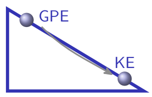
Work, Energy & Power

AS Physics ▪ Topic 5

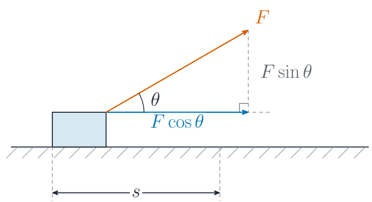
A-Level Physics



What we will cover

1 Work

Work done by a force



Work 功 = force × distance along the force:

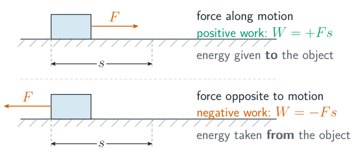
$W = Fs\cos\theta$, $[W] = \mathrm{J} = \mathrm{N\,m}$.

Work is a **scalar** 标量.

Sledge: $20\,\mathrm{N}$ **at** $60°$, $5.0\,\mathrm{m}$

$W = 20 \times 5.0 \times \cos 60° = 50\,\mathrm{J}$

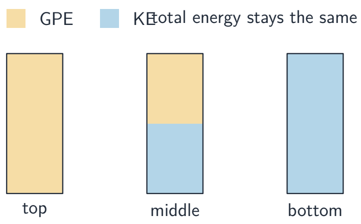
Positive, negative and zero work



- $\theta = 0$: $W = +Fs$ – energy in
- $\theta = 90°$: $W = 0$ – no work (normal force on a flat road)
- $\theta = 180°$: $W = -Fs$ – energy out (**friction** 摩擦力)

2 Conservation of energy

Conservation of energy



Energy is never made or destroyed – it only changes form, or moves between objects. In a **closed system** 封闭系统 the total is constant. On a frictionless **ramp** 斜坡:

$$mgh = \frac{1}{2}mv^2 \;\Rightarrow\; v = \sqrt{2gh}.$$

With friction, some becomes **thermal energy** 热能. List every form at the start and the end: kinetic, gravitational potential, **elastic potential** 弹性势能, **electrical** 电能, thermal, sound, **chemical energy** 化学能.

Where $mg\Delta h$ comes from

In a uniform field, raising a **mass** 质量 m through a height Δh costs **gravitational potential energy** 重力势能: $\boxed{\Delta E_P = mg\Delta h}$

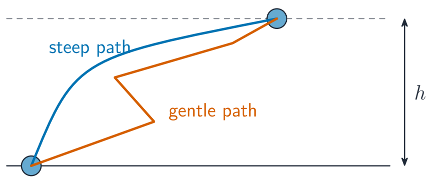
Lift it *slowly*, so no kinetic energy is gained:

- the gravitational force on the mass is mg downwards,
- the force needed to lift it slowly is mg upwards,
- the work done by this force is $W = F\cdot s = mg\cdot\Delta h$,
- this work becomes ΔE_P.

> You need m, Δh and g – never speed or time.

Only the height matters, not the path

same $\Delta E_P = mgh$ – only the height h matters



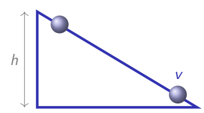
> Steep or zig-zag, the gain is mgh either way: gravitational PE counts only the height risen.

Worked example – speed down a ramp

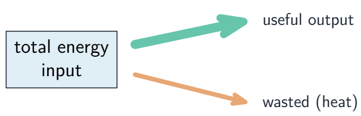
Released from rest, smooth ramp $h = 1.2\,\mathrm{m}$

All GPE becomes KE, so $v = \sqrt{2gh}$ (mass cancels):

$$v = \sqrt{2\times 9.81\times 1.2} \approx 4.9\,\mathrm{m\,s^{-1}}.$$



Efficiency



$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}} \times 100\%$$

$$\eta = \frac{\text{useful output}}{\text{total input}} \times 100\%.$$

Motor lifts $50\,\mathrm{kg}$ at $0.40\,\mathrm{m\,s^{-1}}$, draws $250\,\mathrm{W}$

useful $P = mgv = 50\times 9.81\times 0.40 = 196\,\mathrm{W}$

$$\eta = \frac{196}{250}\times 100\% \approx 78\%.$$

Always $< 100\%$ – the rest is wasted as heat.

> Efficiency: only part of the energy input leaves as useful output; the rest is wasted, usually as heat

3 Power

Power – the rate of energy transfer

$$P = \frac{W}{t} = \frac{\Delta E}{\Delta t}, \qquad [P] = \mathrm{W} = \mathrm{J\,s^{-1}}.$$

Power is a **scalar** 标量.

same work in less time → **more power**

Same work, different power

same work W done → in time t ⇒ power P

same work W done → in time $t/2$ ⇒ power $2P$

$$P = \frac{W}{t} \text{ — less time, more power}$$

Power is work per second: climbing the stairs twice as fast does the *same* work against gravity, at twice the power.

