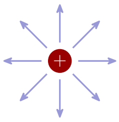
Electric Fields

A2 Physics ▪ Topic 18

A-Level Physics



What we will cover

1 Fields and field lines

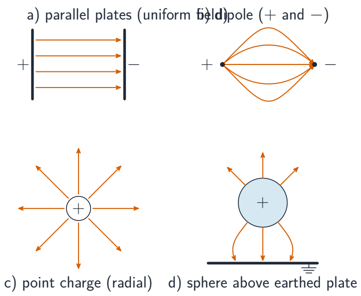
Electric field strength

An **electric field** 电场 is a region where a charge feels a force. Its **field strength** 电场强度 is the force per unit positive charge:

$$E = \frac{F}{q} \Rightarrow F = qE, \qquad [E] = \mathrm{N\,C^{-1}}$$

> Defined on a positive test charge – so a negative charge feels a force against the field.

Field-line patterns



- they run from + to − (or off to infinity)
- they never cross
- closer lines = stronger field

> Two crossing lines would give the force at that point two directions at once – impossible, which is why it never happens.

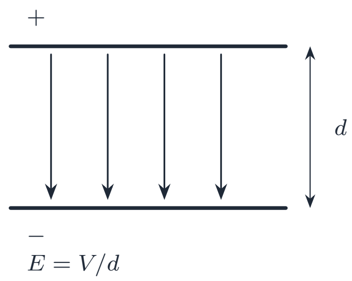
Four field patterns worth recognising

Point charge	Radial lines pointing *out* from a positive **point charge** 点电荷, and *in* towards a negative one.
A dipole	Two opposite charges – a **dipole** 偶极子 – give lines curving from + to −, densest in the gap between them.
Parallel plates	Two parallel charged plates a distance d apart with **potential difference** 电势差 V give a **uniform field** 匀强电场 of size $E = V/d$, apart from edge effects.
Several charges	For more than one charge, add the separate fields as a **vector sum** 矢量和 at each point.

> A charge in a uniform field feels a constant force $F = qE$ and so a constant acceleration $a = qE/m$ – exactly like a mass in a uniform **gravitational field** 引力场.

2 Uniform fields

The uniform field between plates



Plates a distance d apart with p.d. V across them:

$$E = \frac{V}{d}$$

pointing from the high-potential plate to the low.

> This is where $\mathrm{V\,m^{-1}}$ comes from – and it is the same unit as $\mathrm{N\,C^{-1}}$.

Worked example – plates and an electron

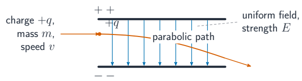
Plates $5.0\,\mathrm{mm}$ apart carry a p.d. of $200\,\mathrm{V}$. Find E, and the force on an electron between them. ($e = 1.6 \times 10^{-19}\,\mathrm{C}$)

$E = \dfrac{V}{d} = \dfrac{200}{5.0 \times 10^{-3}} = 4.0 \times 10^{4}\,\mathrm{V\,m^{-1}}$

$F = qE = (1.6 \times 10^{-19})(4.0 \times 10^{4}) = 6.4 \times 10^{-15}\,\mathrm{N}$

> Convert mm to m first – forgetting it scales E by a thousand. The force is the same everywhere in the gap: that is what uniform means.

A charge crossing a uniform field



Constant force ⇒ constant acceleration $a = qE/m$:

- released at rest: speeds up along the field, gaining KE
- entering at right angles: a **parabolic** 抛物线 path

> Exactly a **projectile** 抛体 problem – steady velocity across, uniform acceleration down. This is how a **cathode-ray tube** 阴极射线管 steered its beam.

Two ways a charge is set moving

Released at rest	It accelerates along the field (positive charge) or against it (negative), gaining **kinetic energy** 动能 qV after falling through a potential difference V.
Fired across	Constant velocity along the plates, constant acceleration across them – the same parabola as a projectile.
In an atom	An **electron** 电子 orbiting a **nucleus** 原子核 of charge $+Ze$ at radius r: the Coulomb attraction supplies the **centripetal force** 向心力, $\dfrac{Ze^2}{4\pi\varepsilon_0 r^2} = \dfrac{mv^2}{r}$.

> Energy method or force method – both work. Use energy when the question gives a potential difference, force when it gives a field.

3 Coulomb's law

Coulomb's law

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

with the **permittivity of free space** 真空介电常数 $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$ and $\dfrac{1}{4\pi\varepsilon_0} \approx 9.0 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$.

> Inverse square, along the joining line – like repels, opposite attracts. That sign choice is the one thing gravity cannot do.

Worked example – two point charges

Find the force between $+2.0\,\mathrm{nC}$ and $+3.0\,\mathrm{nC}$ set $4.0\,\mathrm{cm}$ apart.

$$F = \frac{(9.0 \times 10^9)(2.0 \times 10^{-9})(3.0 \times 10^{-9})}{(0.040)^2}$$

$\approx 3.4 \times 10^{-5}\,\mathrm{N}$, and repulsive (both $+$)

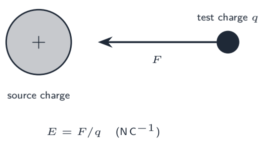
> State the direction as well as the size – "repulsive" is part of the answer. And r goes in as metres, squared.

Spheres as point charges

A charged spherical **conductor** 导体 of total charge Q:

- outside: identical to a point charge Q at its centre (measure r from the centre)
- inside a hollow one: the field is zero

> Zero field inside makes the whole conductor one **equipotential** 等势面 – and is why a car shields you from lightning.



