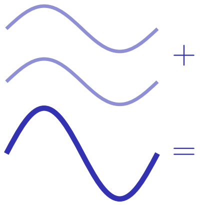
Superposition

AS Physics ▪ Topic 8

A-Level Physics



What we will cover

1. Principle of superposition
2. Stationary waves
3. Diffraction
4. Interference
5. The diffraction grating
6. Recap

1 Superposition

Principle of superposition

Where waves overlap, the displacement is the vector sum of each wave's displacement – and they then pass through each other unchanged.

in phase

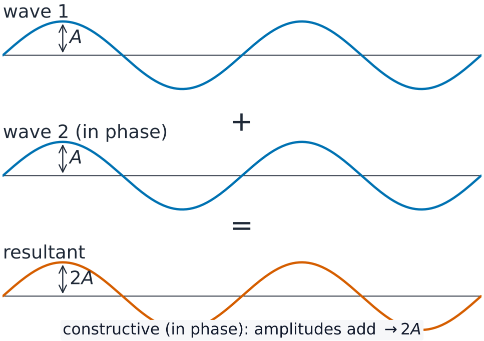
$A_1 + A_2$ – **constructive interference 相长干涉**

out of phase (π)

amplitude $|A_1 - A_2|$ (destructive)

Two equal waves in phase: $I \propto (2A)^2 = 4A^2$ – four times the intensity.

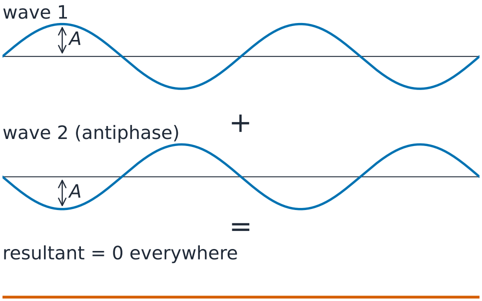
In phase: add



in phase: add

Crest 波峰 on crest: the two add, and the result is **constructive interference** 相长干涉.

Out of phase: cancel



destructive (out of phase by $\lambda/2$): cancel if equal A

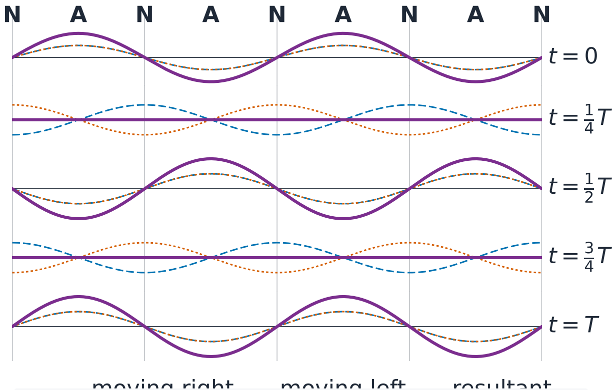
out of phase: cancel

> Crest on **trough** 波谷 – exactly in **antiphase** 反相 – gives **destructive interference** 相消干涉: they cancel.

2

Stationary waves

Stationary (standing) waves

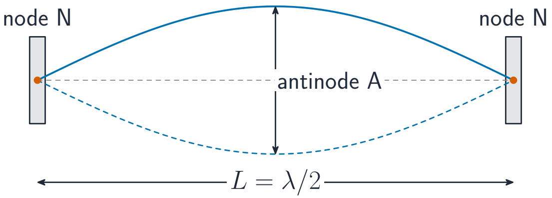


stationary wave: nodes and antinodes from two opposite waves

Two identical **progressive waves** 行波 travelling opposite ways overlap – usually a wave and its own **reflected** 反射 copy from a fixed end.

> It stores energy but does not carry it along – the pattern stays put.

Nodes and antinodes



- **node** 波节 – always zero displacement
- **antinode** 波腹 – largest amplitude
- node to node $= \lambda/2$; node to antinode $= \lambda/4$

String $0.80\,\mathrm{m}$, **fixed ends,** $v = 240\,\mathrm{m\,s^{-1}}$

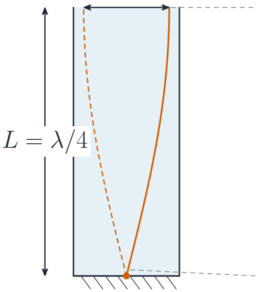
Fundamental: $\lambda = 2L = 1.60\,\mathrm{m}$, so $f = v/\lambda = 240/1.60 = 150\,\mathrm{Hz}$.

