Course Companion

A-Level Physics

Every topic handout in one volume

全部专题讲义合集

exercises.lol

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Physical quantities and units

A-Level Physics

Physical quantities

A **physical quantity** 物理量 has two parts: a number (its **magnitude** 大小) and a **unit** 单位. The number on its own tells you nothing. You must also say what is measured and in which unit.

Example: "the length is 1.5" is not complete. "The length is 1.5 m" is a physical quantity.

Making estimates

You should be able to **estimate** 估算 the size of the physical quantities in this syllabus. Learn these rough values:

- **mass** 质量 of an adult human: ~ 70 kg
- **weight** 重力 of an adult human: ~ 700 N
- height of an adult human: ~ 1.7 m
- mass of an apple: ~ 0.1 kg (so its weight is about 1 N)
- speed of sound in air: $\sim 340\ \text{m s}^{-1}$
- speed of light in a **vacuum** 真空: $3.0 \times 10^8\ \text{m s}^{-1}$
- **acceleration** 加速度 of **free fall** 自由落体: $g \approx 9.81\ \text{m s}^{-2}$

A good estimate has the right **order of magnitude** 数量级 (the right power of ten). For a human, 70 kg is a good guess; 7 kg is not.

SI units 国际单位制

Base units

The SI system has seven **base quantities** 基本量; five of them are used in this topic. You must know these five and their units:

- **mass** — kilogram, kg
- **length** 长度 — metre, m
- time — second, s
- **current** 电流 — ampere 安培, A
- **temperature** 温度 — kelvin 开尔文, K

Every other unit in this syllabus is built from these five.

Derived units

A **derived unit** 导出单位 is made by multiplying or dividing base units. You should be able to write any quantity in this syllabus in base units.

Build a derived unit from the equation that defines it:

- **speed** 速率 = distance / time, so its unit is m s^{-1}
- **acceleration** = change in **velocity** 速度 / time, so its unit is m s^{-2}
- **force** 力 = mass × acceleration, so its unit is kg m s^{-2}. The **newton** 牛顿 is 1 N = 1 kg m s^{-2}.
- **work** 功 and **energy** 能量 = force × distance, so the unit is kg m^2 s^{-2}. The **joule** 焦耳 is 1 J = 1 kg m^2 s^{-2}.
- **power** 功率 = energy / time, so the unit is kg m^2 s^{-3}. The **watt** 瓦特 is 1 W = 1 kg m^2 s^{-3}.
- **pressure** 压强 and **stress** 应力 = force / area, so the unit is kg m^{-1} s^{-2}. The **pascal** 帕斯卡 is 1 Pa = 1 kg m^{-1} s^{-2}.

When a question asks for the SI base units of a quantity, replace each named unit with its base units, then simplify. Example: the SI base units of the watt are kg m^2 s^{-3}.

Checking that the units match

An equation is **homogeneous** 量纲一致 when both sides have the same base units. In plain words: the units on both sides match.

Write each side in base units and compare. Take the equation $v^2 = u^2 + 2as$:

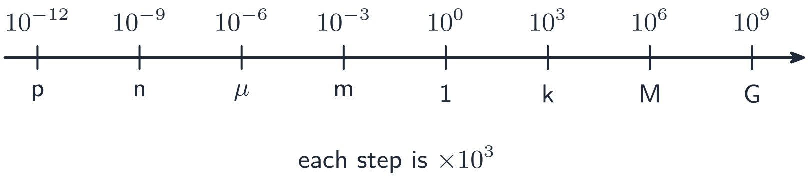
- left side: $(\text{m s}^{-1})^2 = \text{m}^2\ \text{s}^{-2}$
- right side, first term: $(\text{m s}^{-1})^2 = \text{m}^2\ \text{s}^{-2}$
- right side, second term: $\text{m s}^{-2} \cdot \text{m} = \text{m}^2\ \text{s}^{-2}$

Both sides give $\text{m}^2\ \text{s}^{-2}$, so the units match.

Be careful: matching units do **not** prove the whole equation is correct. It could still have a wrong number, or a missing factor of 2. But if the units do **not** match, the equation is wrong for sure.

