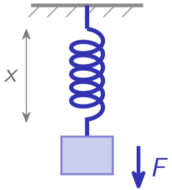
Deformation of Solids

AS Physics ▪ Topic 6

A-Level Physics



What we will cover

1. Forces and deformation
2. Hooke's law
3. Stress, strain & Young modulus
4. Elastic vs plastic
5. Energy stored
6. Recap

1 Forces and deformation

Forces that change shape

A force along the length changes the shape – **deformation** 形变. Two one-dimensional cases:

Tensile 拉伸

stretches → **extension** 伸长量 x

Compressive 压缩

squeezes → a **compression** 压缩, a negative x

Tensile testing machine

The applied force is the **load** 负载; the shape change is measured from the natural length.

2 Hooke's law

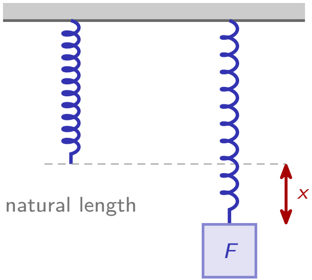
Hooke's law

Extension is proportional to load (at small extensions):

$$F = kx, \qquad k = \frac{F}{x}, \quad [k] = \mathrm{N\,m^{-1}}.$$

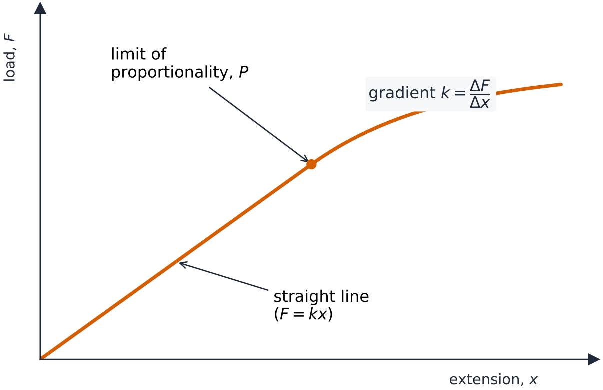
$2.0\,\mathrm{N}$ stretches a spring $4.0\,\mathrm{cm}$

$$k = \frac{2.0}{0.040} = 50\,\mathrm{N\,m^{-1}}$$



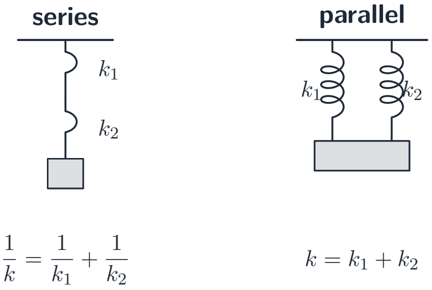
The load–extension graph

- gradient of F vs $x = k$
- straight only up to the **limit of proportionality** 比例极限 P
- beyond P the line curves – the material may stay **elastic** 弹性 a little further, then turns **plastic** 塑性
- plotting *length* L instead of x? The gradient still gives k, since $L = L_0 + F/k$



Springs in series and parallel

- **series** 串联 – the same load passes through both and the extensions add: $\dfrac{1}{k} = \dfrac{1}{k_1} + \dfrac{1}{k_2}$ (softer)
- **parallel** 并联 – each takes half the load and the extensions are equal: $k = k_1 + k_2$ (stiffer)



3 Stress, strain, Young modulus

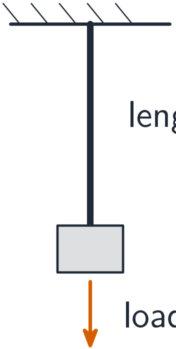
Stress, strain and the Young modulus

- **stress** 应力 $\sigma = \dfrac{F}{A}$ (Pa)
- **strain** 应变 $\varepsilon = \dfrac{x}{L_0}$ (no unit)

$$E = \frac{\sigma}{\varepsilon} = \frac{FL_0}{Ax}.$$

> E is a material property; $k = EA/L_0$ also depends on size.

Stress, strain and the Young modulus, in detail



$$\text{stress } \sigma = \frac{F}{A}$$

$$\text{strain } \varepsilon = \frac{x}{L_0} \quad (x = \text{extension})$$

> Stress is the load per cross-sectional area; strain is the extension per original length

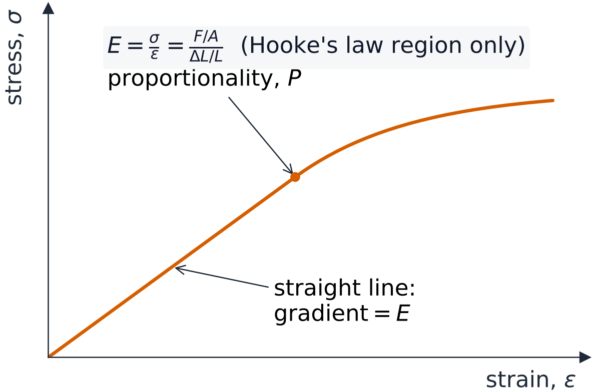
Worked example – the Young modulus

Steel wire: $L_0 = 2.0\,\mathrm{m}$, $A = 1.5\times10^{-7}\,\mathrm{m}^2$, $x = 1.0\,\mathrm{mm}$ **under** $15\,\mathrm{N}$

$$E = \frac{FL_0}{Ax} = \frac{15\times2.0}{(1.5\times10^{-7})(1.0\times10^{-3})}$$