Measuring wavelength from node spacing

Drive a string with a vibrator at frequency f until a stationary pattern appears.

> Measure the distance between two well-separated nodes and divide by the number of half-**wavelengths 波长** between them.

Resonance in pipes



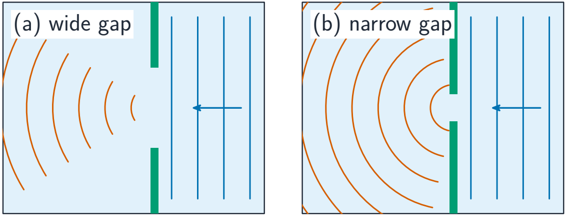
Resonance tube 共鸣管, closed one end:

- closed end = node, open end = antinode
- **fundamental** 基频: $L = \lambda/4$; next $L = 3\lambda/4$
- open both ends: fundamental $L = \lambda/2$

Then $v = f\lambda$ gives the wave speed.

3 Diffraction

Diffraction



gap $\gg \lambda$: little spreading   gap $\approx \lambda$: strong spreading

Diffraction 衍射: a wave spreads after a gap or obstacle. Spread depends on λ vs gap width:

- gap $\gg \lambda$: little spread
- gap $\approx \lambda$: large spread

> In a **ripple tank** 水波槽 the spread grows as the gap narrows. Same idea: you hear round a corner but cannot see round it.

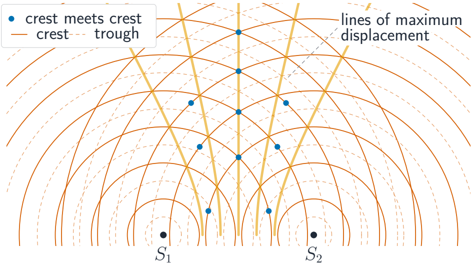
4 Interference

Interference and coherence

Interference 干涉: two **coherent** 相干 waves give a steady pattern. Coherent = constant phase difference (so the same frequency). Two lamps are not.

> Make coherent light by passing one source through a **double-slit** 双缝 pair of **slits** 狭缝: both are lit by the same wavefront.

Interference and coherence, in detail



The pattern only stands still if the sources are coherent: same frequency, constant phase difference.

Four conditions for a clear fringe pattern

1. two **coherent** 相干 sources – a constant phase difference
2. roughly equal amplitudes, or the dark fringes are not dark
3. the waves overlap where you are looking
4. for light, the same plane of **polarisation** 偏振

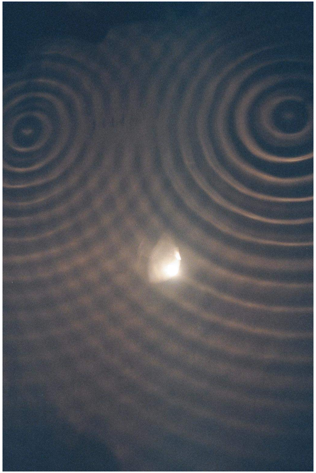
> Miss any one and the fringes wash out – which is why two lamps show nothing.

Path difference

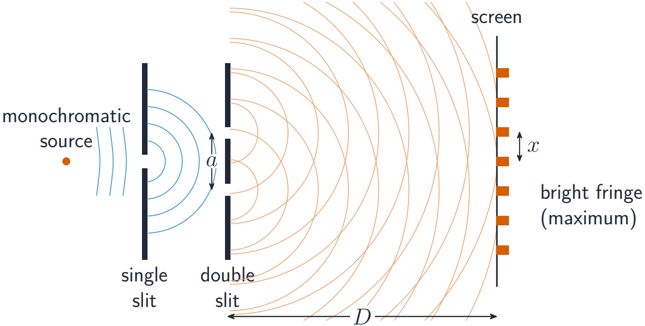
What happens at a point depends on the **path difference** 路程差 Δx:

