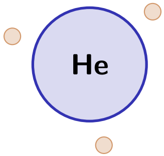
Nuclear Physics

A2 Physics ▪ Topic 23

A-Level Physics



What we will cover

1. Mass–energy equivalence
2. Mass defect and binding energy
3. Fusion and fission
4. Radioactive decay
5. Recap

1 Mass-energy

Mass and energy are the same thing

Mass–energy equivalence 质能等价, from special relativity:

$$E = mc^2, \qquad \Delta E = c^2 \Delta m$$

$c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$. Nuclear masses are tiny but c^2 is huge, so a small mass change is a large energy. The conversion you will use again and again:

$$1\,\mathrm{u} \;\longleftrightarrow\; 931\,\mathrm{MeV}$$

The exchange rate

$$E = mc^2$$

A mass of one atomic mass unit is worth $931.5\,\mathrm{MeV}$ – which is why nuclear energies are quoted in MeV and chemical ones in eV.

> $1\,\mathrm{u} \leftrightarrow 931\,\mathrm{MeV}$. Nuclear masses are tiny and c^2 is enormous, so a mass change too small to weigh releases an energy you can measure.

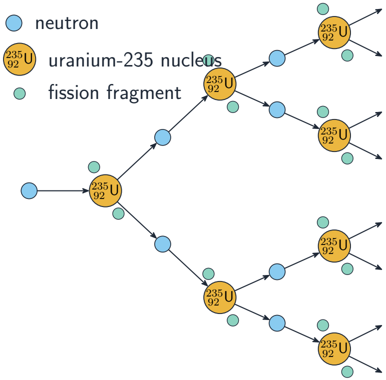
Balancing a nuclear reaction

$$^{14}_{7}\mathrm{N} + {}^{4}_{2}\mathrm{He} \to {}^{17}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H}$$

Nucleon number 核子数 is conserved (top: $14+4=17+1$) and charge is conserved (bottom: $7+2=8+1$).

> To fill in an unknown: identify the species, then balance the top and bottom numbers.

Balancing a nuclear reaction, in detail



In an uncontrolled chain reaction each fission of uranium-235 releases neutrons that cause more

Nuclear fusion and fission

A nuclear power station releases energy by nuclear fission.

2

Binding energy

A nucleus weighs less than its parts

The mass of a **nucleus** 原子核 is less than its Z **protons** 质子 plus N neutrons, $Zm_{\mathrm{p}} + Nm_{\mathrm{n}}$. The difference is the **mass defect** 质量亏损:

$$\Delta m = (Zm_{\mathrm{p}} + Nm_{\mathrm{n}}) - m_{\mathrm{nucleus}}$$

That missing mass was released as **binding energy** 结合能 when the nucleus formed:

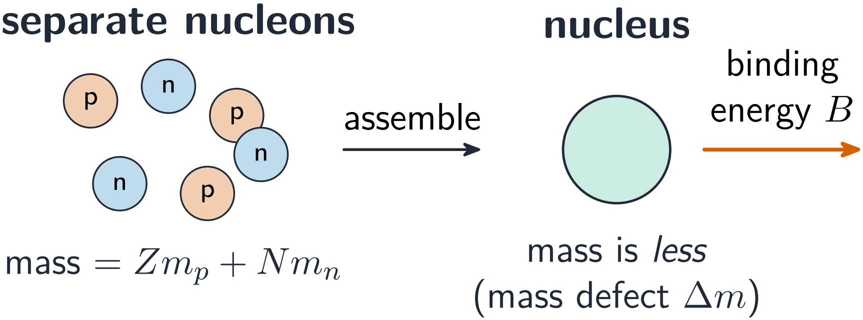
$$B = \Delta m \cdot c^2$$

Helium-4, $\Delta m = 0.0304\,\mathrm{u}$

$B = 0.0304 \times 931 \approx 28\,\mathrm{MeV}$

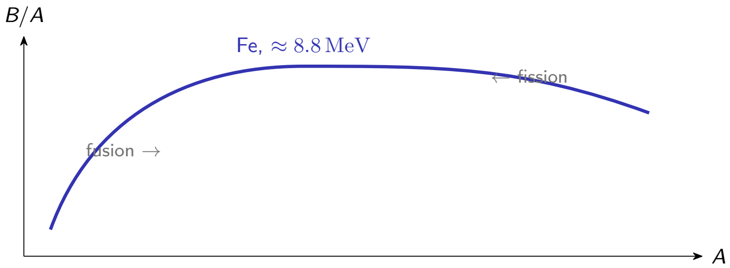
The missing mass

Separated nucleons weigh more than the nucleus they make. That difference is the **mass defect** 质量亏损, and $E = mc^2$ turns it into the binding energy.