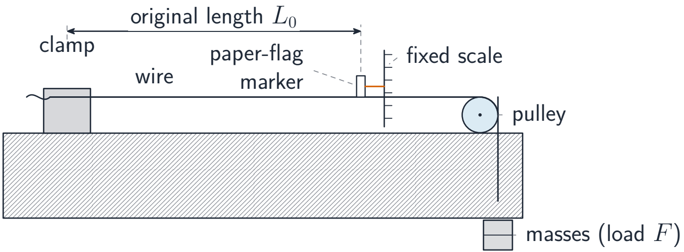
$$= 2.0\times10^{11}\,\mathrm{Pa}.$$

Worked example – the Young modulus, in detail



A stress–strain graph, straight up to the limit of proportionality P

Measuring E for a wire

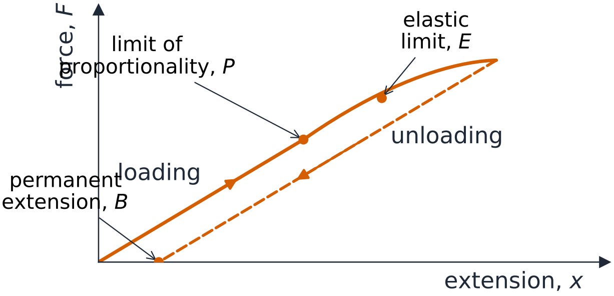


1. long thin wire over a **pulley** 滑轮
2. $A = \pi d^2/4$ from a **micrometer** 螺旋测微器
3. add loads; plot F vs x
4. $E = \text{gradient} \times L_0/A$

Long & thin gives a measurable extension; repeating and measuring the **diameter** 直径 in several places cuts **random error** 随机误差.

4 Elastic vs plastic

Elastic and plastic behaviour



Hooke up to limit of proportionality · plastic beyond yield

1. straight, Hooke obeyed (to P)
2. **elastic** 弹性: returns to first length (to the **elastic limit** 弹性极限)
3. **plastic** 塑性: a permanent extension stays

5 Energy stored

Energy stored in a stretched material

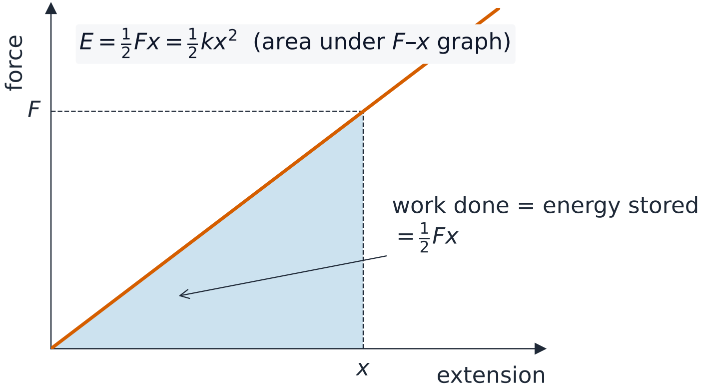
Work done = area under the F–x graph. For a Hooke material:

$$E_P = \frac{1}{2}Fx = \frac{1}{2}kx^2 = \frac{F^2}{2k}.$$

$k = 50\,\mathrm{N\,m^{-1}}$, $x = 0.20\,\mathrm{m}$

$E_P = \frac{1}{2}(50)(0.20)^2 = 1.0\,\mathrm{J}$

Energy stored in a stretched material, in detail



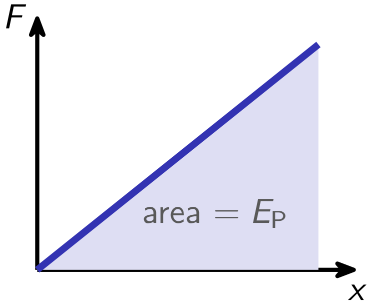
Work done = area under the F–x graph.

Comparing and releasing stored energy

Using $E_P = \frac{1}{2}Fx$:

- same F, softer spring $\to$ bigger x $\to$ more energy
- same x, stiffer spring $\to$ bigger F $\to$ more energy

> Released onto a mass, the **elastic potential energy** 弹性势能 becomes **kinetic energy** 动能: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$ ($+mgh$ if it rises).



Exam tips

- **Hooke's law ($F = kx$)** holds only up to the limit of proportionality.
- **Stress** $= F/A$, **strain** $= x/L$, **Young modulus** $=$ **stress**/**strain** (gradient of the straight part of the stress-strain graph) – watch the units (Pa).
- Energy stored $=$ **area under the force-extension graph** $= \frac{1}{2}Fx$ in the elastic region.
- Distinguish **elastic** (returns to shape) from **plastic** (permanent) deformation.

6 Recap

Topic 6 – key takeaways

1 Hooke's law

$F = kx$ up to the limit of proportionality

2 Young modulus

$E = \sigma/\varepsilon = FL_0/Ax$, a material property

3 Elastic / plastic

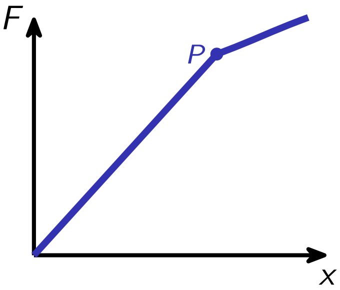
plastic leaves a permanent extension

4 Stored energy

area under F–x: $\frac{1}{2}kx^2$

Check yourself

1. Unit of the spring constant k?
2. Does the Young modulus depend on the wire's shape?
3. Energy in a spring, $k = 200$, $x = 0.10\,\mathrm{m}$?
4. What stays after a plastic deformation?



Answers **1.** $\mathrm{N\,m^{-1}}$ **2.** no (material only) **3.** $\frac{1}{2}kx^2 = 1.0\,\mathrm{J}$ **4.** a permanent extension