bright: $\Delta x = n\lambda$

dark: $\Delta x = (n + \frac{1}{2})\lambda$



Young's double-slit experiment



Fringe spacing 条纹间距:

$$x = \frac{\lambda D}{a}$$

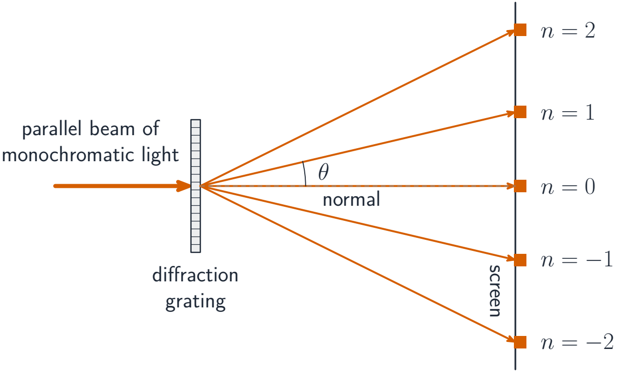
Smaller fringes: bigger a, smaller D, or bluer light.

$a = 0.50\,\mathrm{mm},\ \lambda = 600\,\mathrm{nm},\ D = 2.0\,\mathrm{m}$

$x = (600 \times 10^{-9})(2.0)\,/\,(0.50 \times 10^{-3}) = 2.4\,\mathrm{mm}$

5 Diffraction grating

The diffraction grating



Many slits, spacing d – maxima where

$$d \sin\theta = n\lambda.$$

500 lines/mm, $\lambda = 600\,\mathrm{nm}$, $n = 1$

$d = 2.0\,\mu\mathrm{m},\ \sin\theta = 0.30 \Rightarrow \theta \approx 17°$

A diffraction grating splits monochromatic light into sharp maxima on a screen

Working with a grating

slit spacing

N lines/mm $\Rightarrow d = 10^{-3}/N$ m

highest order

$n_{\max} = \lfloor d/\lambda \rfloor$: at $d/\lambda = 3.27$
you see up to $n = 3$, never $n = 4$

sharper maxima

more slits add $\Rightarrow$ narrower,
brighter lines than two slits

> To find an unknown λ: shine it straight at the grating, measure θ_1 to the first-order **maximum** 极大, then $\lambda = d \sin\theta_1$. Averaging over higher orders cuts the error.

Exam tips

- Two-source interference: **constructive** when path difference $= n\lambda$, **destructive** when $= (n + \frac{1}{2})\lambda$; the sources must be **coherent**.
- Double slit: $\lambda = ax/D$; diffraction grating: $d \sin \theta = n\lambda$ – know every symbol.
- On a **stationary wave** mark nodes and antinodes; adjacent nodes are $\lambda/2$ apart; it stores energy but does not transfer it.
- A stationary wave needs two waves of the **same frequency travelling in opposite directions**.

6 Recap

Topic 8 – key takeaways

1 Superposition

displacements add; in phase $4\times$ the intensity

2 Stationary waves

nodes & antinodes a $\lambda/2$ apart

3 Interference

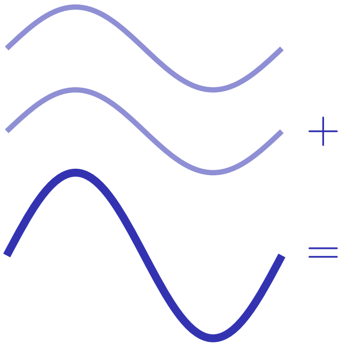
coherent sources; $x = \lambda D/a$

4 Grating

$d \sin\theta = n\lambda$, sharp maxima

Check yourself

1. Distance between two next-door nodes?
2. What does “coherent” mean?
3. Path difference for a bright fringe?
4. Grating $d \sin\theta = n\lambda$ – which n is straight on?



Answers **1.** $\lambda/2$ **2.** constant phase difference **3.** $\Delta x = n\lambda$ **4.** $n = 0$