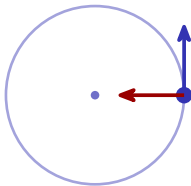


Circular Motion

A2 Physics ■ Topic 12

A-Level Physics



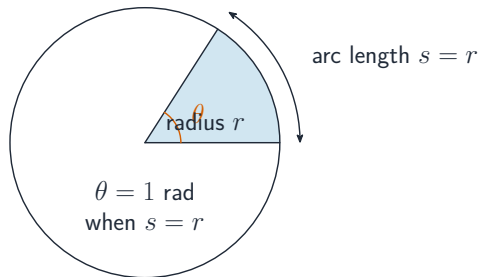
What we will cover

- 1 Angles in radians
- 2 Angular speed
- 3 Centripetal acceleration
- 4 Centripetal force
- 5 Banked & vertical circles
- 6 Recap



1 Radians

Angles in radians



One **radian** 弧度 is the angle whose **arc** 弧 equals the radius:

$$\theta = \frac{s}{r}$$

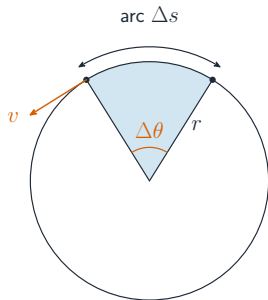
- a ratio of lengths $\Rightarrow$ **no unit**
- full circle = 2π rad; half = π
- $1 \text{ rad} = 180^\circ/\pi \approx 57.3^\circ$

Put the calculator in **radian** mode for this whole topic – degree mode silently gives wrong answers.

2

Angular speed

Angular speed



Angular displacement 角位移 θ is the angle the radius turns through; **angular speed** 角速度 is its rate of change:

$$\omega = \frac{\theta}{t}, \quad [\omega] = \text{rad s}^{-1}$$

One turn ($2\pi \text{ rad}$) takes the **period** 周期 T :

$$\omega = \frac{2\pi}{T} = 2\pi f$$

ω measures how fast the **angle** grows
– not how fast the object moves
through space.

Linear and angular speed

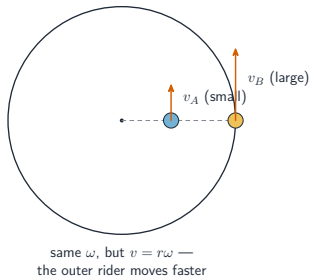
In one period the object covers the circumference $2\pi r$.

$$v = \frac{2\pi r}{T} = r\omega$$

- v is the **tangential** 切向 speed
- same ω , bigger $r \Rightarrow$ bigger v

On a merry-go-round both children share ω – but the one at the **edge** moves faster, because $v = r\omega$.

Linear and angular speed, in detail



In one period the object covers the circumference $2\pi r$:

$$v = \frac{2\pi r}{T} = r\omega$$

Worked example – a fairground ride

A ride of radius 4.0 m turns once every 8.0 s. Find ω and the rider's linear speed.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8.0} = 0.79 \text{ rad s}^{-1}$$

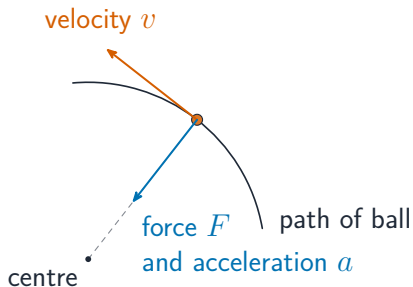
$$v = r\omega = 4.0 \times 0.79 = 3.1 \text{ m s}^{-1}$$

Go through ω **first**: it is the quantity every rider shares. Then $v = r\omega$ gives each rider's own speed from their own radius.

3

Centripetal acceleration

Centripetal acceleration



Constant **speed**, changing **velocity** – the direction keeps turning, so there *is* an acceleration:

$$a = \frac{v^2}{r} = r\omega^2$$

(equal, because $v = r\omega$ – pick the form matching your data).

a is **perpendicular** 垂直 to v at every instant. Any part **along** the motion would change the speed.

4

Centripetal force

Centripetal force



Circular motion at constant speed is still a **changing velocity** – the direction turns – so there is an acceleration, and Newton's second law radially gives:

$$F = ma = \frac{mv^2}{r} = mr\omega^2$$

The **centripetal force** 向心力 is **not a new force**. It is the **net result** of the real forces – always say **which** one provides it.

What provides the centripetal force?

ball on a string:
the **tension**

car on a flat corner:
friction

banked track:
the **normal force**

satellite in orbit:
gravity

electron in an atom:
electrostatic pull

too fast $\Rightarrow$ it **runs out**
and the car skids

The same equation, four different forces

On a string

The tension: $T = mv^2/r$.

Planet or satellite

A planet or satellite 行星或卫星 in orbit 轨道: the gravitational attraction 万有引力, $GMm/r^2 = mv^2/r$.

Electron in an atom

The electrostatic 静电 attraction between the electron and the positive nucleus 原子核.

