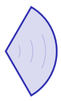
Medical Physics

A2 Physics ▪ Topic 24

A-Level Physics



What we will cover

1 Ultrasound

Piezo-electric – two linked effects

A **piezo-electric** 压电 crystal (quartz, PZT) changes shape when a p.d. is put across it, and makes an e.m.f. when its shape is changed.

apply a p.d. → crystal changes shape

squeeze it → an e.m.f. appears

apply a p.d.

the crystal vibrates – used to make ultrasound

squeeze it

an e.m.f. appears – used to detect ultrasound

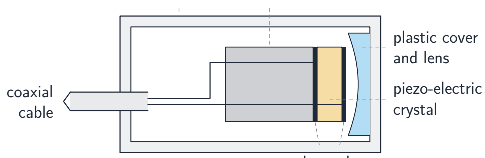
One transducer both sends and listens

An alternating p.d. (a few MHz) makes the crystal vibrate, sending **longitudinal** 纵波 waves above $20\,\mathrm{kHz}$ (1–$10\,\mathrm{MHz}$ for imaging). Returning ultrasound makes it vibrate and produce an e.m.f.

> One transducer switches between sending and listening. **Coupling gel** 耦合剂 pushes out the air – without it almost all the ultrasound would reflect at the skin.

Inside the transducer

> The crystal sits between a damping block (to kill ringing so pulses stay short) and a matching layer (to cut the reflection at the skin).



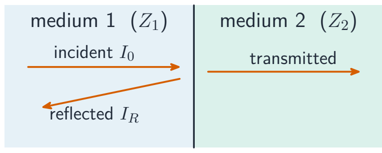
Pulse-echo: depth from the time delay

1. send a short pulse
2. at each tissue boundary, part reflects and part goes on
3. detect each echo
4. depth $d = ct/2$ (there and back); amplitude ⇒ strength of the reflection

Echo after $60\,\mu\text{s}$; $c = 1500\,\text{m}\,\text{s}^{-1}$

$$d = \frac{ct}{2} = \frac{1500 \times 60 \times 10^{-6}}{2} = 0.045\,\text{m}\ (4.5\,\text{cm})$$

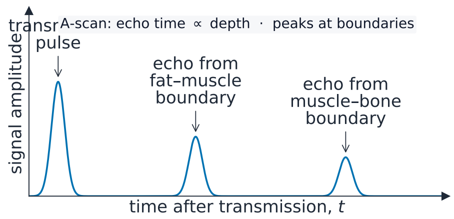
Pulse-echo: depth from the time delay, in detail



$$\frac{I_R}{I_0} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$ — a bigger impedance difference gives a bigger echo

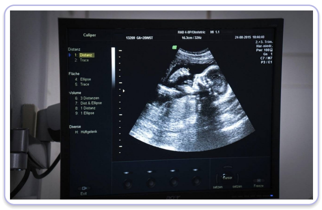
At a boundary, part of the pulse reflects and part transmits – a bigger impedance difference giv

Pulse-echo imaging



An A-scan shows the transmitted pulse and the echoes from each tissue boundary

What the operator sees



A B-scan builds a 2-D image from many pulse-echo lines swept across the body.

Specific acoustic impedance

$$Z = \rho c$$

Unit $\text{kg}\,\text{m}^{-2}\,\text{s}^{-1}$. Bone has large Z; air has small Z; soft tissue is in between. At a boundary the intensity reflection coefficient is

$$\frac{I_R}{I_0} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

Very different Z: almost all reflected (skin/air – hence the gel). Very similar Z: the boundary cannot be seen.

Attenuation – intensity falls exponentially

$$I = I_0 e^{-\mu x}$$

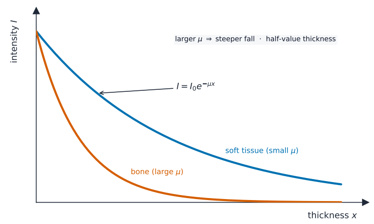
μ is the **attenuation coefficient** 衰减系数. The same form applies to X-rays.

Impedance, reflection and attenuation

acoustic impedance	$Z = \rho c$ – **density** 密度 times the speed of sound. Soft tissue: $1060 \times 1540 \approx 1.6 \times 10^6$ kg m^{-2}s^{-1}
reflection at a boundary	the fraction reflected depends on the difference in Z between the two media
best for imaging	different enough to give an echo, but not so different that nothing passes on
why gel is used	air's Z is so low that almost everything would reflect at the skin
attenuation	$I = I_0 e^{-\mu x}$; higher-energy X-rays penetrate further, so their μ is smaller
contrast	bone has a much larger μ than soft tissue, which is why it shows white

To find a thickness, rearrange to $x = \frac{1}{\mu} \ln \frac{I_0}{I}$. Half-value thickness is the special case $I = I_0/2$.

Attenuation – intensity falls exponentially, in detail



Intensity falls exponentially with thickness; a larger attenuation coefficient (bone) falls off

Production

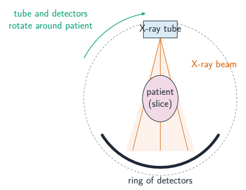
X-rays come from an **X-ray tube** X 射线管:

a heated **cathode 阴极** emits electrons by **thermionic emission 热电子发射**.

