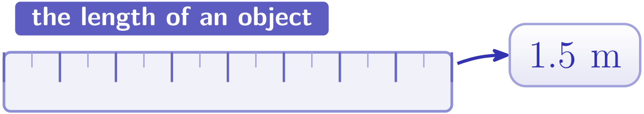
Physical Quantities & Units

AS Physics ▪ Topic 1

A-Level Physics



What we will cover

1. Physical quantities
2. SI units & prefixes
3. Errors & uncertainties
4. Scalars & vectors
5. Recap & self-check

1 Physical quantities

A number *and* a unit

A **physical quantity** 物理量 has two parts:

- a **magnitude** 大小 – the number
- a **unit** 单位 – what the number means

The number alone tells you nothing.

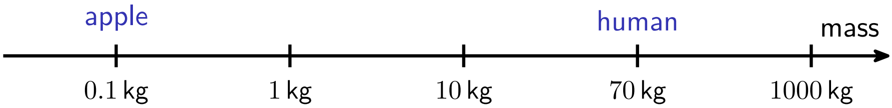
$$\underbrace{1.5}_{\text{magnitude}} \quad \underbrace{\mathrm{m}}_{\text{unit}}$$

“1.5” ✗

“1.5 m” ✓

Estimate the everyday

A good estimate has the right **order of magnitude** 数量级.



body weight $\approx 700\,\mathrm{N}$

sound $\approx 340\,\mathrm{m\,s^{-1}}$

light $\approx 3.0 \times 10^{8}\,\mathrm{m\,s^{-1}}$

$g \approx 9.81\,\mathrm{m\,s^{-2}}$

2 SI units

SI base quantities

The SI system has **seven base quantities** 基本量; these **five** are used here:

kg	m	s	A	K
mass	length	time	current	temperature

Combine them with $\times$ and $\div$ to get **every other unit** – N, J, W, Pa, ...

The seven estimates to know by heart

human **mass** 质量	$70\,\mathrm{kg}$
human **weight** 重力	$700\,\mathrm{N}$
human height	$1.7\,\mathrm{m}$
an apple	$0.1\,\mathrm{kg}$
speed of sound in air	$340\,\mathrm{m\,s^{-1}}$
speed of light in a **vacuum** 真空	$3.0\times10^{8}\,\mathrm{m\,s^{-1}}$
free fall 自由落体	$g\approx 9.81\,\mathrm{m\,s^{-2}}$

An apple weighs about $1\,\mathrm{N}$. That one number lets you check every other force estimate.

Build any derived unit

Each **derived unit** 导出单位 is just base units, from its defining equation:

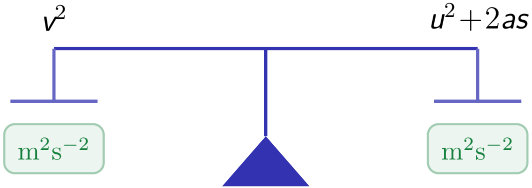
N	**J**	**W**	**Pa**
newton 牛顿	**joule** 焦耳	**watt** 瓦特	**pascal** 帕斯卡
force	energy	power	pressure
$\mathrm{kg\,m\,s^{-2}}$	$\mathrm{kg\,m^{2}\,s^{-2}}$	$\mathrm{kg\,m^{2}\,s^{-3}}$	$\mathrm{kg\,m^{-1}\,s^{-2}}$

Read the unit straight off the defining equation: **speed** 速率 $= \mathrm{m\,s^{-1}}$, **acceleration** 加速度 $= \mathrm{m\,s^{-2}}$, and **stress** 应力 is force per area, so it takes the pascal too.

Do the units balance?

Homogeneous 量纲一致 = both sides have the same base units. Test $v^2 = u^2 + 2as$:

- LHS $= \mathrm{m^2 s^{-2}}$
- $u^2 = \mathrm{m^2 s^{-2}}$; $2as = \mathrm{m^2 s^{-2}}$



> Matching units don't prove an equation right – but mismatched units prove it wrong.

Prefixes – powers of ten

A **prefix** 词头 scales a unit by a power of ten.

n	μ	m	1	k	M	G
nano	micro	milli	base	kilo	mega	giga
10^{-9}	10^{-6}	10^{-3}	10^{0}	10^{3}	10^{6}	10^{9}

each step is $\times 10^3$

← smaller

larger →

Prefixes on the page

10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{0}	10^{3}	10^{6}	10^{9}
p	n	μ	m	1	k	M	G

each step is $\times 10^3$

Each step is a factor of 1000, so converting is a shift of three places – never a new calculation.