Charge in a field

A charged particle in a magnetic field 磁场: $Bqv = mv^2/r$, which is how a mass spectrometer sorts by mass.

One **revolution** 转 is 2π radians in a **period** 周期 T , so $\omega = 2\pi/T$ – and the centripetal force is never a new force, only the name of whichever one points inwards.

Worked example – a ball on a string

A 0.20 kg ball is whirled in a horizontal circle of radius 0.50 m at 3.0 m s^{-1} . Find the tension.

The tension **is** the centripetal force:

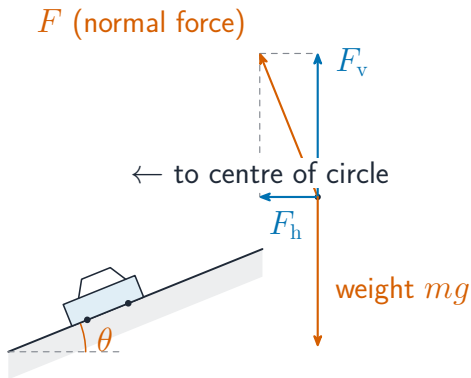
$$F = \frac{mv^2}{r} = \frac{0.20 \times 3.0^2}{0.50} = 3.6 \text{ N}$$

Name the real force first (**tension**), then set it equal to mv^2/r . Never add a separate “centripetal force” on top of the tension – you would count it twice.

5

Banked and vertical circles

The banked corner



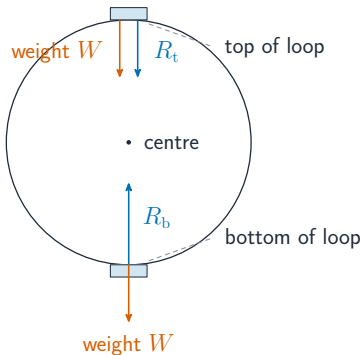
Tilt the track and the **normal contact force** 支持力 leans inwards:

- vertical part balances the weight
- horizontal part turns the car

$$\tan \theta = \frac{v^2}{rg}$$

At that angle the corner needs **no friction at all** – which is why race tracks and railways are banked.

Vertical circles



The speed is **not** constant (gravity does work), but the **net inward** force is still mv^2/r .

- bottom: $T - mg = \frac{mv^2}{r}$ – tension **largest**
- top: $T + mg = \frac{mv^2}{r}$ – tension **smallest**

At the top both the weight **and** the tension point to the centre – they add, they do not cancel.

Worked example – the top of a loop

A car loops a vertical circle of radius 2.0 m. Find the minimum speed at the top that keeps it on the track. ($g = 9.81 \text{ m s}^{-2}$)

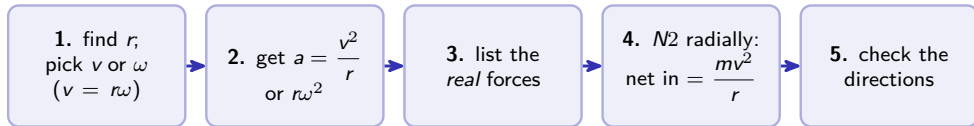
Slowest $\Rightarrow$ the track pushes with **zero** force, so gravity alone turns the car:

$$mg = \frac{mv_{\min}^2}{r} \Rightarrow v_{\min} = \sqrt{gr}$$

$$v_{\min} = \sqrt{9.81 \times 2.0} \approx 4.4 \text{ m s}^{-1}$$

“Just keeps contact” always means **set the contact force to zero**. The mass cancels – the answer is the same for a car or a coin.

How to structure the answer



For a period or frequency, reach for $\omega = 2\pi/T$ or $T = 2\pi r/v$.

Exam tips

- Work in **radians**; angular speed $\omega = 2\pi/T = v/r$.
- **Centripetal acceleration** $a = v^2/r = \omega^2 r$; **the net force acts towards the centre** – it is provided by tension/gravity/friction, not an extra force.
- Always state **what provides the centripetal force** in the situation given.

6

Recap

Topic 12 – key takeaways

1 Radians

$\theta = s/r$, work in rad, not degrees

2 Speeds

$\omega = 2\pi/T$ and $v = r\omega$

3 Acceleration

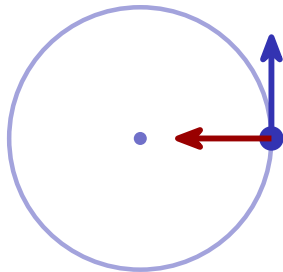
$a = v^2/r = r\omega^2$, always to the centre

4 Force

$F = mv^2/r$, provided by a *real* force

Check yourself

- 1 How many radians in a half-circle?
- 2 A wheel turns once in 0.50 s. Find ω .
- 3 Which way does the centripetal acceleration point?
- 4 What provides the centripetal force on a satellite?



Answers 1. π 2. $\omega = 2\pi/0.50 \approx 13 \text{ rad s}^{-1}$ 3. to the centre 4. gravity