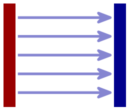
Capacitance

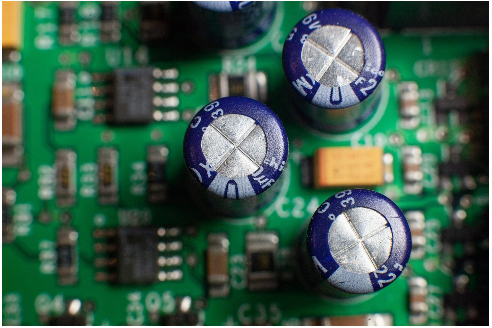
A2 Physics ▪ Topic 19

A-Level Physics



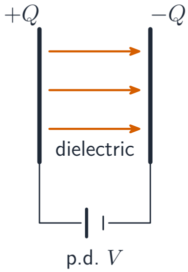
What we will cover

1. Capacitance $C = Q/V$
2. Combining capacitors
3. Energy stored
4. Discharge through a resistor
5. The time constant RC
6. Recap



1 Capacitance

What a capacitor does



$$C = \frac{Q}{V}$$

A **capacitor** 电容器 stores charge: two **conductor** 导体 plates with an **insulator** 绝缘体 between them – a **dielectric** 电介质, or just a **vacuum** 真空 – with a **potential difference** 电势差 V across the gap.

- connected to a battery, $+Q$ builds on one plate, $-Q$ on the other

> The two charges are equal and opposite – the capacitor stores charge separation, not net charge.

Capacitance $C = Q/V$

$$C = \frac{Q}{V}, \qquad [C] = \mathrm{F} = \mathrm{C\,V^{-1}}$$

- set by size and dielectric, so C is constant
- double Q and V doubles too – the ratio holds
- an isolated sphere: $C = 4\pi\varepsilon_0 r$

> A farad is huge – real capacitors run from pF to mF.

Worked example – charge stored

A $100\,\mu\mathrm{F}$ capacitor is charged to $12\,\mathrm{V}$. Find the charge stored.

$$Q = CV = (100 \times 10^{-6})(12)$$

$$= 1.2 \times 10^{-3}\,\mathrm{C} = 1.2\,\mathrm{mC}$$

Put the μF in as $\times 10^{-6}\,\mathrm{F}$. A stored mC is a lot of separated charge for such a small device.

2 Combining capacitors

Parallel: the charges add

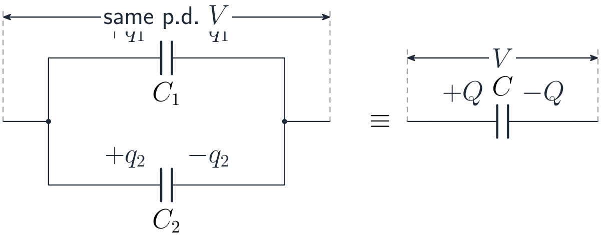
Same p.d. V across each, so the charges sum:

$$C_{\text{parallel}} = C_1 + C_2 + \dots$$

A parallel bank has larger capacitance than any one.

> More plate area $\Rightarrow$ more room for charge – adding in parallel is like widening the plates.

Parallel: the charges add, in detail



$Q = q_1 + q_2$

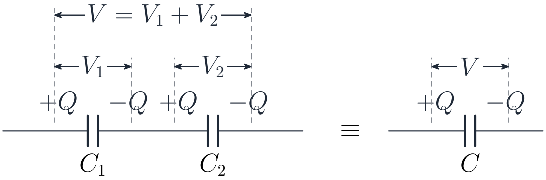
$C = C_1 + C_2$

Same p.d. V across each, so the charges sum:

$$C_{\text{parallel}} = C_1 + C_2 + \dots$$

A parallel bank has larger capacitance than any one.

Series: the p.d.s add



Same charge Q through each, so the p.d.s sum:

$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

A series chain has smaller capacitance than any one.

> The opposite of resistors – because $C = Q/V$ has V on the bottom, while $R = V/I$ has I on the bottom.