Prefixes

A **prefix** 词头 is a letter put in front of a unit to make it bigger or smaller by powers of ten. You must know these:



The SI prefixes climb in steps of a thousand, from pico to giga

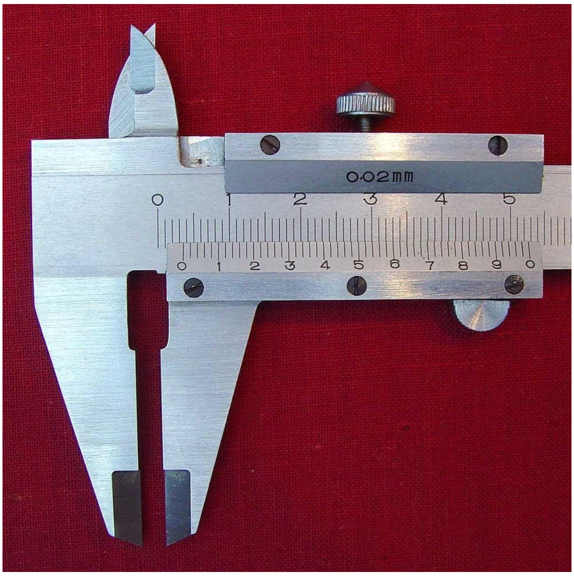
Prefix	**Symbol**	**Factor**
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

To change a prefixed unit into base units, replace the prefix with its factor, then simplify. Example: change 0.25 kN mm^{-2} into N m^{-2}:

$$0.25\ \text{kN mm}^{-2} = 0.25 \times \frac{10^3\ \text{N}}{(10^{-3}\ \text{m})^2} = 0.25 \times \frac{10^3}{10^{-6}}\ \text{N m}^{-2} = 2.5 \times 10^8\ \text{N m}^{-2}.$$

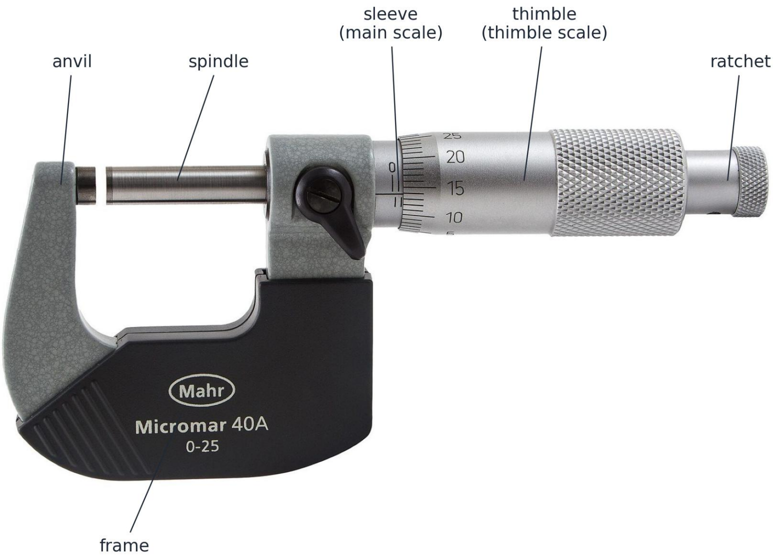
Take special care with squared units like mm^2: you must square the factor too.

Errors and uncertainties

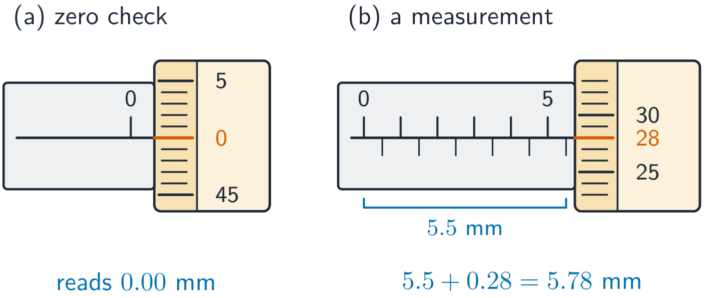


A vernier caliper measures length precisely, with a small uncertainty.

Every **measurement** 测量 has some **uncertainty** 不确定度 — we are never fully sure of the value. A good experimenter knows where the uncertainty comes from, makes a fair estimate of it, and carries it through to the final answer.