> Square the prefix too: $1\,\mathrm{mm}^2 = (10^{-3}\,\mathrm{m})^2 = 10^{-6}\,\mathrm{m}^2$, not 10^{-3}.

Worked example – converting prefixes

$0.25\,\mathrm{kN\,mm^{-2}} \rightarrow \mathrm{N\,m^{-2}}$

Replace each prefix – and square the mm:

$$0.25\,\frac{10^3}{(10^{-3})^2} = 2.5 \times 10^8\,\mathrm{N\,m^{-2}}.$$

$0.25\,\mathrm{kN\,mm^{-2}}$

↓

replace prefixes
$\times 10^3 \ / \ 10^{-6}$

↓

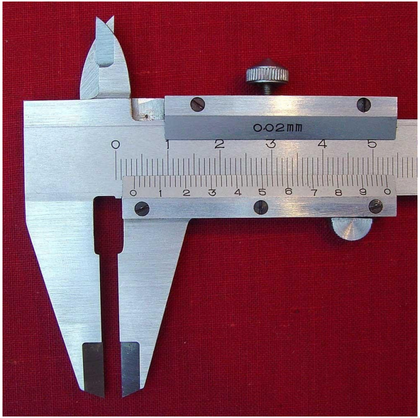
$\boxed{2.5 \times 10^8\,\mathrm{N\,m^{-2}}}$

3 Errors and uncertainties

Every measurement is uncertain

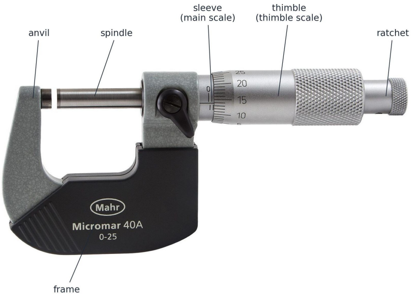
Every measurement carries some **uncertainty** 不确定度. A good experimenter:

- knows where it comes from
- estimates it fairly
- carries it to the answer

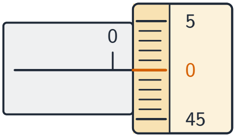


Calipers measure with a small uncertainty

The micrometer – to $0.01\,\mathrm{mm}$

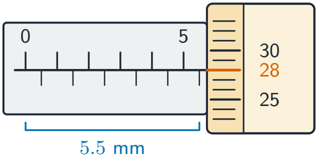


(a) zero check



reads 0.00 mm

(b) a measurement

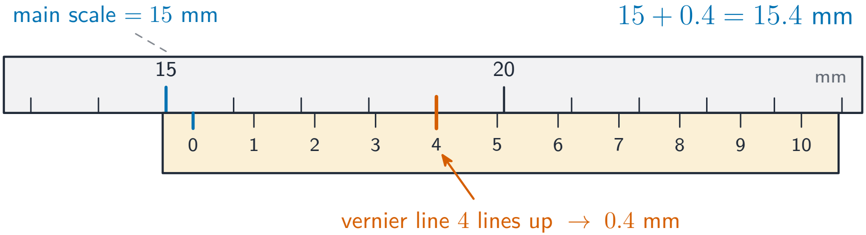


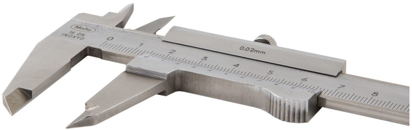
$5.5 + 0.28 = 5.78$ mm

Reading = sleeve (main scale) + thimble scale.

> A **micrometer** 千分尺 reads to $0.01\,\mathrm{mm}$: sleeve + thimble.

The vernier caliper – to $0.1\,\mathrm{mm}$





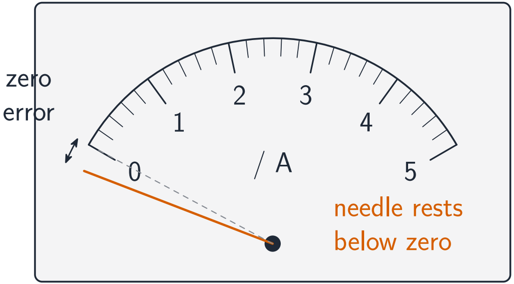
Whole mm from the main scale + tenths from the lined-up vernier mark.

