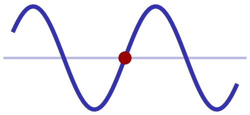
Oscillations

A2 Physics ▪ Topic 17

A-Level Physics

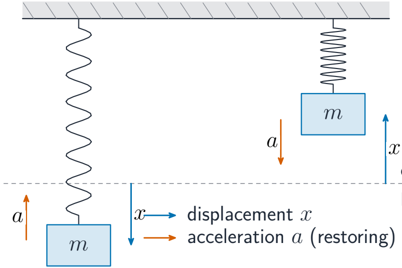


What we will cover

1. SHM: $a = -\omega^2 x$
2. Displacement, velocity, acceleration
3. Graphs
4. Energy
5. Damping & resonance
6. Recap

1 Defining SHM

Simple harmonic motion



A body moves with **simple harmonic motion** 简谐运动 when its acceleration is

- **proportional to its displacement** from a fixed **equilibrium** 平衡 point, and
- **directed back towards that point**.

Many systems do this near a stable equilibrium: a mass on a **spring** 弹簧, a **pendulum** 单摆 (small swing), a floating block, the charge on a **capacitor** 电容器 in an LC circuit, atoms in a solid.

The words you must use precisely

displacement x – from equilibrium, signed

amplitude x_0 – the largest $|x|$, always $+$

period T – one full oscillation

frequency $f = 1/T$, in Hz

angular frequency $\omega = 2\pi f = 2\pi/T$

phase difference – a fraction of a cycle, in rad

Given any one of ω, f, T you can reach the other two – start every question by getting ω.

Worked example – maximum acceleration

A mass oscillates with amplitude $0.050\,\mathrm{m}$ at $2.5\,\mathrm{Hz}$. Find its maximum acceleration.

$\omega = 2\pi f = 2\pi \times 2.5 = 15.7\,\mathrm{rad\,s^{-1}}$

$|a|$ is largest at the extremes, where $|a| = \omega^2 x_0$:

$a_{\max} = 15.7^2 \times 0.050 \approx 12\,\mathrm{m\,s^{-2}}$

Fastest at the centre, most accelerated at the ends – the two extremes are opposite. Confusing them is the classic error.

2 x, v and a

Displacement, velocity, acceleration

Starting from $x = 0$ moving positive:

$$x = x_0 \sin \omega t$$

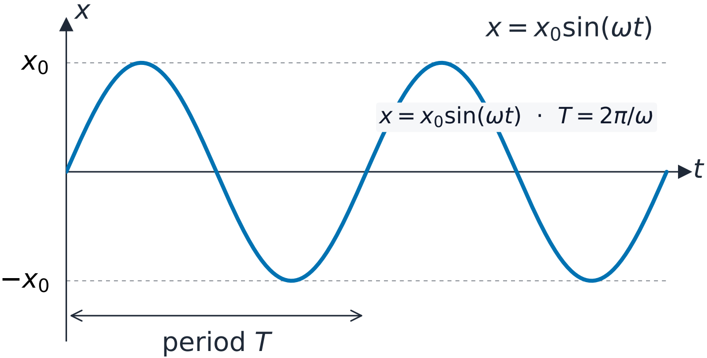
$$v = x_0 \omega \cos \omega t = v_0 \cos \omega t$$

$$a = -x_0 \omega^2 \sin \omega t = -\omega^2 x$$

so $v_0 = x_0 \omega$ and $a_{\max} = \omega^2 x_0$.

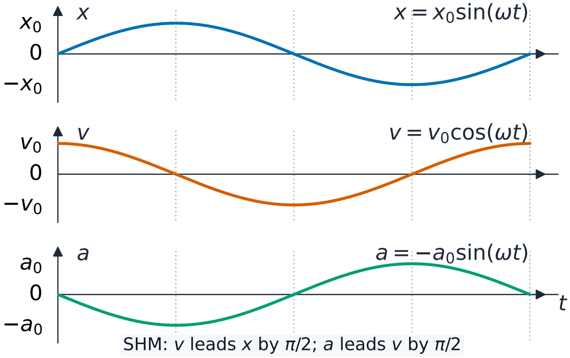
> Starting at the extreme instead? Use $x = x_0 \cos \omega t$. Pick the one that matches $t = 0$.

Displacement, velocity, acceleration, in detail



Displacement is a sine curve of amplitude x_0 when the motion starts at the centre, and a cosine when it starts at the extreme. Pick the one that matches $t = 0$.

