-1)=(8-3)/1=5\,\text{m}\,\text{s}^{-1}$.

(b) $v(h)=[(1+h)^2+2(1+h)-3]/h=4+h$ for $h\neq 0$; $\lim_{h\to 0} v(h)=4\,\text{m}\,\text{s}^{-1}$.

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