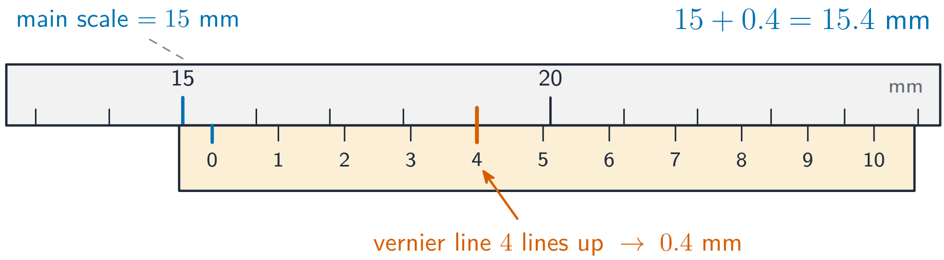
The main parts of a real micrometer screw gauge, which measures to the nearest 0.01 mm



Reading a micrometer: add the main scale reading to the thimble reading

Reading a real micrometer: read the mm and half-mm on the sleeve, then add the thimble scale

Vernier calipers measure to the nearest 0.1 mm — the sliding scale gives the extra digit

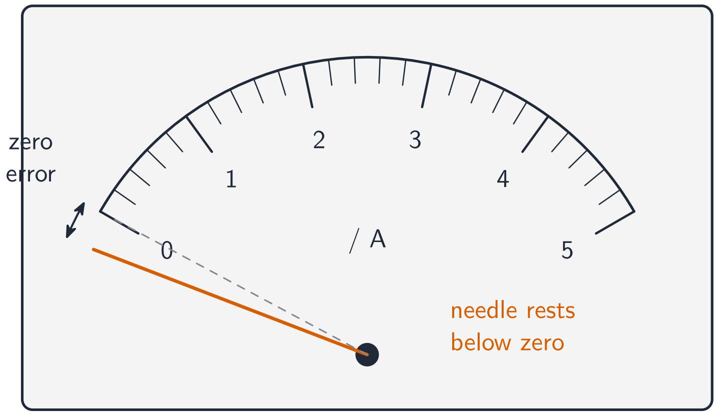


Reading a vernier caliper: whole millimetres from the main scale, plus the tenths from the vernier line that lines up

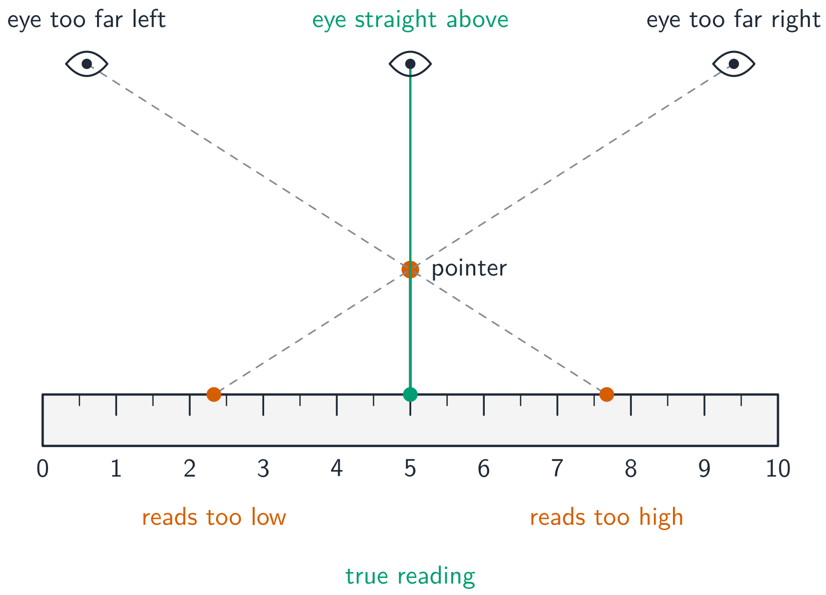
Systematic and random errors

A **systematic error** 系统误差 changes every reading by the same amount, in the same direction. You cannot find it by repeating the measurement. Common causes:

- a **zero error** 零点误差 (the scale does not read zero when the true value is zero)
- a **calibration** 校准 error (the scale itself is wrong)
- **parallax** 视差 (your eye is always to one side of the scale)



An ammeter with a zero error: the needle reads below zero before any current flows



Parallax error: different viewing angles give different scale readings

A systematic error makes the **accuracy** 准确度 worse, but it does not change the **precision** 精密度.

