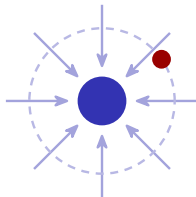


Gravitational Fields

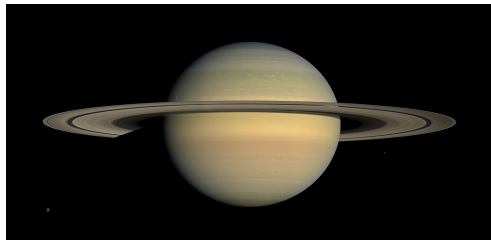
A2 Physics ■ Topic 13

A-Level Physics



What we will cover

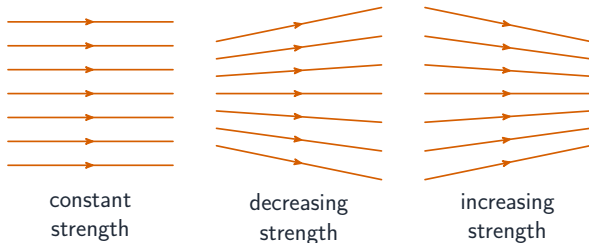
- 1 Fields & field lines
- 2 Newton's law of gravitation
- 3 Field strength $g = GM/r^2$
- 4 Orbits & Kepler's third law
- 5 Gravitational potential
- 6 Recap



1

Fields and field lines

Gravitational field strength



A **gravitational field** 重力场 is a region where a mass feels a force. Its **field strength** 重力场强度 is the force **per unit mass** on a small **test mass** 检验质量:

$$g = \frac{F}{m}, \quad [g] = \text{N kg}^{-1}$$

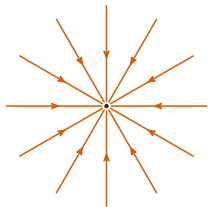
N kg^{-1} is m s^{-2} – g is also the acceleration of free fall. It is a **vector**, pointing at the source.

Definition

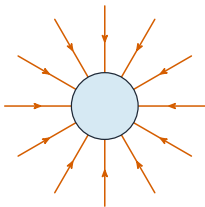
A **gravitational field** 重力场 is a region where a **mass** 质量 feels a **force** 力 from other masses.

The **gravitational field strength** 重力场强度 g at a point is the **gravitational force per unit mass** on a small **test mass** 检验质量 placed there:

Radial and uniform fields



point mass



uniform sphere

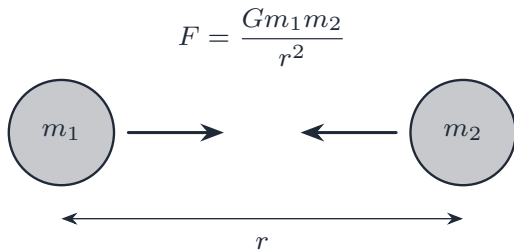
- round a point mass or sphere:
radial 径向, pointing **inwards**
- over a small patch of ground:
parallel and evenly spaced – a
uniform field 匀强场
- **closer lines** = stronger field

Outside a uniform sphere the field is **identical** to a point mass of the same total mass at its centre – which is why you may treat the Earth as a point.

2

Newton's law of gravitation

Newton's law of gravitation



Two point masses attract along the line joining them:

$$F = \frac{Gm_1m_2}{r^2}$$

with $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.

Inverse square: double the separation and the force falls to a **quarter**, not a half.

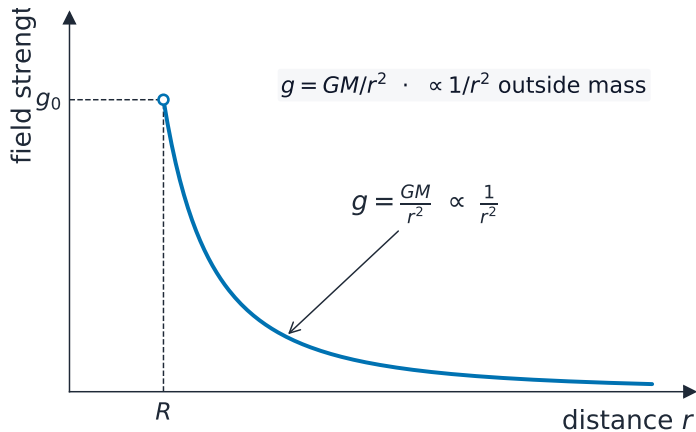
Field strength from a point mass

Put $F = GMm/r^2$ into $g = F/m$; the test mass cancels:

$$g = \frac{GM}{r^2}$$

g depends only on the **source** mass and the distance – never on the mass you put there. It falls off as $1/r^2$.

