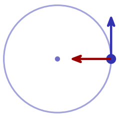
Circular Motion

A2 Physics ▪ Topic 12

A-Level Physics



What we will cover

1 Angles in radians

2 Angular speed

3 Centripetal acceleration

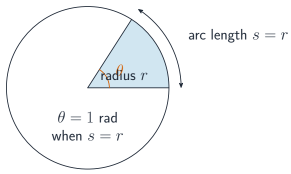
4 Centripetal force

5 Banked & vertical circles

6 Recap

1 Radians

Angles in radians



One **radian** 弧度 is the angle whose **arc** 弧 equals the radius:

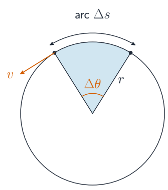
$$\theta = \frac{s}{r}$$

- a ratio of lengths ⇒ no unit
- full circle $= 2\pi\,\mathrm{rad}$; half $= \pi$
- $1\,\mathrm{rad} = 180°/\pi \approx 57.3°$

> Put the calculator in radian mode for this whole topic – degree mode silently gives wrong answers.

2 Angular speed

Angular speed



Angular displacement 角位移 θ is the angle the radius turns through; **angular speed** 角速度 is its rate of change:

$$\omega = \frac{\theta}{t}, \qquad [\omega] = \mathrm{rad\,s^{-1}}$$

One turn ($2\pi\,\mathrm{rad}$) takes the **period** 周期 T:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

> ω measures how fast the angle grows – not how fast the object moves through space.

Linear and angular speed

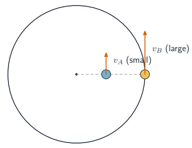
In one period the object covers the circumference $2\pi r$:

$$v = \frac{2\pi r}{T} = r\omega$$

- v is the **tangential** 切向 speed
- same ω, bigger $r \Rightarrow$ bigger v

> On a merry-go-round both children share ω – but the one at the edge moves faster, because $v = r\omega$.

Linear and angular speed, in detail



same ω, but $v = r\omega$ —
the outer rider moves faster

> In one period the object covers the circumference $2\pi r$:
>
> $$v = \frac{2\pi r}{T} = r\omega$$
>
> .

Worked example – a fairground ride

A ride of radius $4.0\,\mathrm{m}$ turns once every $8.0\,\mathrm{s}$. Find ω and the rider's linear speed.

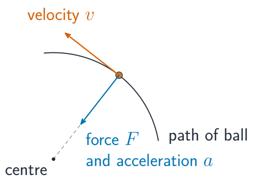
$\omega = \frac{2\pi}{T} = \frac{2\pi}{8.0} = 0.79\,\mathrm{rad\,s^{-1}}$

$v = r\omega = 4.0 \times 0.79 = 3.1\,\mathrm{m\,s^{-1}}$

> Go through ω first: it is the quantity every rider shares. Then $v = r\omega$ gives each rider's own speed from their own radius.

3 Centripetal acceleration

Centripetal acceleration



Constant speed, changing velocity – the direction keeps turning, so there *is* an acceleration:

$$a = \frac{v^2}{r} = r\omega^2$$

(equal, because $v = r\omega$ – pick the form matching your data).

> a is **perpendicular** 垂直 to v at every instant. Any part along the motion would change the speed.

4 Centripetal force

Centripetal force

Circular motion at constant speed is still a changing velocity – the direction turns – so there is an acceleration, and Newton's second law radially gives:

$$F = ma = \frac{mv^2}{r} = mr\omega^2$$

The **centripetal force** 向心力 is not a new force. It is the net result of the real forces – always say which one provides it.

What provides the centripetal force?

ball on a string:
the **tension**

car on a flat corner:
friction

banked track:
the **normal force**

satellite in orbit:
gravity

electron in an atom:
electrostatic pull

too fast ⇒ it **runs out**
and the car skids

The same equation, four different forces

On a string — The tension: $T = mv^2/r$.

Planet or satellite — A **planet or satellite** 行星或卫星 in **orbit** 轨道: the **gravitational attraction** 万有引力, $GMm/r^2 = mv^2/r$.

Electron in an atom — The **electrostatic** 静电 attraction between the electron and the positive **nucleus** 原子核.

