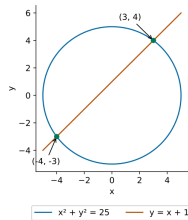


Rational equations, radical branches and systems

Original skill preparation; versioned source scope

Method and conditions



- A candidate root must satisfy the original equation.
- Factorisation; substitution; domain restrictions.

Worked method 1

Read and reason

A candidate root must satisfy the original equation. In $\sqrt{x+6} = x$, the right side must be nonnegative. Squaring gives $x^2 - x - 6 = 0$, so candidates are 3, -2. Only 3 survives the original check: $\sqrt{9} = 3$.

Original context 1

Read and reason

Task context

Read and reason

Each task below states its own complete data. Values may differ from the worked example.

Attempt 1.1

Read and reason

Solve simultaneously $x^2 + y^2 = 25$ and $y = x - 1$. Check each ordered pair in both equations.

Reasoning 1.1

Read and reason

Substitute to get $x^2 + (x - 1)^2 = 25$, or $x^2 - x - 12 = 0$. Roots are 4, -3 , giving $(4, 3)$ and $(-3, -4)$. Both give squared sum 25 and satisfy $y = x - 1$.

Attempt 2.1

Read and reason

Solve $\sqrt{3x+4} = x$ and describe all points satisfying $y \geq x+1$ and $y < 4-x$.

Reasoning 2.1

Read and reason

Squaring with $x \geq 0$ gives $(x - 4)(x + 1) = 0$; only $x = 4$ works. The region is on/above $y = x + 1$ and below $y = 4 - x$. It exists only when $x + 1 < 4 - x$, so $x < 3/2$; their meeting point is excluded.

Attempt 3.1

Read and reason

Simplify $F(x) = (x^2 - 4x)/(x^2 - 16)$. State every original restriction and explain why F is not identical to $x/(x + 4)$ on that latter formula's whole domain.

Reasoning 3.1

Read and reason

Factor numerator $x(x - 4)$ and denominator $(x - 4)(x + 4)$. The original domain excludes $x = 4, -4$. Cancellation gives $F(x) = x/(x + 4)$ for $x \neq 4, -4$. The latter formula alone permits 4, where it gives $1/2$, while $F(4)$ is undefined.

Transfer and limits

Read and reason

Factorisation; substitution; domain restrictions.

Check yourself

Read and reason

Close the worked reasoning. Explain why the method fits the final transfer task, reconstruct its conclusion, and state a condition that would change your answer. Then check the full reasoning in the reference.