4 Potential and kinetic energy

Kinetic energy: where $\frac{1}{2}mv^2$ comes from

$$\boxed{E_K = \tfrac{1}{2}mv^2}$$

Push a mass m from rest up to speed v over a displacement s.

- from $v^2 = u^2 + 2as$ with $u = 0$: $s = \dfrac{v^2}{2a}$
- the work done is $W = Fs = ma \cdot \dfrac{v^2}{2a}$
- the a cancels: $W = \frac{1}{2}mv^2$

All of that work becomes kinetic energy, so $E_K = \frac{1}{2}mv^2$.

When you know the momentum, not the speed

$p = mv$ and $E_K = \frac{1}{2}mv^2$

$$\downarrow$$

$$\boxed{E_K = \frac{p^2}{2m}}$$

A momentum change from p_1 to p_2 at constant mass changes the kinetic energy by $\dfrac{p_2^2 - p_1^2}{2m}$.

Reach for this whenever a question gives you **momentum** 动量 and asks for energy – there is no need to find v first.

Power at a steady speed is force times velocity

$$\boxed{P = Fv}$$

In a time Δt the force moves through $v\,\Delta t$, doing $Fv\,\Delta t$ of work. Divide by Δt.

Car at $25\,\mathrm{m\,s^{-1}}$ against $600\,\mathrm{N}$

At constant speed the driving force equals the resistive force, so $P = 600 \times 25 = 15\,\mathrm{kW}$.

- a car on a flat road: the engine balances the total resistive force, $P = F_{\text{resist}}\,v$
- **drag** 阻力 grows like v^2, so doubling the speed roughly quadruples the power
- lifting a **weight** 重力 mg at steady v: useful $P = mg\,v$ – the same $P = Fv$ with $F = mg$
- a hovering aircraft needs large power – air must be pushed down all the time

Three quantities a power equation can give you

A force	At a steady speed the useful power is $P = Fv$, so a measured power and speed give the force opposing motion.
A lifting speed	For an electric motor lifting a load at **voltage** 电压 V and **current** 电流 I with efficiency η, the useful output power is ηVI – and $\eta VI = mgv$ gives the speed.
A tension	Same equation, read the other way: the **tension** 张力 in the cable is the useful power divided by the speed.
Which forces do no work	A force perpendicular to the motion does none – the **normal contact force** 法向接触力 on a car on a flat road, or any centripetal force.

> Work is $Fs\cos\theta$ with θ measured from the **horizontal** 水平 displacement – so a force at 90° transfers no energy at all.

Using energy methods

1. write energies at the start and end
2. add work by outside forces (push in, friction out)
3. set $E_{\text{start}} + W_{\text{in}} = E_{\text{end}} + W_{\text{lost}}$

- into a **spring** 弹簧: $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$ at greatest **compression** 压缩, where k is the **spring constant** 劲度系数
- box pushed up a ramp at constant velocity: E_K is unchanged, so the push pays for ΔE_P *and* the friction
- a ball bouncing from h_1 to h_2: the fraction of energy kept is $h_2/h_1 = (v_{\text{up}}/v_{\text{down}})^2$
- a **projectile** 抛体 thrown to the same height at any angle lands at the same speed – only the height counts; use **components** 分量 for the direction

Using energy methods, in detail

> A block's kinetic energy becomes elastic potential energy as it compresses the spring

Exam tips

- **Work** = **force** × **distance moved in the direction of the force** – use $Fs\cos\theta$ when they are at an angle.
- Use **conservation of energy**: loss in GPE = gain in KE (+ work done against resistance).
- **Power** = **work** / **time** = Fv; efficiency = useful output ÷ total input.
- GPE uses the **vertical** height gained, not the distance along a slope.

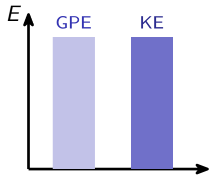
5 Recap

Topic 5 – key takeaways

1. **Work** — $W = Fs\cos\theta$; energy in or out
2. **Energy** — conserved; only changes form
3. **Power** — $P = W/t = Fv$, efficiency $< 100\%$
4. **Stores** — $E_P = mg\Delta h$, $E_K = \frac{1}{2}mv^2$

Check yourself

1. Work done by a force at 90° to the motion?
2. Speed at the foot of a smooth $5.0\,\mathrm{m}$ drop?
3. Power to move at $20\,\mathrm{m\,s^{-1}}$ against $400\,\mathrm{N}$?
4. Why is a real machine never 100% efficient?



Answers **1.** zero **2.** $\sqrt{2g \cdot 5} \approx 9.9\,\mathrm{m\,s^{-1}}$ **3.** $Fv = 8\,\mathrm{kW}$ **4.** some energy always wasted as heat