4 Field of a point charge

The field of a point charge

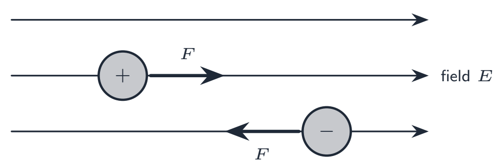
Coulomb's law gives the force between two charges. Divide by the test charge and only the source is left:

$$E = \frac{F}{q} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Out of a positive Q, into a negative one, falling as $1/r^2$.

> The same shape as $g = GM/r^2$ – but charges can cancel, so for several of them add the fields as a vector sum.

The field of a point charge, in detail



> The field of a point charge Coulomb's law gives the force between two charges.

5 Electric potential

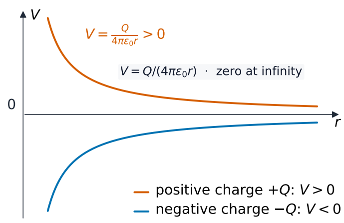
Electric potential

Electric potential 电势 is the work done per unit positive charge bringing a test charge in from **infinity** 无穷远:

$$V = \frac{W}{q} = \frac{Q}{4\pi\varepsilon_0 r}$$

> Note $1/r$, not $1/r^2$. V is a scalar and takes the sign of Q – so several charges just add, signs and all, with no directions to resolve.

Electric potential, in detail



> Potential falls as $1/r$, not $1/r^2$, and takes the sign of its charge – so several charges simply add, signs and all.

Worked example – potential near a charge

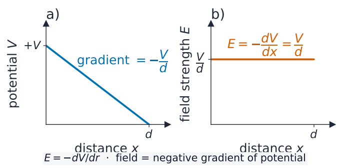
Find V at $4.0\,\mathrm{cm}$ from a point charge of $+3.0\,\mathrm{nC}$.

$$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{(9.0\times10^9)(3.0\times10^{-9})}{0.040}$$

$\approx 6.8\times10^2\ \mathrm{V}$

> Same charge, same distance as the Coulomb example – but r appears to the first power here. Mixing up $1/r$ and $1/r^2$ is the standard slip.

Field is the potential gradient



The field is the negative potential gradient:

$$E = -\frac{dV}{dx}$$

- between plates V falls evenly $\Rightarrow$ $E = V/d$, constant
- for a point charge $-\frac{dV}{dr} = \frac{Q}{4\pi\varepsilon_0 r^2} = E$

> The minus sign says the field points downhill in potential – from high V to low.

Electric potential energy

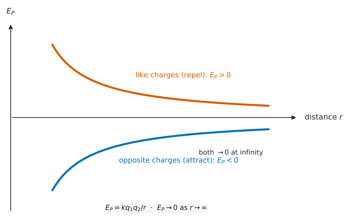
A charge q where the potential is V has $E_P = qV$; for two point charges

$$E_P = \frac{Qq}{4\pi\varepsilon_0 r}$$

- like: $E_P > 0$ – energy released if they fly apart
- opposite: $E_P < 0$ – a **bound** 束缚 well

> Both tend to zero as $r \to \infty$ – that is the choice of zero, exactly as for gravity.

Electric potential energy, in detail



A charge q sitting where the potential is V has $E_P = qV$, so for two point charges

$$E_P = \frac{Qq}{4\pi\varepsilon_0 r}.$$

> Like charges give $E_P > 0$, opposite charges $E_P < 0$; both tend to zero at infinity.

Gravitational vs electric

Quantity	**Gravitational**	**Electric**
Source	mass M (always $+$)	charge Q (either sign)
Field strength	$g = GM/r^2$	$E = Q/(4\pi\varepsilon_0 r^2)$
Force	$F = mg$	$F = qE$
Gravitational potential 引力势 vs potential	$\phi = -GM/r$	$V = Q/(4\pi\varepsilon_0 r)$
PE of two	$-GMm/r$	$Qq/(4\pi\varepsilon_0 r)$
Nature	always attractive	attract *or* repel

> Identical machinery, one difference: gravity's minus sign is permanent (mass only attracts), while the electric sign is inherited from the charge.

Exam tips

- **Coulomb's law** $F = Q_1Q_2/4\pi\varepsilon_0 r^2$ (inverse-square, but can attract or repel).
- Distinguish a **uniform field** ($E = V/d$, between plates) from a **radial field** ($E = Q/4\pi\varepsilon_0 r^2$).
- Electric potential $V = Q/4\pi\varepsilon_0 r$ (sign follows the charge); the field points from high to low potential.

6 Recap

Topic 18 – key takeaways

1 Coulomb
$F = Q_1Q_2/4\pi\varepsilon_0 r^2$ – attract or repel

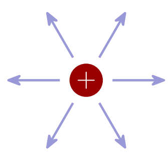
2 Two fields
uniform $E = V/d$; radial $E = Q/4\pi\varepsilon_0 r^2$

3 Potential
$V = Q/4\pi\varepsilon_0 r$ – scalar, signed, $1/r$

4 The link
$E = -dV/dx$ – the field points downhill in V

Check yourself

1. Which way does a field line point?
2. Halve the plate gap at fixed V – what happens to E?
3. Is potential a vector or a scalar?
4. What is the field inside a hollow charged sphere?



> **Answers** **1.** the force on a $+$ charge **2.** doubles **3.** scalar **4.** zero