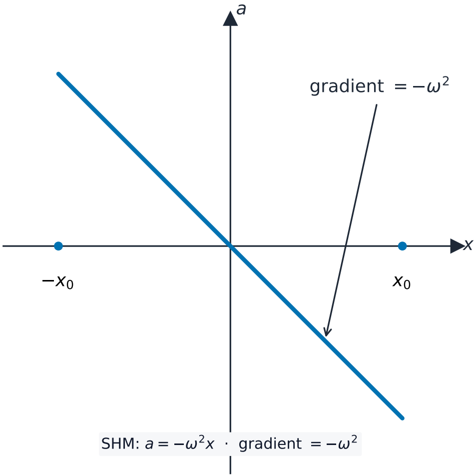
The three graphs together



- x: sine, amplitude x_0
- v: cosine, leads x by $\pi/2$, amplitude ωx_0
- a: $-$sine, π out of phase with x, amplitude $\omega^2 x_0$

> Each differentiation advances the phase by $\pi/2$ and multiplies the amplitude by ω – that is the whole pattern.

Reading ω off the a–x graph



$a = -\omega^2 x$ is a straight line through the origin with gradient $-\omega^2$. So

$$\omega = \sqrt{|\text{gradient}|}, \qquad T = \frac{2\pi}{\omega}$$

> A straight line through the origin with negative gradient *is* the test for SHM – and a very common exam route to T.

Velocity without time

A useful relation that does not use *time*:

$$v = \pm\omega\sqrt{x_0^2 - x^2}$$

- at $x = 0$: $v = \pm\omega x_0$ – the maximum speed
- at $x = \pm x_0$: $v = 0$ – at rest for an instant

> The $\pm$ is real: the particle crosses equilibrium twice a cycle, once each way.

$x_0 = 0.050\,\mathrm{m}$, $\omega = 15.7\,\mathrm{rad\,s^{-1}}$. Find v at $x = 0.030\,\mathrm{m}$.

$v = 15.7\sqrt{0.050^2 - 0.030^2}$

$= 15.7 \times 0.040 \approx 0.63\,\mathrm{m\,s^{-1}}$

3 Energy

Energy in SHM

An oscillator keeps swapping energy between two forms: **kinetic energy** 动能 $\frac{1}{2}mv^2$, and potential energy (elastic for a spring, gravitational for a pendulum).

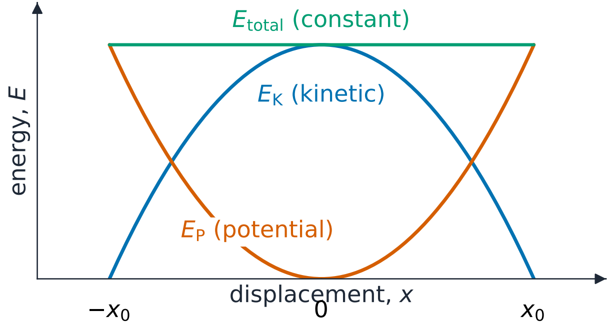
$$E_{\mathrm{K}} = \tfrac{1}{2}m\omega^2(x_0^2 - x^2)$$
$$E_{\mathrm{P}} = \tfrac{1}{2}m\omega^2 x^2$$

E_{K} is a downward parabola, E_{P} an upward one – and their sum is flat:

$$E_{\mathrm{total}} = \tfrac{1}{2}m\omega^2 x_0^2$$

> **Kinetic energy** 动能 and potential energy trade places while the total stays put – **conservation of energy** 能量守恒 for a mass on a **spring** 弹簧. $E \propto x_0^2$: double the amplitude for four times the energy.

Energy in SHM, in detail



SHM: $E_k + E_p =$ const · max E_k at centre

Kinetic and potential energy swap over a cycle while the total energy stays constant

Worked example – total energy

A $0.20\,\mathrm{kg}$ mass oscillates with $x_0 = 0.050\,\mathrm{m}$ and $\omega = 15.7\,\mathrm{rad\,s^{-1}}$. Find the total energy.

The total equals the maximum KE, as it passes $x = 0$ at $v_{\max} = \omega x_0$:

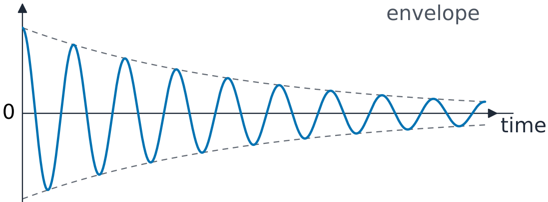
$$E = \tfrac{1}{2} m\omega^2 x_0^2 = \tfrac{1}{2}(0.20)(15.7)^2(0.050)^2$$

$$\approx 0.062\,\mathrm{J}$$

Evaluate the total where one form vanishes – at $x = 0$ it is all kinetic, at $x = x_0$ all potential. Either gives the same number.

