

Sequences, finite sums, exponentials and logarithms

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Scope and prerequisites

ACT framework, February 2026 revision. Original classroom cases are not an official form; preserve the scored/field-test boundaries of each source form.

- Distinguish arithmetic difference from geometric ratio
- Find a term or finite sum while keeping indexing consistent
- Convert logarithmic equations to exponential form on the valid domain

Prerequisites: Index notation; exponent rules; finite sequences.

Explain and choose the method

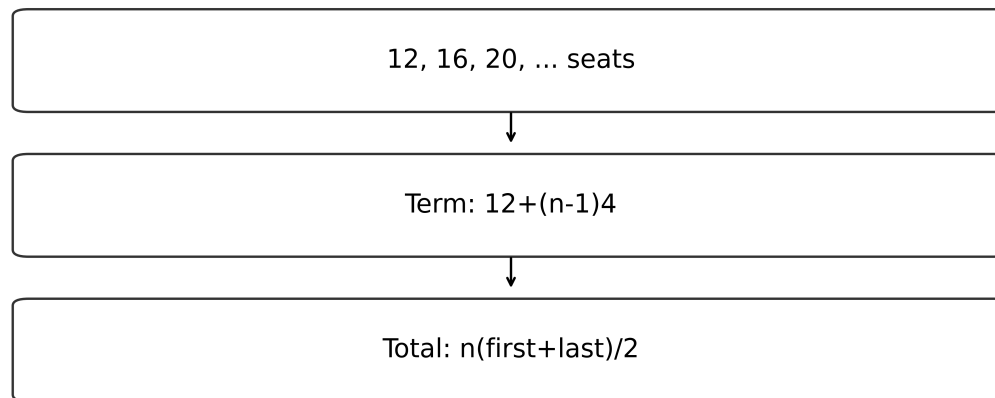
An arithmetic sequence has constant difference d : $a_n = a_1 + (n-1)d$. Its finite sum is $n(a_1 + a_n)/2$. A geometric sequence has constant ratio r : $a_n = a_1 r^{n-1}$. For $r \neq 1$, a finite sum is $a_1(1-r^n)/(1-r)$; when $r=1$ it is simply $n a_1$. Match the requested term or total.

An exponential model $A b^t$ represents multiplication by b per stated time unit. A percentage growth r uses factor $1+r$; repeated changes multiply factors rather than adding percentages. If doubling takes several time units, divide t by that interval in the exponent.

For base $b > 0$ with $b \neq 1$, $\log_b(a) = c$ means $b^c = a$ and requires $a > 0$. Solve a logarithmic equation by translating it to an exponential relationship, then check the argument. Logarithms of products add under valid positive arguments; logarithms do not distribute over addition.

Use exact powers when available. ACT may test a simple logarithmic relationship without requiring numerical logarithm tables. A calculated approximation must still respect the question's representation and domain. Keep a sequence's index, an exponential model's time and a logarithm's argument conceptually separate.

Arithmetic sequence : $a_n = a_1 + (n-1)d$. Its sum is $S_n = n(a_1 + a_n)/2$. Geometric sequence : $a_n = a_1 r^{n-1}$ and $S_n = a_1(1-r^n)/(1-r)$ for $r \neq 1$. A logarithm reverses exponentiation: $\log_b a = c$ means $b^c = a$, with $a > 0$, $b > 0$, $b \neq 1$.



Original diagram of the worked relationship; read the full wording and qualifications.

Existing worked example: Arithmetic 4,7,10,... has $a_5=16$ and sum of first five $=5(4+16)/2=50$. Geometric 3,6,12,24 has four-term sum 45. $\log_2(x-1)=4$ gives $x-1=16$, so $x=17$, which satisfies $x>1$. A population $50 \cdot 2^{(t/3)}$ reaches 200 after six time units.

Complete original context

Every transfer question states all data it needs.

Independent practice and checked reasoning

■ Transfer 1

Seats in successive rows number 12,16,20,... . Find the eighth row and the total in the first eight rows. State the model assumption.

Reasoning: Constant difference is 4 seats per row. $a_8 = a_1 + 7d = 12 + 7(4) = 40$ seats. $S_8 = 8(a_1 + a_8)/2 = 8(12 + 40)/2 = 208$ seats. This assumes the same four-seat increment through all eight rows.

■ Transfer 2

A geometric sequence starts 5,15,45,... . Find its fifth term and sum of the first five terms. Explain what changes if the ratio is 1.

Reasoning: $a_5 = a_1 r^4 = 5(3)^4 = 405$. $S_5 = a_1(1-r^5)/(1-r) = 5(1-243)/(1-3) = 605$. When $r = 1$, the fraction has zero denominator and the correct sum is na_1 , here 25 for five identical terms.

■ Transfer 3

Solve $\log_2(x - 3) = 5$ and explain why $\log_2(x + 3)$ cannot generally be split into $\log_2 x + \log_2 3$.

Reasoning: The domain requires $x > 3$. Exponential form gives $x - 3 = 32$, so $x = 35$ and the argument is positive. The sum of logs corresponds to the product $3x$, not $x + 3$; at $x = 1$, the proposed equality would equate 2 with $\log_2 3$.

■ Transfer 4

A culture model starts with 120 cells and grows by a factor of 1.5 every two hours. Write $N(t)$ for hours t , find $N(4)$, and distinguish factor from percentage growth.

Reasoning: $N(t) = 120(1.5)^{t/2}$ cells. $N(4) = 120(1.5)^2 = 270$ cells. The growth is 50% per two hours, not 150%; the four-hour factor is 2.25. The hourly factor would be $\sqrt{1.5}$.

Limits and next use

Do not use n rather than $n-1$ for the n th term, confuse a finite sum with its last term, or expand $\log(a+b)$ as $\log a + \log b$.

All tasks here are public original practice with authored guidance. They are not official questions or fresh diagnostics. Existing protected tests and mocks remain separate.