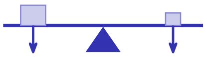
Forces, Density & Pressure

AS Physics ▪ Topic 4

A-Level Physics



What we will cover

1 Turning effects: moments

2 Equilibrium of forces

3 Density

4 Pressure

5 Upthrust & floating

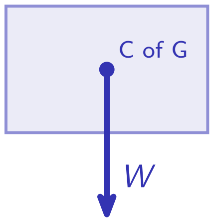
6 Recap

1 Turning effects

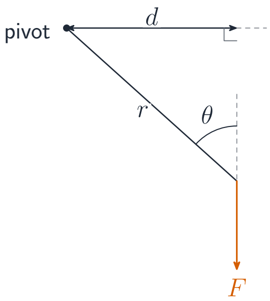
Centre of gravity

The whole **weight** 重力 can be treated as acting at one point – the **centre of gravity** 重心, which is the **centre of mass** 质心 in a uniform field.

- uniform regular shape → at the middle
- draw the weight arrow there on a **free-body diagram** 受力图



Moment of a force



Moment 力矩 = force × the perpendicular distance d from the **pivot** 支点 to the force's **line of action** 作用线:

$$M = Fd = Fr\sin\theta,$$

where r is the **lever arm** 力臂. Only the part of F perpendicular to it turns anything.

Spanner: $F = 50\,\mathrm{N}$, $r = 0.40\,\mathrm{m}$, $\theta = 30°$

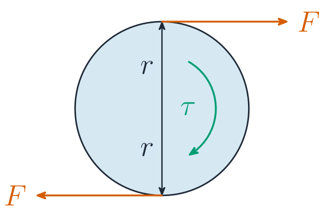
$M = 50 \times 0.40 \times \sin 30° = 10\,\mathrm{N\,m}$

Couple and torque

A **couple** 力偶: two equal, opposite forces, lines of action a distance apart.

$$\tau = Fd.$$

> A couple turns only – zero resultant force. Same forces in the *same* direction is **not** a couple.



2 Equilibrium

Two conditions for equilibrium

Both must hold at once:

$\sum F = 0$	$\sum M = 0$
no acceleration of the **translation** 平动	no **angular acceleration** 角加速度, about any point

Zero force can still turn; zero moment can still move in a line.

Principle of moments

Not turning ⇒ total clockwise = total anticlockwise moment.

Beam pivoted at centre

$40\,\mathrm{N}$ at $1.5\,\mathrm{m}$ balances $30\,\mathrm{N}$ at d:

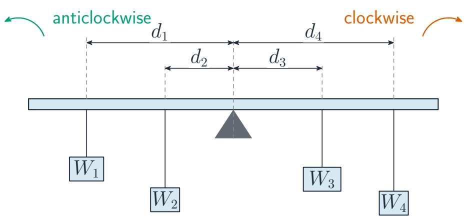
$$40(1.5) = 30\,d \;\Rightarrow\; d = 2.0\,\mathrm{m}.$$

How to solve a balance problem

1. **Choose a pivot** — Usually where an *unknown* force acts – its distance is then zero, so it drops out of the moment equation.
2. **List the forces** — Every force, with its **perpendicular** distance from that pivot. Include the rod's own weight at its centre of gravity.
3. **Balance moments** — $\sum M_{\text{clockwise}} = \sum M_{\text{anticlockwise}}$ about the chosen pivot.
4. **Add $\sum F = 0$** — Use it when a second equation is needed – typically to find the force at the pivot itself.

> A uniform rod's weight acts at its centre: place the pivot there and it contributes no moment at all.

Principle of moments, in detail

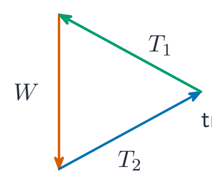


> Not turning ⇒ total clockwise = total anticlockwise moment.

Vector triangle

Three coplanar forces in equilibrium form a **closed vector triangle** 矢量三角形 – tip to tail, back to the start.

- solve with the sine / cosine rule, or
- or split each force into **horizontal** 水平 and **vertical** 竖直 **components** 分量: $\sum F_x = 0$, $\sum F_y = 0$



the three forces form a **closed** triangle → equilibrium

The closed vector triangle

Tip to tail	Three coplanar forces in equilibrium, drawn tip-to-tail, return to the start – a **closed vector triangle**.
Solve it	Use the **sine rule** 正弦定理 or **cosine rule** 余弦定理 for unknown sizes and directions, or draw it to scale on graph paper.
Or components	The alternative: split each force into horizontal and vertical parts and set $\sum F_x = 0$, $\sum F_y = 0$.

> If the triangle does *not* close, the forces are not in equilibrium – the gap is the resultant.