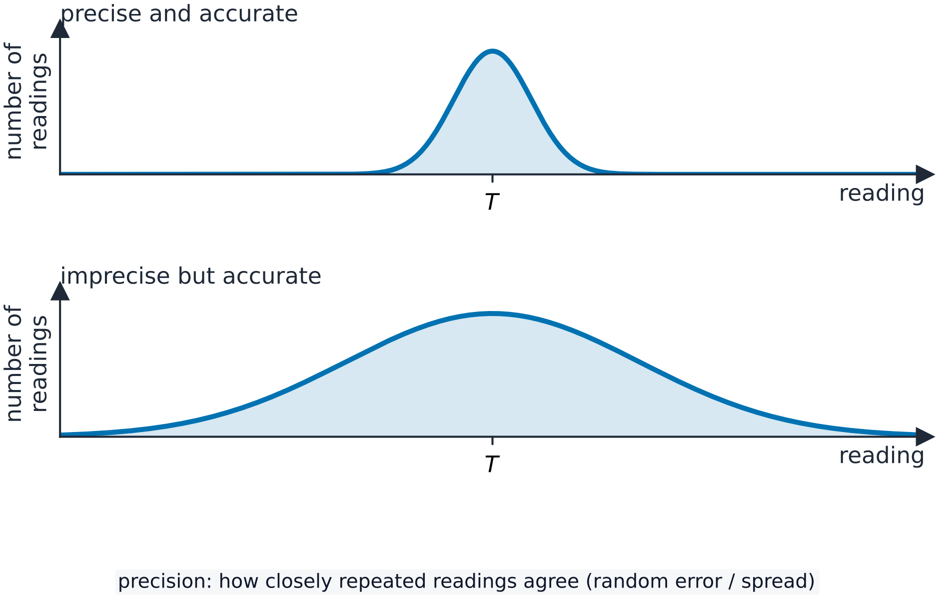
A **random error** 随机误差 makes readings jump above and below the true value, with no pattern. Causes include how carefully you read the scale, changing conditions, and the smallest step the **instrument** 仪器 can show. If you repeat the measurement many times and take the **mean** 平均值 (the average), random errors partly cancel out.

A random error makes the precision worse. But with enough repeats, the mean can still be accurate.

Precision and accuracy

Precision is how close repeated readings are to each other. Precise readings are grouped very close together.

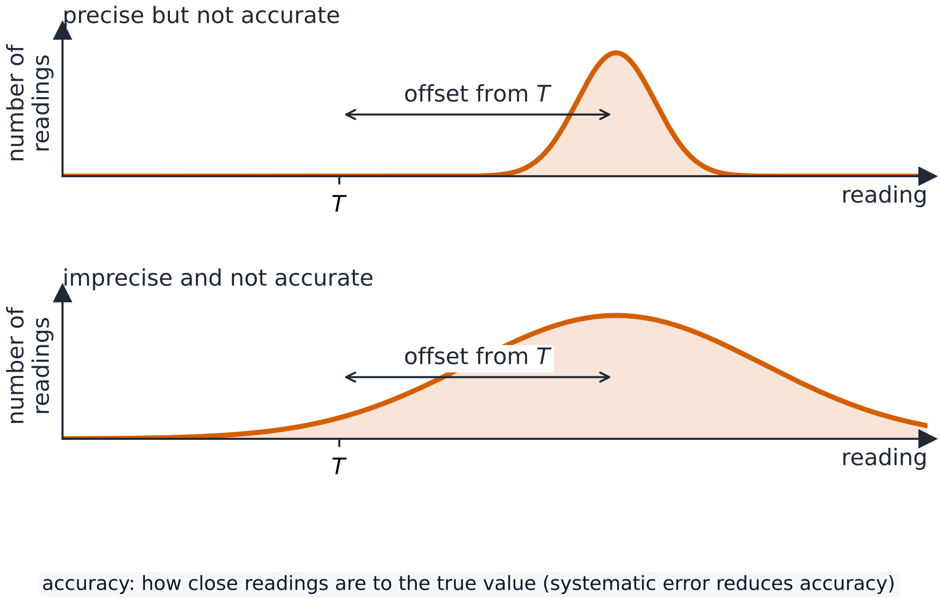
Accuracy is how close a reading (or the mean of several readings) is to the true value.