4 Damping and resonance

Damped oscillations

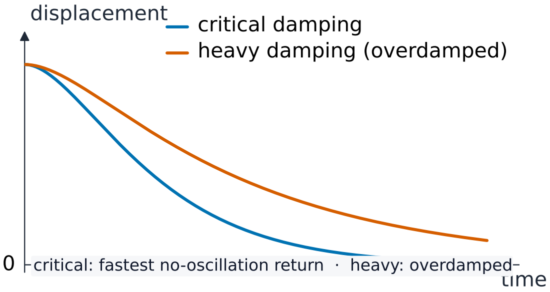


light damping: amplitude decays under envelope

A resistive force (**air resistance** 空气阻力, **friction** 摩擦力) removes energy as heat, so the amplitude shrinks. **Light damping** 轻阻尼: the amplitude dies away slowly over many cycles, still near the natural frequency.

> A car's suspension is light-to-medium damped – bumps die away, but the ride stays smooth.

Critical and heavy damping



- **critical damping** 临界阻尼: back to equilibrium in the shortest time, no overshoot, no oscillation
- **heavy damping** 过阻尼: no oscillation either, but slower than critical

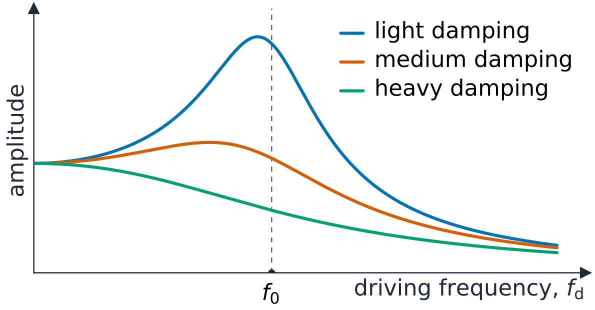
A **galvanometer** 检流计 needle is critically damped so it settles at once; a strong door closer is heavily damped.

Forced oscillations and resonance

A **forced oscillation** 受迫振动 runs at the driving frequency f_d, not its own.
Resonance 共振: $f_\mathrm{d} = f_0$, the **natural frequency** 固有频率 – largest amplitude, most efficient transfer.

> Damping shapes the **resonance curve** 共振曲线: lighter $\rightarrow$ sharper and higher; heavier $\rightarrow$ broader, lower, and nudged to a lower frequency.

Forced oscillations and resonance, in detail



resonance: max amplitude near f_0 · lighter damping ⇒ sharper peak

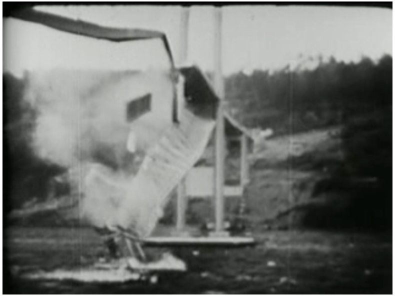
Forced oscillations and resonance A forced oscillation runs at the driving frequency f_d, not its own.

Resonance in the world

- a swing pushed at the right rate builds a big amplitude
- a wine glass shattered by its own ringing note
- a building shaken by an earthquake at a matching frequency

> Engineers design a building's natural frequencies to avoid the main earthquake range – resonance is as often something to dodge as to exploit.

Resonance that brought a bridge down



> Tacoma Narrows, 1940: a driving frequency close to a natural frequency, with too little damping to limit the amplitude.

Exam tips

- The **SHM condition is** $a = -\omega^2 x$ (acceleration proportional to displacement, directed back to equilibrium).
- Learn $x = x_0 \sin \omega t$ (or cos), $v_{max} = \omega x_0$, $a_{max} = \omega^2 x_0$; KE and PE interchange with the **total energy constant**.
- Velocity is **zero at the extremes** and maximum at the centre.
- **Resonance** occurs when the driving frequency equals the natural frequency; damping lowers and broadens the peak.

5 Recap

Topic 17 – key takeaways

1 The condition

$a = -\omega^2 x$ – proportional, and back to equilibrium

2 Maxima

$v_{\max} = \omega x_0$ at the centre; $a_{\max} = \omega^2 x_0$ at the ends

3 Energy

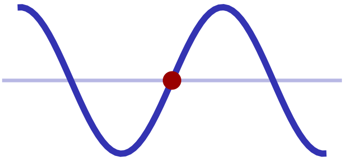
$E = \frac{1}{2} m\omega^2 x_0^2$, constant; $E \propto x_0^2$

4 Resonance

peak at $f_d = f_0$; damping lowers and broadens it

Check yourself

1. Where in the cycle is the speed greatest?
2. What is the gradient of an a–x graph?
3. Triple the amplitude – what happens to E?
4. Which damping settles fastest with no overshoot?



Answers **1.** at equilibrium, $x = 0$ **2.** $-\omega^2$ **3.** $\times 9$ **4.** critical