Field strength from a point mass, in detail



Field strength falls off as $1/r^2$ with distance from a point mass

Worked example – g at the Earth's surface

Earth: $M = 6.0 \times 10^{24} \text{ kg}$, $R = 6.4 \times 10^6 \text{ m}$.

Find g at the surface.

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{(6.4 \times 10^6)^2}$$

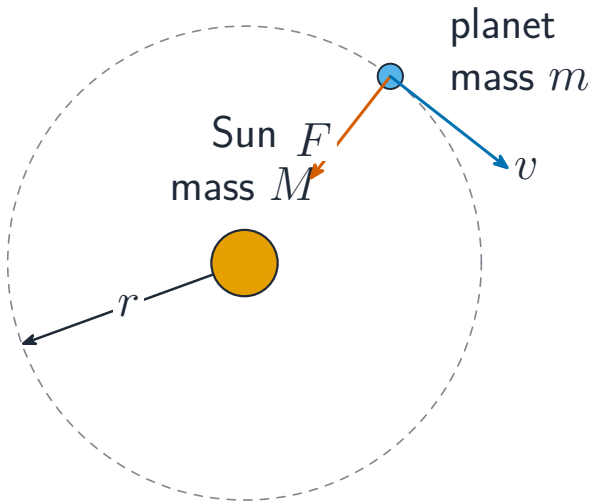
$$\approx 9.8 \text{ N kg}^{-1}$$

Why is g constant in the lab? Climbing 10 m changes r by one part in a million of $R = 6.4 \times 10^6 \text{ m}$.

3

Orbits

Orbital motion



Gravity **is** the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

The satellite's mass **cancels**.
A bolt and a space station
at the same radius orbit at
the same speed – which is
why astronauts float along-
side their craft.

Worked example – orbital speed

**A satellite orbits Earth at $r = 7.0 \times 10^6$ m.
Find its speed. ($GM = 4.0 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$)**

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{4.0 \times 10^{14}}{7.0 \times 10^6}}$$
$$\approx 7.6 \times 10^3 \text{ m s}^{-1}$$

r is measured from the **centre** of the Earth, not the surface – a height above ground must have R added to it first.

Kepler's third law

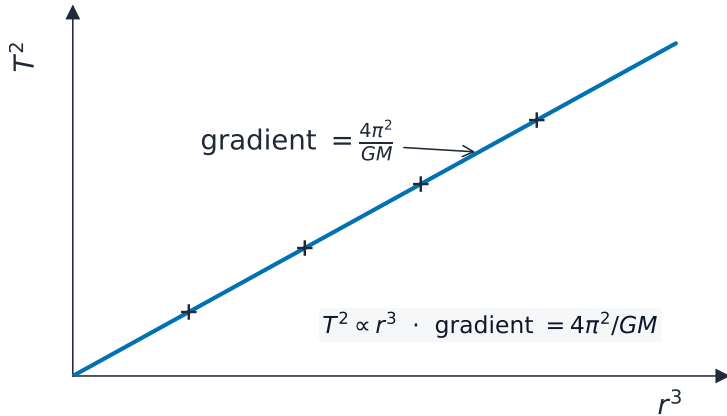
From $T = 2\pi r/v$:

$$T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow T^2 = \frac{4\pi^2}{GM} r^3$$

So $T^2 \propto r^3$ – a straight line through the origin.

The **gradient** of T^2 against r^3 is $4\pi^2/(GM)$, so orbital data **weighs the central body**.

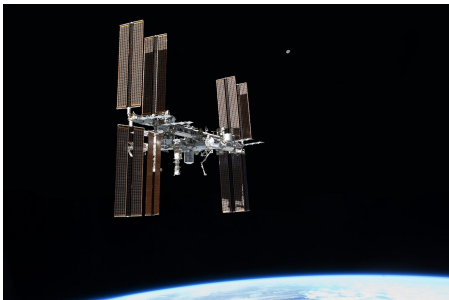
Kepler's third law, in detail



Kepler's third law: $T^2 \propto r^3$, a straight line through the origin

Geostationary orbit

It must have the same **angular speed** 角速度 as the Earth, in the same direction, in the equatorial plane – otherwise it drifts north–south during the day.



A **geostationary** 地球同步 satellite sits over one spot, so it must have:

- period **24 h** – the Earth's own
- direction **west to east**
- plane: above the **equator** 赤道

Then $T^2 = 4\pi^2 r^3 / (GM)$ fixes
 $r \approx 4.2 \times 10^7$ m.

Off the equator it would drift north–south each day, so a fixed dish could not track it.