3 Energy stored

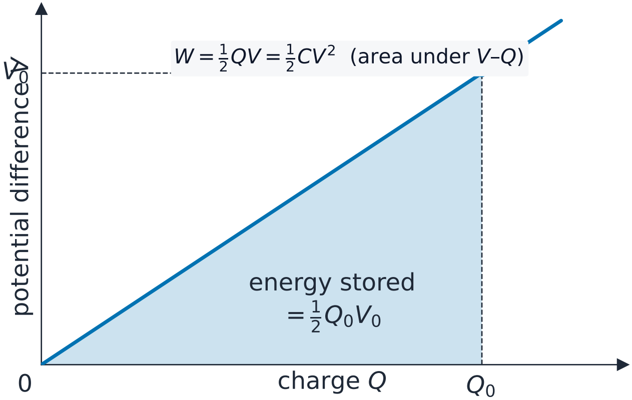
Energy stored in a capacitor

Each extra dq is pushed against the p.d. q/C already there, so

$$W = \int_0^Q \frac{q}{C}\,dq = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

> On the V–Q line the energy is the area beneath – the triangle $\frac{1}{2}QV$. The $\frac{1}{2}$ is the average p.d. during charging.

Energy stored in a capacitor, in detail



The energy stored is the area under the potential–charge line (the triangle $\frac{1}{2}QV$)

Reading the Q–V graph

A plot of V against Q is a straight line through the origin with gradient $1/C$.

> The energy stored is the **area** under the line up to a given charge Q, which is the triangle $\frac{1}{2}QV$.

Worked example – energy stored

Find the energy in a $100\,\mu\text{F}$ capacitor charged to $12\,\text{V}$.

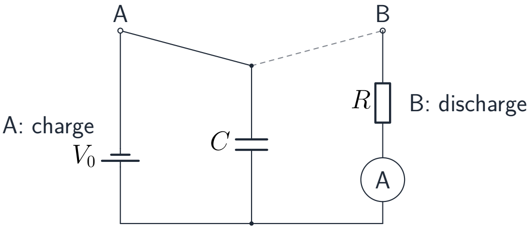
$W = \frac{1}{2}CV^2 = \frac{1}{2}(100 \times 10^{-6})(12)^2$

$= 7.2 \times 10^{-3}\,\text{J} = 7.2\,\text{mJ}$

Charging through a wire wastes the other half: the battery gives out $QV = CV^2$, the capacitor keeps $\frac{1}{2}CV^2$, and the rest heats the wire – whatever its resistance.

4 Discharge through a resistor

Setting up the discharge



Close the switch and C drives current through R. By **Kirchhoff's second law** 基尔霍夫第二定律 around the loop $V_C = V_R$; using $V_C = Q/C$, $V_R = IR$ and $I = -dQ/dt$:

$$\frac{Q}{C} = -R\frac{dQ}{dt}$$

> The rate of loss is proportional to what is left – and that always gives an **exponential decay** 指数衰减.

The discharge equations

Charge, p.d. and current all decay with the same time constant RC:

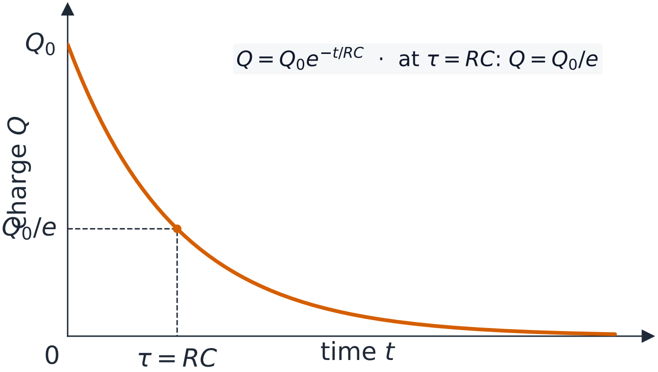
$$Q = Q_0 e^{-t/RC}$$

$$V = V_0 e^{-t/RC}, \quad I = I_0 e^{-t/RC}$$

with $I_0 = V_0/R$.

> After one time constant RC the charge is down to $Q_0/e \approx 37\%$ of the start.