Binding energy per nucleon peaks at iron

B/A (MeV per nucleon) measures how tightly each nucleon is held.



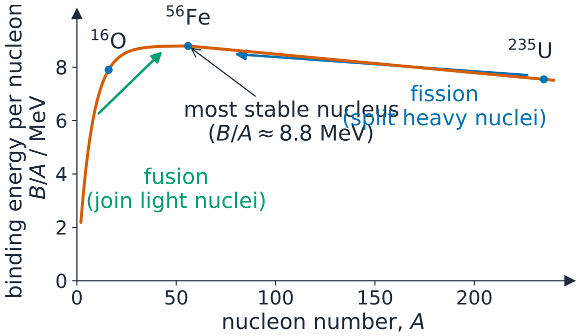
> Iron-56 is the most stable nucleus. Light nuclei sit to the left; heavy nuclei sit to the right.

Reading the binding-energy curve

the shape	dome-shaped: rising steeply from hydrogen, peaking at iron, then falling slowly
why it peaks	the **strong nuclear force** 强核力 is short-range, so beyond about iron the electrostatic repulsion of the protons wins
to the left of iron	fusion moves nuclei up the curve and releases energy
to the right	fission moves them up too – both directions run downhill in energy
mass-energy equivalence 质能等价	$E = \Delta mc^2$ converts the mass defect into the energy released
in every equation	nucleon number, **conservation of charge** 电荷守恒 and energy must all balance

Binding energy per nucleon measures stability, not total energy. Iron is the most stable nuclide, which is why stars stop there.

Binding energy per nucleon vs nucleon number



binding energy per nucleon peaks near ^{56}Fe

Binding energy per nucleon peaks near iron ($A \approx 56$); lighter and heavier nuclei are less tightly bound

3 Fusion and fission

Energy is released towards the iron peak

fusion

核聚变

two light nuclei join

$$^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + n$$

needs millions of kelvin to beat electrostatic repulsion

fission

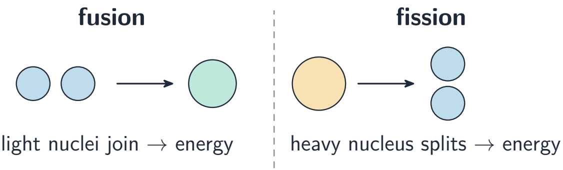
核裂变

a heavy nucleus splits

extra neutrons can cause more fissions – a **chain reaction** 链式反应

needs a **critical mass** 临界质量

Both directions run downhill to iron



Fusion 聚变 joins light nuclei; **fission** 裂变 splits heavy ones. Both move nucleons towards the iron peak, and both release the difference in binding energy.

Calculating the energy released

1. total mass of the reactants
2. total mass of the products
3. $\Delta m = m_{\text{reactants}} - m_{\text{products}}$ (positive when energy is released)
4. $\Delta E = c^2 \Delta m$, or in u: $\Delta E\,(\text{MeV}) = \Delta m\,(\text{u}) \times 931$

Mass decreases by $0.020\,\text{u}$

$\Delta E = 0.020 \times 931 \approx 19\,\text{MeV}$

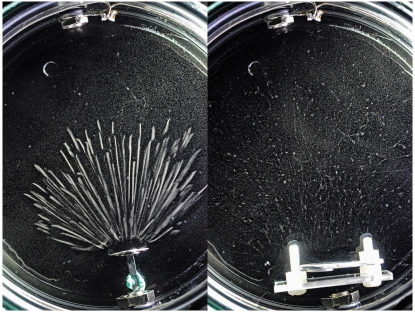
4 Radioactive decay

Random and spontaneous

Spontaneous – no outside trigger. Rate is not changed by temperature, pressure or chemical state.

Random – you cannot predict when a given nucleus will decay, only the probability. Evidence: the count rate fluctuates.

Seeing single decays



Each track is one decay – unpredictable individually, precisely regular in bulk. A **Geiger counter** 盖革计数器 beside a source clicks at uneven intervals, never a steady stream, which is the evidence that decay is random.

Activity, decay constant, exponential

$$A = \lambda N$$

A is **activity** 活度 (decays per unit time), in **becquerel** 贝可勒尔 (Bq). λ is the **decay constant** 衰变常数 – probability per unit time that a nucleus decays.

$$N = N_0 e^{-\lambda t}, \qquad A = A_0 e^{-\lambda t}$$

The same fraction decays in each interval – that is what makes it **exponential decay** 指数衰减.