> Whole mm from the main scale, then tenths from the vernier mark that lines up.

Systematic vs random errors

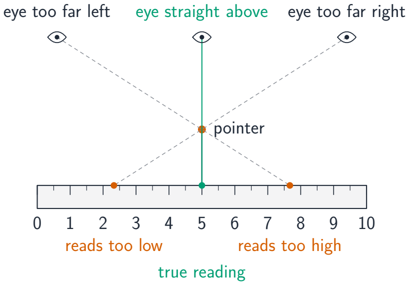
systematic error 系统误差 – same shift every time (**zero error** 零点误差, **calibration** 校准, **parallax** 视差).

Spoils the accuracy; repeating cannot find it.

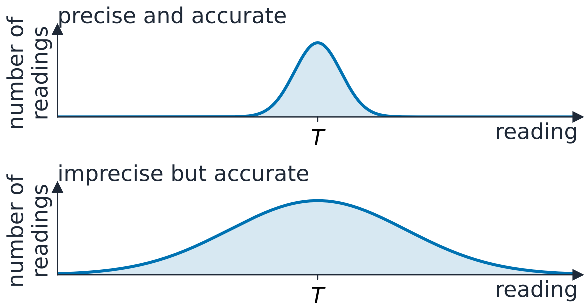


random error 随机误差 – scatters above and below with no pattern.

Spoils the precision; from how you read the scale, changing conditions, and the smallest step the **instrument** 仪器 shows. The **mean** 平均值 of many repeats cancels it.



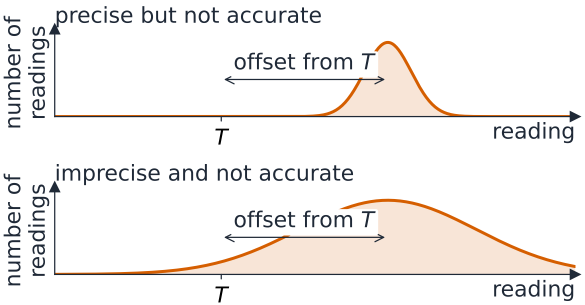
Precision 精密度: how narrow is the spread?



precision: how closely repeated readings agree (random error / spread)

precision *精密度: how narrow*

Accuracy 准确度: is the peak on the true value?



accuracy: how close readings are to the true value (systematic error reduces accuracy)

accuracy 准确度: *peak on true value?*

Combining uncertainties

A **derived quantity** 导出量 is one you *calculate* from measured values. Each is $x \pm \Delta x$; its **percentage uncertainty** 百分比不确定度 $= \frac{\Delta x}{|x|} \times 100\%$.

- **add / subtract** $\Rightarrow$ add **absolute uncertainty** 绝对不确定度
- **multiply / divide** $\Rightarrow$ add the %
- **power** a^n $\Rightarrow$ multiply % by $|n|$

$$y = \frac{a\,b}{c}$$

$$\downarrow$$

$$\boxed{\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}}$$

Adding or subtracting – add the *absolute* uncertainties

$$\boxed{\Delta y = \Delta a + \Delta b}$$

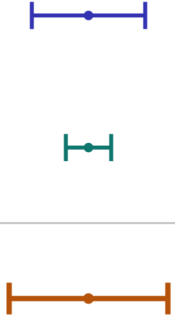
Both $y = a + b$ and $y = a - b$ add them.

> A difference keeps the whole Δ but has a far smaller value, so its % is the worst of the three.

$a = (50.0 \pm 0.5)\,\mathrm{mm}$ 1.0%

$b = (20.0 \pm 0.2)\,\mathrm{mm}$ 1.0%

$a - b = (30.0 \pm 0.7)\,\mathrm{mm}$ 2.3%



one scale for all three – the *difference* is the widest

Multiplying or dividing – add the *percentage* uncertainties

$$\boxed{\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}}$$

Density $\rho = \dfrac{m}{V}$ from $m = (250 \pm 1)\,\mathrm{g}$ and $V = (100 \pm 2)\,\mathrm{cm}^3$.