The discharge equations, in detail



Charge decays exponentially during discharge, falling to Q_0/e after one time constant RC

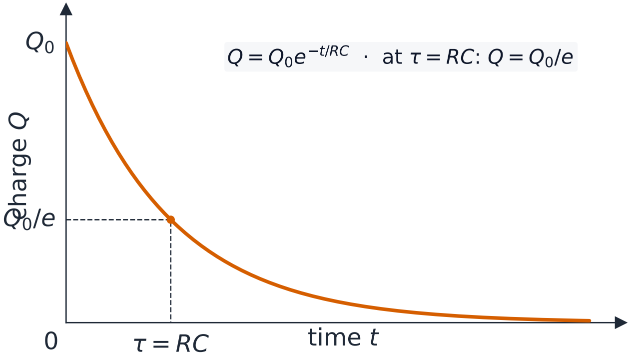
The time constant and half-life

$$\tau = RC, \qquad t_{1/2} = RC\ln 2 \approx 0.69\,RC$$

- units check: $\quad \times \mathrm{F} = \mathrm{s}$
- bigger R or $C \Rightarrow$ slower discharge
- every RC, the charge falls by the same factor $1/e$

> $\ln$ turns the exponential into a straight line: plotting $\ln Q$ against t gives a gradient of $-1/RC$.

The time constant and half-life, in detail



$$\tau = RC, \qquad t_{1/2} = RC \ln 2 \approx 0.69\, RC$$

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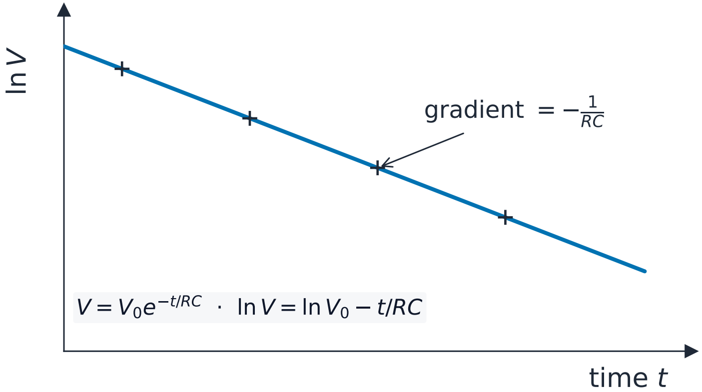
Time constant



A graph of $\ln V$ against time is a straight line of gradient $-1/(RC)$

Reading a discharge curve in the exam

Start value	Read V_0 or Q_0 off the graph at $t = 0$.
Time constant	Read the time taken to fall to V_0/e (about 37% of the start): that time *is* $\tau = RC$.
The missing component	Given R, find $C = \tau/R$ – or given C, find R.
A later value	Predict any later value with $V = V_0 e^{-t/RC}$; or plot $\ln V$ against t for a straight line of gradient $-1/RC$.

Every discharge question is one of these four. Identify which before you touch the algebra.

Exam tips

- $C = Q/V$; energy stored $W = \frac{1}{2}QV = \frac{1}{2}CV^2$ (the $\frac{1}{2}$ is the area under the Q-V graph).
- Combine capacitors the **opposite way to resistors** (parallel add, series reciprocal).
- Discharge is exponential: $Q = Q_0 e^{-t/RC}$; the time constant $\tau = RC$ is when the charge falls to 37%.

5 Recap

Topic 19 – key takeaways

1 Capacitance

$C = Q/V$, in farads – a constant of the device

2 Combining

parallel $C_1 + C_2$; series $1/C_1 + 1/C_2$

3 Energy

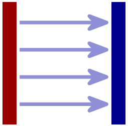
$\frac{1}{2}QV = \frac{1}{2}CV^2$ – area under V–Q

4 Discharge

$Q = Q_0 e^{-t/RC}$ – time constant $\tau = RC$

Check yourself

1. Two $2\,\mu\mathrm{F}$ caps in parallel: total C?
2. Same two in series: total C?
3. Why the factor $\frac{1}{2}$ in the energy?
4. What fraction is left after one RC?



Answers **1.** $4\,\mu\mathrm{F}$ **2.** $1\,\mu\mathrm{F}$ **3.** average p.d. is $V/2$ **4.** $1/e \approx 37\%$