2 X-rays

How an X-ray tube works

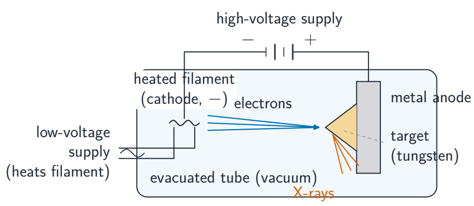
1. heated **cathode** 阴极 emits electrons by **thermionic emission** 热电子发射
2. a high p.d. accelerates them across a vacuum to a metal **target** 靶 anode (often tungsten)
3. they slow sharply: most KE becomes heat; a small part is X-ray photons (**Bremsstrahlung** 轫致辐射). Knocking out inner electrons gives **characteristic** 特征 lines.



In a CT scan the X-ray tube and detectors rotate around the patient to image a slice from many a

The X-ray tube

Most of the electrons' kinetic energy becomes heat; a small part leaves as X-ray photons, so the anode is angled and cooled.



Minimum wavelength and imaging

The most energy one photon can have is eV:

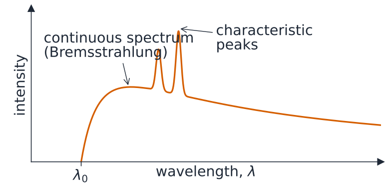
$$hf_{\max} = eV, \qquad \lambda_{\min} = \frac{hc}{eV}$$

Bone (high Z) attenuates more and looks lighter; soft tissue looks darker. A **contrast medium** 造影剂 (barium meal) makes soft tissues stand out. Half-value thickness $x_{1/2} = \ln 2/\mu$ halves the intensity.

$x = 3.0\,\mathrm{cm}$, $\mu = 40\,\mathrm{m}^{-1}$

$I/I_0 = e^{-40\times 0.030} = e^{-1.2} \approx 0.30$

Minimum wavelength

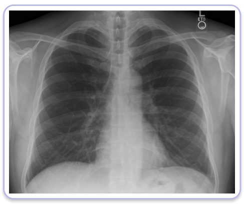


X-ray: continuous bremsstrahlung + characteristic peaks

A typical X-ray spectrum – a continuous Bremsstrahlung curve cut off at λ_0, with sharp characteristic peaks

Why bone shows white

Dense bone (high Z) absorbs strongly and leaves the film unexposed; the air-filled lungs let X-rays through and darken it.



CT – many angles, then a computer

Computed tomography 计算机断层扫描:

1. tube and detectors rotate, imaging one thin slice from many angles
2. a computer combines them into a 2-D cross-section
3. the patient is moved; the next slice is imaged
4. slices stack into a 3-D image

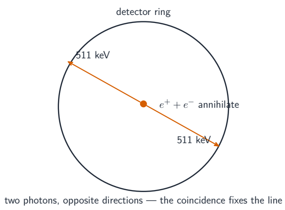
Overlapping soft tissues that hide each other on a single X-ray are separated by the many-angle reconstruction.

3 PET scanning

A β^+ tracer, then annihilation

A **tracer** 示踪剂 (often FDG, fluorine-18 on a glucose analogue) is taken up more by active tissue. In PET it is a β^+ emitter – it gives out a **positron** 正电子. The positron meets an electron and they **annihilate** 湮灭: two photons, opposite directions, each $511\,\mathrm{keV}$ ($= m_e c^2$).

$$2hf = 2m_e c^2$$



PET: annihilation gives two 511 keV photons in opposite directions; a coincidence fixes the line

A coincidence fixes the line

The two photons hit detector rings around the patient. Two simultaneous arrivals (a coincidence) fix the line the annihilation lay on. Many coincidences from many angles build a 3-D map of the tracer – tissues with high metabolic activity.

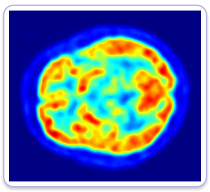
> Comparing arrival times can refine the position along that line (time-of-flight PET).

Tracers and annihilation

a tracer 示踪剂	a substance containing radioactive nuclei, introduced into the body
how it works	it is taken up more by the tissue being studied, so that tissue emits more strongly
the choice	a γ or β^+ emitter with a short half-life, so the dose falls quickly
an antiparticle 反粒子	the same mass, opposite charge – the positron is the electron's
annihilation 湮灭	a particle meets its antiparticle and their mass turns into electromagnetic energy
the signature	two γ photons of 0.51 MeV leave in opposite directions, and detecting both fixes the line the event lay on

> PET works because momentum conservation forces the two photons back to back. That geometry is what locates the tracer.

A finished PET image



> **Positron emission tomography** 正电子发射断层成像 (PET) maps function, not shape: warm colours mark where the tracer collected – the tissue with the fastest **metabolism** 代谢.

Exam tips

- **Ultrasound** reflects at boundaries (acoustic impedance); a coupling gel reduces reflection at the skin.
- **X-rays** are attenuated as $I = I_0 e^{-\mu x}$; contrast media (barium, iodine) absorb strongly.
- **PET** uses a positron emitter; annihilation gives two γ-rays in opposite directions, located by the detector ring.

4 Recap

Topic 24 – key takeaways

1. **Ultrasound**
   piezo; $d = ct/2$; gel; $Z = \rho c$
2. **X-rays**
   $\lambda_{\min} = hc/eV$; $I = I_0 e^{-\mu x}$
3. **CT**
   many angles of one slice, then stack
4. **PET**
   two $511\,\mathrm{keV}$ photons; coincidence line

Check yourself

1. Why $d = ct/2$, not ct?
2. Why put gel between the transducer and the skin?
3. What sets $\lambda_{\min}$ of an X-ray spectrum?
4. Why do PET photons fly in opposite directions?

Answers **1.** there and back **2.** air/skin reflects almost everything **3.** $hf_{\max} = eV$
4. momentum conservation (pair at rest)