

Functions, composition, transformations and inverses

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Scope and prerequisites

ACT framework, February 2026 revision. Original classroom cases are not an official form; preserve the scored/field-test boundaries of each source form.

- Evaluate and convert between function rules, tables and graphs
- Track input order and domain through composition
- Find an inverse after choosing a one-to-one domain

Prerequisites: Function notation; substitution; equation solving.

Explain and choose the method

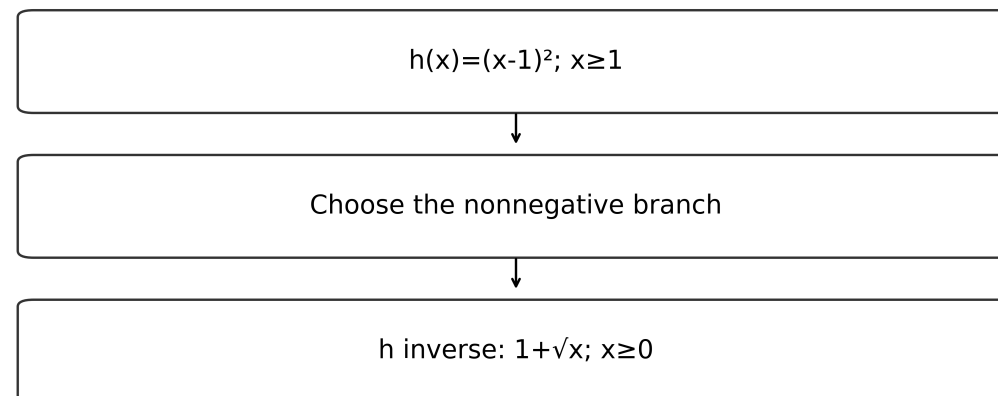
A function gives one output for each allowed input. A graph passing a vertical line at two different output points is not a function of x . Domain lists allowed inputs; range lists attained outputs. Convert a table to a rule only when the relationship is supported, and interpret each variable's role.

In $f(g(x))$, the inner function g acts first and its output must be in f 's domain. For $f(u)=1/(u-1)$ and $g(x)=x^2+1$, composition gives $1/x^2$ and excludes $x=0$. Domain checking belongs to the original functions as well as the simplified result.

An output change $f(x)+k$ shifts vertically by k ; $f(x-h)$ shifts right by h . An outside multiplier changes output scale, while an inside multiplier changes how quickly the input reaches old points. Identify a known point to verify the transformation instead of guessing direction from a sign.

An inverse exchanges input and output and requires a one-to-one relationship on the chosen domain. Solve $y=f(x)$ for x , then exchange names. A quadratic must usually be restricted to one side of its vertex. The inverse function f^{-1} is not the reciprocal $1/f$. Check composition in both directions on the allowed domain.

Composition applies the inside rule first. If $f(u) = 1/(u - 2)$ and $g(x) = x^2 + 2$, $f(g(x)) = 1/x^2$, with $x \neq 0$. An inverse reverses input and output on a one-to-one domain. Restrict $h(x) = (x - 1)^2$ to $x \geq 1$ before taking $h^{-1}(x) = 1 + \sqrt{x}$.



Original diagram of the worked relationship; read the full wording and qualifications.

Existing worked example: $f(x)=2x+3$ and $g(x)=x^2$ give $f(g(x))=2x^2+3$, but $g(f(x))=(2x+3)^2$. For $h(x)=(x-2)^2$ restricted to $x \geq 2$, the inverse is $2+\sqrt{x}$ with domain $x \geq 0$. A vertical shift of h by 4 has vertex $(2,4)$; the unrestricted quadratic would not have a single-valued inverse.

Complete original context

Every transfer question states all data it needs.

Independent practice and checked reasoning

■ Transfer 1

Let $f(u) = 1/(u - 3)$ and $g(x) = x^2 + 3$. Find $f(g(x))$ with domain. Find $g(f(4))$ and explain why these compositions differ.

Reasoning: $f(g(x)) = 1/(x^2 + 3 - 3) = 1/x^2$, excluding $x = 0$. For the other order, $f(4) = 1$ and $g(1) = 4$. Composition order changes both operation and possibly domain; $f(g(4)) = 1/16$ is different.

■ Transfer 2

Restrict $h(x) = (x + 2)^2$ to $x \leq -2$. Find its inverse, including domain and range. Explain why the positive-root branch is wrong.

Reasoning: Solve $y = (x + 2)^2$ with $x + 2 \leq 0$ to get $x = -2 - \sqrt{y}$. Thus $h^{-1}(x) = -2 - \sqrt{x}$, domain $x \geq 0$, range $y \leq -2$. The plus branch returns values outside the chosen original domain.

■ Transfer 3

The graph of $f(x) = x^2$ is changed to $g(x) = 3f(x+4) - 2$. Find its vertex and range, checking one mapped point.

Reasoning: $g(x) = 3(x+4)^2 - 2$ has vertex $(-4, -2)$ and range $y \geq -2$. Old point $(1, 1)$ maps to $(-3, 1)$ because $3(1) - 2 = 1$; substitution confirms $g(-3) = 1$.

Limits and next use

Preserve composition order and the restricted inverse branch. Superscript -1 in function notation does not mean a reciprocal.

All tasks here are public original practice with authored guidance. They are not official questions or fresh diagnostics. Existing protected tests and mocks remain separate.