4

Gravitational potential

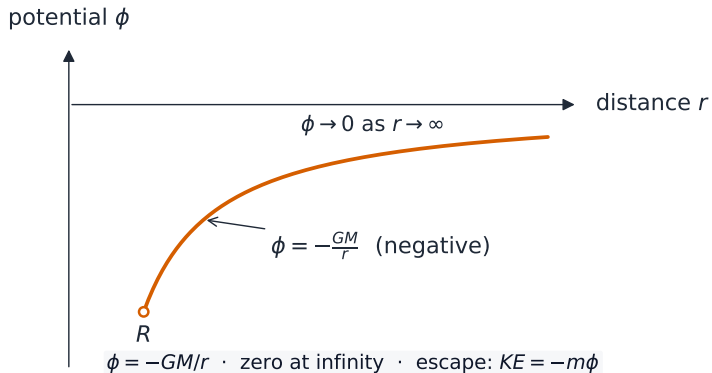
Gravitational potential

Potential 引力势 is the work done **per unit mass** bringing a test mass in from **infinity** 无穷远:

$$\phi = \frac{W}{m} = -\frac{GM}{r}, \quad [\phi] = \text{J kg}^{-1}$$

Zero at infinity, so ϕ is **negative** everywhere else – gravity did the work for you on the way in. ϕ is a **scalar**: just add them.

Gravitational potential, in detail



The gravitational potential well: $\phi = -GM/r$ is negative, rising to zero at infinity

Link with $\Delta E_p = mg\Delta h$

For small height changes near the surface, r barely changes, so $\Delta E_p \approx mg\Delta h$.

For large changes (a satellite moving to a higher orbit) use $-GMm/r$ at each radius and take the difference:

Potential energy and escape

A mass m sitting where the potential is ϕ :

$$E_P = m\phi = -\frac{GMm}{r}$$

Raising a satellite from r_1 to r_2 :

$$\Delta E_P = GMm \left(\frac{1}{r_1} - \frac{1}{r_2} \right) > 0$$

To escape, the kinetic energy must match the well's depth:

$$\frac{1}{2}mv_{\text{esc}}^2 = \frac{GMm}{r} \Rightarrow v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

$mg\Delta h$ is only the **small-height** shortcut, where r hardly changes. Over orbital distances you must difference $-GMm/r$ at each radius.

Escape needs **no rocket left over**: reach v_{esc} and you coast to infinity, arriving with zero speed.

The energy of two point masses

Potential energy

The **gravitational potential energy** 引力势能 of two point masses is $E_p = -\frac{GMm}{r}$ – negative, because the zero is placed at infinity.

The constant

$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ is the **universal gravitational constant** 万有引力常量 – the same everywhere, which is what “universal” asserts.

Escape speed

Escape needs $\frac{1}{2}mv^2 \geq \frac{GMm}{r}$, so $v_{\text{esc}} = \sqrt{2GM/r}$ – independent of the escaping mass.

The minus sign is not decoration: it says work must be *done* to separate the masses, and it is what makes a bound orbit have negative total energy.

Worked example – escape velocity

Find the escape velocity from the Earth's surface. ($GM = 4.0 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$, $R = 6.4 \times 10^6 \text{ m}$)

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(4.0 \times 10^{14})}{6.4 \times 10^6}}$$
$$\approx 1.1 \times 10^4 \text{ m s}^{-1} = 11 \text{ km s}^{-1}$$

The mass **cancels**: a pebble and a rocket need the same 11 km s^{-1} . Note the **2** – escape speed is $\sqrt{2} \times$ the circular orbital speed at the same radius.

Exam tips

- **Newton's law of gravitation** $F = GMm/r^2$ (inverse-square); field strength $g = GM/r^2$.
- Distinguish **gravitational potential** ($\phi = -GM/r$, always negative, zero at infinity) from field strength.
- For an orbit set gravity = centripetal force to get $T^2 \propto r^3$; a geostationary orbit has $T = 24$ h.

5

Recap

Topic 13 – key takeaways

1 Force

$F = GMm/r^2$ – inverse square,
always attractive

2 Field

$g = GM/r^2$, a vector; the test
mass cancels

3 Orbits

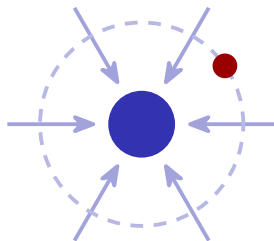
gravity = centripetal $\Rightarrow T^2 \propto r^3$

4 Potential

$\phi = -GM/r$, a scalar, zero at
infinity

Check yourself

- 1 Triple the separation – what happens to F ?
- 2 Why is ϕ always negative?
- 3 What is the period of a geostationary orbit?
- 4 Does escape velocity depend on the object's mass?



Answers 1. falls to $1/9$ 2. zero is set at infinity 3. 24 h 4. no – m cancels