Half-life

Half-life 半衰期 $t_{1/2}$ is the time for N (or A, or the count rate) to fall to half:

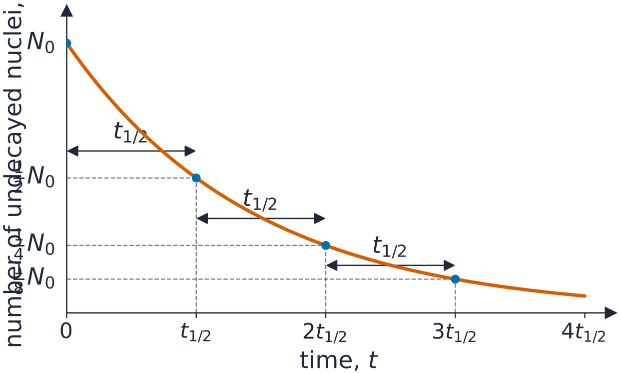
$$\lambda = \frac{\ln 2}{t_{1/2}} \approx \frac{0.693}{t_{1/2}}$$

After n half-lives the surviving fraction is $(1/2)^n$. After 5 half-lives, about 3% remains.

$t_{1/2} = 6.0$ hours

$6.0\,\mathrm{h} = 21600\,\mathrm{s}$, so $\lambda = 0.693/21600 \approx 3.2 \times 10^{-5}\,\mathrm{s}^{-1}$

Half-life, in detail



$N = N_0 e^{-\lambda t}$ · $T_{1/2} = \ln 2/\lambda$

The number of undecayed nuclei falls by half in each half-life

Activity, decay constant and half-life

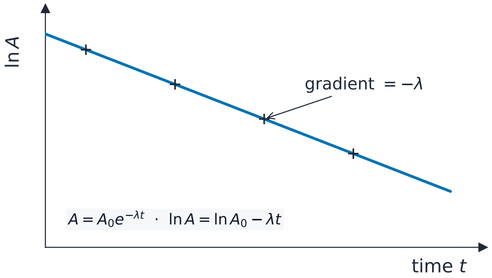
activity	$A = \lambda N$ – decays per second, in becquerel
λ	the decay constant: the probability per second that any one nucleus decays
fixed per nuclide	λ never changes for a given **radionuclide** 放射性核素 – so a larger sample has proportionally larger activity
the link	$\lambda = \dfrac{\ln 2}{t_{1/2}}$
the units	convert the half-life to seconds first: $6.0\ \text{h} = 21\,600\ \text{s}$
the decay law	$N = N_0 e^{-\lambda t}$, and the same form for A and for count rate

Decay is random and spontaneous: unaffected by temperature, pressure or chemical state. Only the number of nuclei present changes the activity.

Finding λ from data

$$\lambda = \frac{1}{t}\ln\frac{A_0}{A}, \qquad \ln A = \ln A_0 - \lambda t$$

A plot of $\ln A$ against t is a straight line of gradient $-\lambda$.



A graph of $\ln A$ against time is a straight line of gradient $-\lambda$

Exam tips

- $E = mc^2$; **the mass defect** is the mass lost when nucleons bind, released as binding energy.
- **Binding energy per nucleon** peaks near iron – both fusion (light nuclei) and fission (heavy nuclei) move towards it and release energy.
- Activity $A = \lambda N$; decay is exponential $N = N_0 e^{-\lambda t}$; half-life $t_{1/2} = \ln 2/\lambda$.
- Balance nuclear equations by conserving **nucleon number and proton number**.

5 Recap

Topic 23 – key takeaways

1 $E = mc^2$

$1\,\mathrm{u} \leftrightarrow 931\,\mathrm{MeV}$; balance A and Z

2 **Binding**

B/A peaks at iron; fusion and fission move towards it

3 **Decay**

$A = \lambda N$; $N = N_0 e^{-\lambda t}$

4 **Half-life**

$t_{1/2} = \ln 2/\lambda$; random and spontaneous

Check yourself

1. Why does a small Δm release a lot of energy?
2. Which nucleus sits at the peak of the B/A curve?
3. Fusion and fission both release energy – why?
4. After 3 half-lives, what fraction remains?

Answers **1.** c^2 is huge **2.** iron-56 **3.** both move towards the iron peak **4.** $1/8$