> The bigger % wins: improve V, not m.

$$\frac{\Delta m}{m} = \frac{1}{250}$$ 0.4%

$$\frac{\Delta V}{V} = \frac{2}{100}$$ 2.0%

$$\frac{\Delta \rho}{\rho}$$ 2.4%

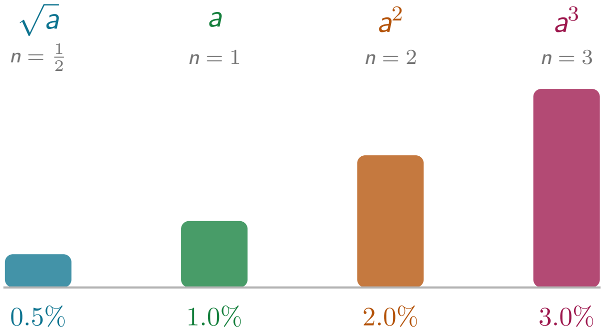
$$\boxed{\rho = (2.50 \pm 0.06)\,\mathrm{g\,cm^{-3}}}$$

Powers – multiply the percentage by the index

$$\boxed{\frac{\Delta y}{|y|} = |n| \frac{\Delta a}{|a|}}$$

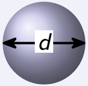
A 1% error in a length becomes 3% in a volume – three edges each carry it.

> Next slide: d carries 0.38%, so $V \propto d^3$ carries 1.14%.



each bar is the % uncertainty in y when a carries 1%

Worked example – uncertainty in a volume



$d = (5.26 \pm 0.02)\,\mathrm{cm}$

$V = \frac{\pi}{6} d^3 = 76.2\,\mathrm{cm}^3$

1

% uncertainty in d

$$\frac{0.02}{5.26} \times 100\%$$

$$= 0.38\%$$

2

$V \propto d^3$: triple it

$$3 \times 0.38\%$$

$$= 1.14\%$$

3

back to absolute

$$1.14\% \times 76.2$$

$$\Delta V = 0.9\,\mathrm{cm}^3$$

$$V = (76.2 \pm 0.9)\,\mathrm{cm}^3$$

Significant figures

> Give the answer the same number of **significant figures** 有效数字 as the least precise measurement you used – usually 2 or 3.

- too many $\Rightarrow$ looks too exact
- too few $\Rightarrow$ loses information

$$76.2\,\underline{34\ldots}$$

drop extra

$$\boxed{76.2\,\mathrm{cm}^3 \quad (3\text{ s.f.})}$$

4 Scalars and vectors

Size only, or size *and* direction?

Scalar 标量

$5\ \mathrm{kg}$

magnitude only

mass · time · energy · speed
distance · **density** 密度
· **electric charge** 电荷

Vector 矢量

magnitude + direction

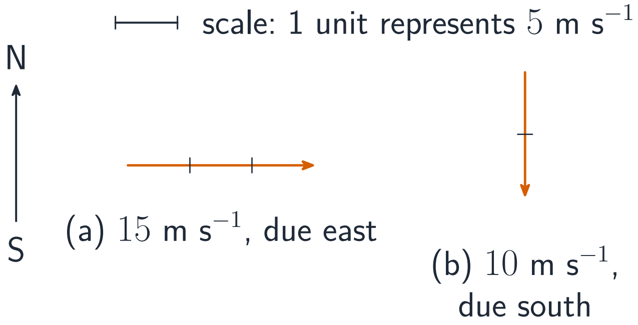
displacement 位移 · velocity
force · momentum 动量

> Quick test – can you ask "in which direction?" If yes, it is a vector.

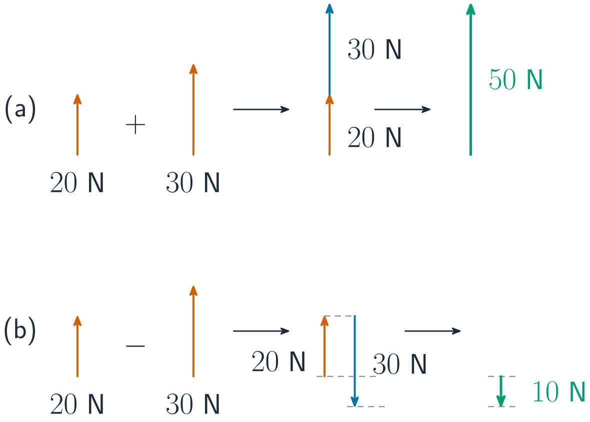
Drawing vectors to scale

A vector is an arrow:

