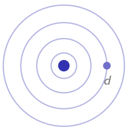
Astronomy and Cosmology

A2 Physics ▪ Topic 25

A-Level Physics



What we will cover

1 Luminosity and flux

Total power, then power per unit area

Luminosity 光度 L is the total power a star radiates in all directions. Unit: W. At distance d that power has spread over a sphere $4\pi d^2$, so the **radiant flux intensity** 辐射通量密度 is

$$F = \frac{L}{4\pi d^2}$$

The **inverse-square law** 平方反比定律: doubling d cuts the flux to a quarter. A telescope measures F; if L is known,

$$d = \sqrt{\frac{L}{4\pi F}}$$

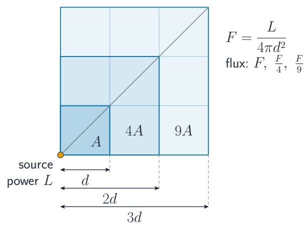
Worked example – flux at the Earth

Sun: $L = 3.8 \times 10^{26}\,\mathrm{W}$, $d = 1.5 \times 10^{11}\,\mathrm{m}$

$$F = \frac{L}{4\pi d^2} = \frac{3.8 \times 10^{26}}{4\pi(1.5 \times 10^{11})^2}$$

$$\approx 1.4 \times 10^{3}\,\mathrm{W\,m^{-2}}$$

Worked example – flux at the Earth, in detail



The same power spreads over a larger area as distance grows, so flux falls as $1/d^2$

2 Standard candles

An object whose luminosity is known from its type

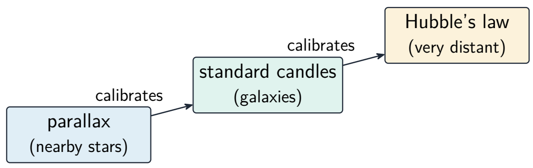
Find one in a distant galaxy, measure F, get $d = \sqrt{L/(4\pi F)}$. Reaches far beyond **parallax** 视差.

Cepheids
造父变星
pulsation period is tightly linked to luminosity – the period gives L

Type Ia supernovae
a white dwarf at critical mass always has about the same peak luminosity

Each rung calibrates the next

> Parallax reaches the nearest stars; Cepheids reach nearby galaxies; supernovae reach the far ones. Every rung is calibrated by the one below it.



each rung is calibrated by the one below, reaching ever further

3 Stellar temperature

Wien – hotter means bluer

A blackbody spectrum peaks at $\lambda_{\max}$ set by temperature:

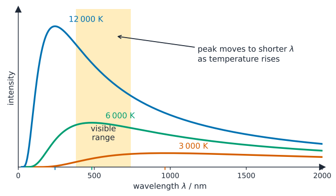
$$\lambda_{\max} T = b \approx 2.90 \times 10^{-3}\,\mathrm{m\,K}$$

Cool red star $\sim 3000\,\mathrm{K}$ peaks in the infrared; the Sun $\sim 5800\,\mathrm{K}$ near $500\,\mathrm{nm}$; a hot blue-white star $\sim 20\,000\,\mathrm{K}$ in the ultraviolet.

$\lambda_{\max} = 500\,\mathrm{nm}$

$T = b/\lambda_{\max} = 2.90 \times 10^{-3}/(500 \times 10^{-9}) \approx 5800\,\mathrm{K}$

Wien – hotter means bluer, in detail



Wien: $\lambda_{\max} T =$ const · hotter ⇒ peak shifts left

> A hotter black body radiates more, and its peak wavelength shifts towards the blue (Wien's law)

Wien's displacement law

The Pillars of Creation in the Eagle Nebula – clouds of gas and dust lit by hot, newly formed stars

Stefan–Boltzmann – $L \propto r^2 T^4$

A star as a blackbody sphere: The **Stefan–Boltzmann law** 斯特藩-玻尔兹曼定律, with the **Stefan–Boltzmann constant** 斯特藩-玻尔兹曼常数 σ:

$$L = 4\pi\sigma r^2 T^4, \qquad \sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$$

Twice the radius $\Rightarrow$ four times L. Twice the temperature $\Rightarrow$ sixteen times L. To estimate a radius: $\lambda_{\max} \to T$ (Wien); F and $d \to L$; then

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

Estimating a star's radius

Wien, then Stefan–Boltzmann

1. $\lambda_{\max} \Rightarrow T$ $(\lambda_{\max} T = 2.9 \times 10^{-3}\,\mathrm{m\,K})$
2. $L = 4\pi d^2 F$ from flux and distance
3. $L = 4\pi r^2 \sigma T^4 \Rightarrow r = \sqrt{\dfrac{L}{4\pi\sigma T^4}}$

Two laws and a distance are enough to size a star nobody can resolve.

Luminosity, and the law behind it

Luminosity	The **luminosity** 光度 L of a star is its **total power** 总功率 of radiation – the **energy** 能量 radiated per second in all directions.
Stefan–Boltzmann law	The **Stefan–Boltzmann law**: $L = 4\pi r^2 \sigma T^4$, where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is the **Stefan–Boltzmann constant**.
Why it gives a radius	Wien's law gives T from the peak wavelength, the flux and distance give L – so the only unknown left is r.

$L \propto T^4$: doubling the surface temperature multiplies the power by sixteen, which is why hot stars are so much brighter than their size suggests.