Charge in a field — A charged particle in a **magnetic field** 磁场: $Bqv = mv^2/r$, which is how a mass spectrometer sorts by mass.

One **revolution** 转 is 2π radians in a **period** 周期 T, so $\omega = 2\pi/T$ – and the centripetal force is never a new force, only the name of whichever one points inwards.

Worked example – a ball on a string

A $0.20\,\mathrm{kg}$ ball is whirled in a horizontal circle of radius $0.50\,\mathrm{m}$ at $3.0\,\mathrm{m\,s^{-1}}$. Find the tension.

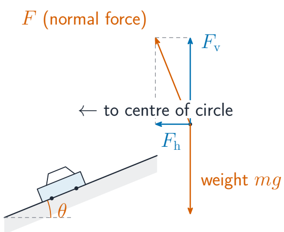
The tension is the centripetal force:

$$F = \frac{mv^2}{r} = \frac{0.20 \times 3.0^2}{0.50} = 3.6\,\mathrm{N}$$

Name the real force first (tension), then set it equal to mv^2/r. Never add a separate "centripetal force" on top of the tension – you would count it twice.

5 Banked and vertical circles

The banked corner



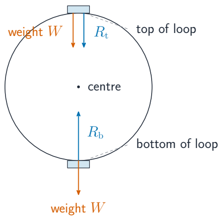
Tilt the track and the **normal contact force** 支持力 leans inwards:

- vertical part balances the weight
- horizontal part turns the car

$$\tan\theta = \frac{v^2}{rg}$$

At that angle the corner needs no friction at all – which is why race tracks and railways are banked.

Vertical circles



The speed is not constant (gravity does work), but the net inward force is still mv^2/r.

- bottom: $T - mg = \dfrac{mv^2}{r}$ – tension largest
- top: $T + mg = \dfrac{mv^2}{r}$ – tension smallest

> At the top both the weight and the tension point to the centre – they add, they do not cancel.

Worked example – the top of a loop

A car loops a vertical circle of radius $2.0\,\mathrm{m}$. Find the minimum speed at the top that keeps it on the track. ($g = 9.81\,\mathrm{m\,s^{-2}}$)

Slowest ⇒ the track pushes with zero force, so gravity alone turns the car:

$$mg = \frac{mv_{\min}^2}{r} \;\Rightarrow\; v_{\min} = \sqrt{gr}$$

$$v_{\min} = \sqrt{9.81 \times 2.0} \approx 4.4\,\mathrm{m\,s^{-1}}$$

> "Just keeps contact" always means set the contact force to zero. The mass cancels – the answer is the same for a car or a coin.

How to structure the answer

1. find r; pick v or ω ($v = r\omega$) → **2.** get $a = \dfrac{v^2}{r}$ or $r\omega^2$ → **3.** list the *real* forces → **4.** *N2* radially: net in $= \dfrac{mv^2}{r}$ → **5.** check the directions

> For a period or frequency, reach for $\omega = 2\pi/T$ or $T = 2\pi r/v$.

Exam tips

- Work in **radians**; angular speed $\omega = 2\pi/T = v/r$.
- **Centripetal acceleration $a = v^2/r = \omega^2 r$; the net force acts towards the centre** – it is provided by tension/gravity/friction, not an extra force.
- Always state **what provides the centripetal force** in the situation given.

6 Recap

Topic 12 – key takeaways

1 Radians
$\theta = s/r$, work in rad, not degrees

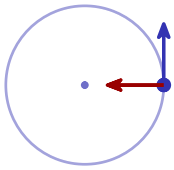
2 Speeds
$\omega = 2\pi/T$ and $v = r\omega$

3 Acceleration
$a = v^2/r = r\omega^2$, always to the centre

4 Force
$F = mv^2/r$, provided by a *real* force

Check yourself

1. How many radians in a half-circle?
2. A wheel turns once in $0.50\,\mathrm{s}$. Find ω.
3. Which way does the centripetal acceleration point?
4. What provides the centripetal force on a satellite?



Answers **1.** π **2.** $\omega = 2\pi/0.50 \approx 13\,\mathrm{rad\,s^{-1}}$ **3.** to the centre **4.** gravity