- direction of arrow = direction
- length (to scale) = magnitude

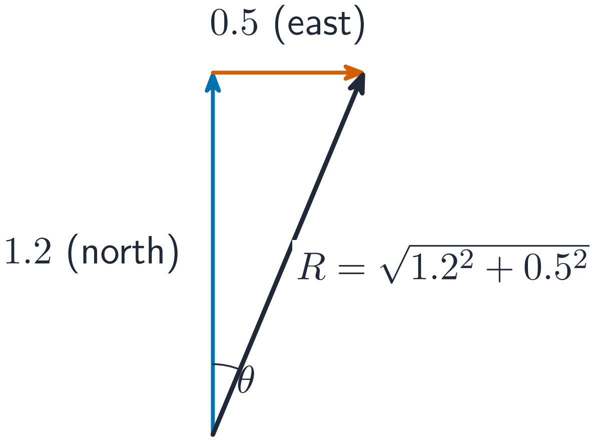


Adding tip to tail



- add **coplanar** 共面 vectors tip to tail
- the **resultant** 合矢量 runs first tail $\rightarrow$ last tip
- to subtract $\vec{X} - \vec{Y}$, add the reverse of $\vec{Y}$

Vectors at right angles



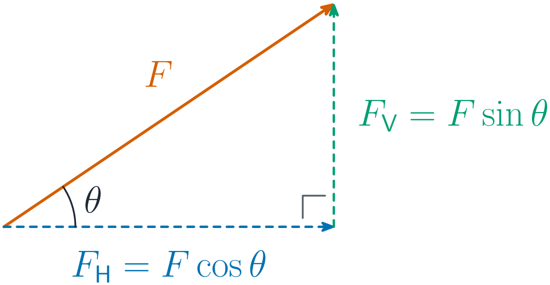
For **perpendicular** 垂直 vectors:

$$|\vec{R}| = \sqrt{X^2 + Y^2}, \quad \tan\theta = \frac{Y}{X}.$$

Swimmer

$1.2\,\mathrm{m\,s^{-1}}$ north, current $0.5\,\mathrm{m\,s^{-1}}$ east: $|\vec{R}| = 1.3\,\mathrm{m\,s^{-1}}$, $\theta \approx 23^\circ$ E of N.

Resolving into components



Split into two **perpendicular** 垂直 **components** 分量 – usually horizontal and vertical, or along and across a surface:

$$v_H = v\cos\theta, \quad v_V = v\sin\theta.$$

On an **inclined plane** 斜面:

$$W_{\parallel} = W\sin\theta, \quad W_{\perp} = W\cos\theta.$$

Exam tips

- Give every answer a **unit**, and check **homogeneity** – both sides of an equation must have the same base units.
- Distinguish **random error** (reduce by repeating and averaging) from **systematic error** (a zero or calibration error that repeats do not remove).
- Combine uncertainties: **add absolute** uncertainties when adding/subtracting, **add percentage** uncertainties when multiplying/dividing (and multiply the % by any power).
- Distinguish **precision** (small spread) from **accuracy** (close to the true value).
- Resolve a vector into perpendicular components ($F\cos\theta$, $F\sin\theta$); add vectors tip-to-tail or by components.

5 Recap

Topic 1 – key takeaways

1 Magnitude + unit

every quantity is a number *and* a unit

2 Derived units

built from 5 base units; check homogeneity

3 Precision vs accuracy

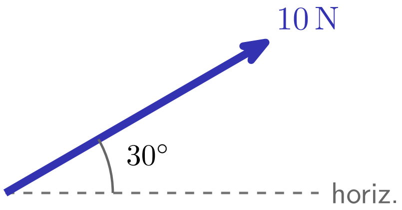
spread of repeats vs the true value

4 Vectors

add tip-to-tail; resolve into components

Check yourself

1. SI base units of the **pascal**?
2. Close, but far from true – precise or accurate?
3. $y = ab^2$ – how do the % combine?
4. $10\,\mathrm{N}$ at $30°$ – horizontal component?



for question 4

Answers **1.** $\mathrm{kg\,m^{-1}s^{-2}}$ **2.** precise, not accurate **3.** $\frac{\Delta y}{y} = \frac{\Delta a}{a} + 2\frac{\Delta b}{b}$ **4.** $8.7\,\mathrm{N}$