3 Density and pressure

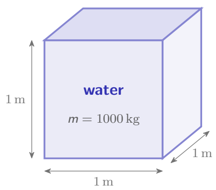
Density

Density 密度 = mass per unit volume (a scalar):

$$\rho = \frac{m}{V}, \qquad [\rho] = \mathrm{kg\,m^{-3}}.$$

water 1000 | air ~ 1.2 | iron ~ 7800

all in $\mathrm{kg\,m^{-3}}$



$\rho = 1000\,\mathrm{kg}$ *per* $\mathrm{m^3}$ *– a cubic metre of water is a tonne*

Some useful densities to know

Water	1000 kg m^{-3} – and 1 g cm^{-3} is the same number: the conversion factor is 1000.
Air (room conditions)	~ 1.2 kg m^{-3} – about $800\times$ less dense than water, which is why upthrust in air is usually ignored.
Iron / steel	~ 7800 kg m^{-3} – roughly $8\times$ water, so a steel hull floats only because it encloses air.

> Carry these three: they let you sanity-check an answer without a data sheet.

Pressure

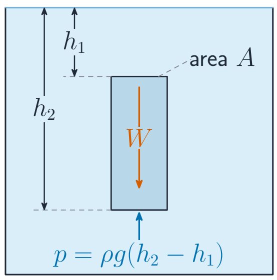
Pressure 压强 = force per unit area, normal to the surface:

$$p = \frac{F}{A}, \qquad [p] = \mathrm{Pa} = \mathrm{N\,m^{-2}}.$$

A scalar – it acts equally in all directions in a fluid. A is the **cross-sectional area** 横截面积 the force presses on.

> A barometer reads **atmospheric pressure** 大气压 – about $1.0 \times 10^5\,\mathrm{Pa}$ at sea level.

Hydrostatic pressure



Extra pressure with depth Δh:

$$\Delta p = \rho g \Delta h.$$

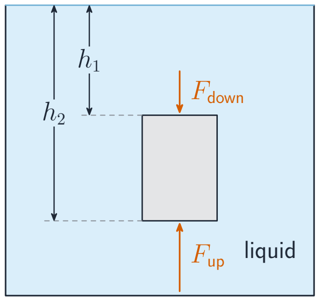
> Depends only on density and depth – a liquid is nearly **incompressible** 不可压缩, so the shape of the container does not matter.

Pool $2.5\,\mathrm{m}$ deep

$\Delta p = 1000 \times 9.81 \times 2.5 \approx 2.5 \times 10^4\,\mathrm{Pa}$

4 Upthrust

Upthrust – Archimedes' principle



For a **submerged** 浸没 object the pressure on the bottom beats the pressure on the top ⇒ a net **upthrust** 浮力:

$$F_{\text{up}} = \rho_{\text{fluid}}\, g\, V.$$

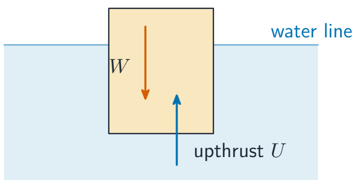
Archimedes' principle 阿基米德原理: upthrust = weight of fluid pushed aside.

$V = 2.0 \times 10^{-3}\,\mathrm{m}^3$ in water

$F_{\text{up}} = 1000 \times 9.81 \times 2.0 \times 10^{-3} \approx 20\,\mathrm{N}$

Floating and weighing under water

floats when $U = W$



- floats when upthrust = weight (V = volume below the surface)
- held down by a string: $F_{\text{up}} = W + T$, where T is the **tension** 张力
- **newton meter** 弹簧测力计 reading $= W - F_{\text{up}}$

Exam tips

- Take **moments** about a chosen pivot; for equilibrium, total clockwise = total anticlockwise moments **and** the resultant force is zero.
- A body in equilibrium under three forces gives a **closed triangle** of forces.
- Fluid pressure $= \rho g h$; **upthrust** = **weight of fluid displaced** (Archimedes).
- Distinguish mass, weight and density, and give the base unit each time.

5 Recap

Topic 4 – key takeaways

1 Moments

$M = Fd$; a couple gives torque $\tau = Fd$

2 Equilibrium

$\sum F = 0$ and $\sum M = 0$ together

3 Pressure

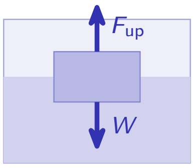
$p = F/A$; in a fluid $\Delta p = \rho g \Delta h$

4 Upthrust

$F = \rho g V$ – weight of fluid displaced

Check yourself

1. Unit of the moment of a force?
2. Are two equal forces the same way a couple?
3. Pressure $3\,\mathrm{m}$ down in water (to 1 s.f.)?
4. What does the upthrust on a body equal?



Answers **1.** $\mathrm{N\,m}$ **2.** no (they have a resultant force) **3.** $\approx 3 \times 10^4\,\mathrm{Pa}$ **4.** weight of fluid displaced