Precision: how narrow the distribution is around the true value T

A set of readings can be:

- precise and accurate — close together and near the true value
- precise but not accurate — close together, but away from the true value (a systematic error)
- accurate but not precise — spread out, but the mean is near the true value
- neither — spread out and away from the true value



Accuracy: whether the peak of the distribution is centred on the true value T

When a question gives a table of repeated readings, look at the **spread** (precision) and the **mean** (accuracy) separately.

Uncertainty in a derived quantity

A measurement is often written as $x \pm \Delta x$. Here Δx is the **absolute uncertainty** 绝对不确定度. The **percentage uncertainty** 百分比不确定度 is

$$\text{percentage uncertainty in } x = \frac{\Delta x}{|x|} \times 100\%.$$

A **derived quantity** 导出量 is one you calculate from measured values. Its uncertainty is found by simple rules:

- **Adding or subtracting** — add the absolute uncertainties. If $y = a+b$ or $y = a-b$, then $\Delta y = \Delta a + \Delta b$.
- **Multiplying or dividing** — add the percentage uncertainties. If $y = \dfrac{a \cdot b}{c}$, then

$$\frac{\Delta y}{|y|} = \frac{\Delta a}{|a|} + \frac{\Delta b}{|b|} + \frac{\Delta c}{|c|}.$$

- **Powers** — multiply the percentage uncertainty by the power. If $y = a^n$, then $\dfrac{\Delta y}{|y|} = |n| \cdot \dfrac{\Delta a}{|a|}$.

Worked example. A ball's **diameter** 直径 is measured as $d = (5.26 \pm 0.02)$ cm. The **volume** 体积 of a sphere is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(d/2)^3$, so $V \propto d^3$ (V depends on d cubed).

The percentage uncertainty in d is

$$\frac{0.02}{5.26} \times 100\% \approx 0.38\%.$$

Because $V \propto d^3$, the percentage uncertainty in V is three times this, about 1.14%. The volume is $\frac{4}{3}\pi(2.63)^3 \approx 76.2$ cm^3. So the absolute uncertainty is $0.0114 \times 76.2 \approx 0.87$ cm^3. The final answer is $V = (76.2 \pm 0.9)$ cm^3.

Significant figures

When you write a calculated quantity, give it the **same number of significant figures** 有效数字 as the least precise measurement you used — usually two or three in this syllabus. Too many significant figures makes the answer look more exact than it really is. Too few loses useful information.

Scalars and vectors

A **scalar** 标量 has size only. A **vector** 矢量 has both size and direction.

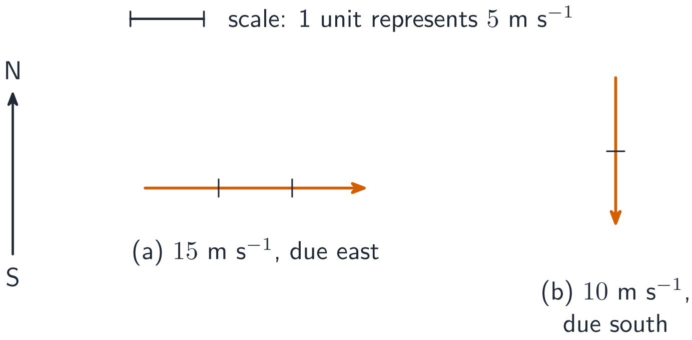
Examples from the syllabus:

- scalars: mass, time, temperature, energy, work, power, distance, speed, pressure, **density** 密度, **electric charge** 电荷
- vectors: **displacement** 位移, velocity, acceleration, force (including weight), **momentum** 动量

Quick test: if it makes sense to ask "in which direction?", the quantity is a vector. You cannot ask "in which direction is the temperature?", so temperature is a scalar. You can ask "in which direction is the velocity?", so velocity is a vector.