From flux to a star's radius, and the distance ladder

the Stefan–Boltzmann law 斯特藩-玻尔兹曼定律	$L = 4\pi r^2 \sigma T^4$, with σ the **Stefan–Boltzmann constant** 斯特藩-玻尔兹曼常量
the route	Wien's law gives T from the peak wavelength; flux and distance give L; then rearrange for r
why it matters	this is how astronomers estimate the radii of stars they cannot resolve as a disc
the ladder	each method is calibrated by the one below it
the rungs	radar in the solar system, then parallax, then standard candles, then Hubble's law
the evidence for the Big Bang	the expansion, galactic redshift, the **cosmic microwave background** 宇宙微波背景, and the hydrogen/helium abundance

Each rung reaches further and is less precise. An error low on the ladder propagates all the way up it, which is why calibration matters so much.

4 Redshift and Hubble

Spectral lines shifted to the red

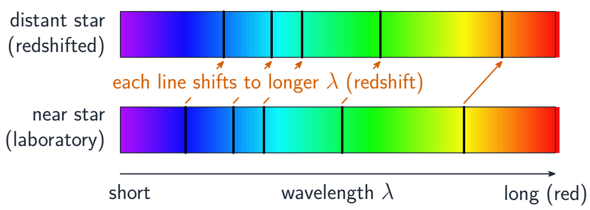
Lines from distant galaxies sit at longer wavelengths than in the lab – **redshift** 红移. Read as a Doppler shift, the galaxy is receding. For $v \ll c$:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$$

Almost every distant galaxy is redshifted (a few near ones are **blueshifted** 蓝移 by local motion). Galaxies are, on average, moving apart – the Universe is expanding.

The same lines, moved

> A galaxy's lines keep their pattern but sit at longer wavelengths – which is how the shift is recognised as a shift, and not as different elements.



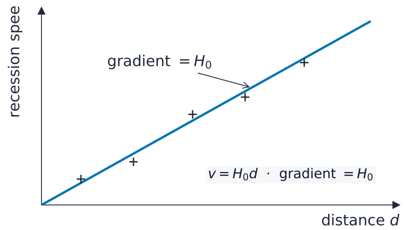
Hubble's law

$$v \approx H_0 \cdot d$$

where H_0 is the **Hubble constant** 哈勃常数, $\approx 2.3 \times 10^{-18}\,\mathrm{s}^{-1}$. Always SI units. Running the expansion backwards, distances shrink to zero at $t = -1/H_0$ – the **Big Bang** 大爆炸. For steady expansion the age is

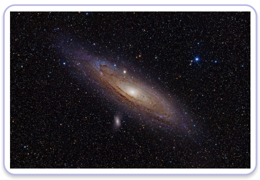
$$T_{\mathrm{age}} \approx \frac{1}{H_0} \approx 14 \text{ billion years}$$

Hubble's law, in detail



> Hubble's law: a galaxy's recession speed is proportional to its distance, $v = H_0 d$

How far the ladder reaches



Andromeda – Cepheids put it well outside our own galaxy.

Almost every smudge here is a galaxy, most of them receding.

> Each rung calibrates the next: parallax, then **Cepheid** 造父变星 variables, then redshift.

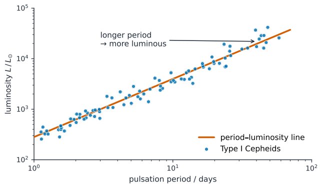
Worked example – a receding galaxy

Emitted $4.62 \times 10^{-7}\,\mathrm{m}$, seen at $4.91 \times 10^{-7}\,\mathrm{m}$. $H_0 = 2.3 \times 10^{-18}\,\mathrm{s}^{-1}$.

$\Delta\lambda = 0.29 \times 10^{-7}\,\mathrm{m}$, so $v \approx (\Delta\lambda/\lambda)c \approx 1.9 \times 10^{7}\,\mathrm{m\,s^{-1}}$.

Distance $d = v/H_0 \approx 8.3 \times 10^{24}\,\mathrm{m}$.

Standard candles



Cepheid: longer period ⇒ brighter · standard candles

> For Cepheid variables the pulsation period sets the luminosity, making them standard candles

Exam tips

- $F = L/4\pi d^2$ (inverse-square): a standard candle has known luminosity L, so the distance follows from the measured flux F.
- **Wien's law** ($\lambda_{max} \propto 1/T$) gives surface temperature; **Stefan's law** $L = 4\pi r^2 \sigma T^4$.
- **Redshift** $z = \Delta\lambda/\lambda \approx v/c$; Hubble's law $v = H_0 d$ leads to the age of the universe (evidence for the Big Bang).

5 Recap

Topic 25 – key takeaways

1. **Flux** — $F = L/(4\pi d^2)$; inverse-square
2. **Candles** — Cepheids, Type Ia; d from known L
3. **Star** — Wien $\lambda_{\max} T = b$; $L = 4\pi\sigma r^2 T^4$
4. **Hubble** — $v = H_0 d$; age $\approx 1/H_0$

Check yourself

1. Double the distance: what happens to flux?
2. What makes a Cepheid a standard candle?
3. A hotter star: longer or shorter $\lambda_{\max}$?
4. Hubble's law points to what origin of the Universe?

> **Answers** **1.** falls to a quarter **2.** period gives luminosity **3.** shorter **4.** the Big Bang