Adding and subtracting vectors

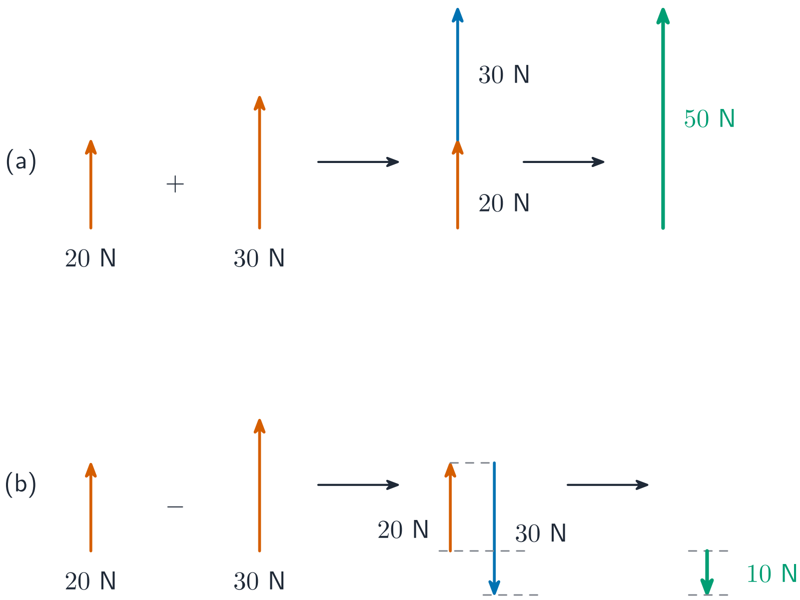
A vector is drawn as an arrow: the **direction** of the arrow gives the direction of the quantity, and the **length** of the arrow (drawn to scale) gives the magnitude.



Vectors represented as arrows drawn to scale

To add two **coplanar** 共面 vectors (vectors in the same flat plane), draw them **tip to tail**. The **resultant** 合矢量 goes from the tail of the first arrow to the tip of the second.

To find $\vec{X} - \vec{Y}$, add the reverse of $\vec{Y}$: $\vec{X} + (-\vec{Y})$. The reverse of $\vec{Y}$ has the same size as $\vec{Y}$ but points the opposite way.



Adding and subtracting parallel vectors

If the two vectors are at right angles (90°), the size of the resultant is

$$|\vec{R}| = \sqrt{X^2 + Y^2},$$

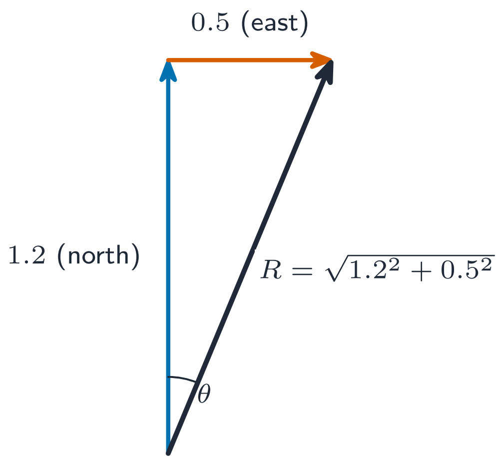
and its direction comes from $\tan\theta = Y/X$.

Worked example. A swimmer heads north at 1.2 m s^{-1} across a river that flows east at 0.5 m s^{-1}. Find the size and direction of the resultant velocity.

The two velocities are perpendicular, so

$$|\vec{R}| = \sqrt{1.2^2 + 0.5^2} = \sqrt{1.69} = 1.3 \text{ m s}^{-1},$$

at an angle $\tan\theta = 0.5/1.2$, giving $\theta \approx 23°$ east of north.



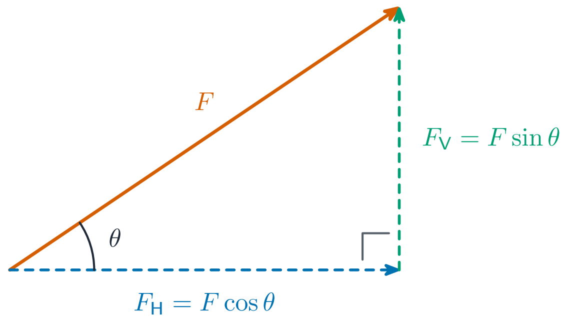
Two perpendicular vectors add to a resultant of size $\sqrt{X^2 + Y^2}$ at angle θ

If two vectors have the same size F with an angle 2α between them, the resultant has size $2F\cos\alpha$ and lies along the line that cuts the angle in half.

Splitting a vector into perpendicular parts

Any vector can be split into two **perpendicular** 垂直 (at right angles) **components** 分量. Usually these are **horizontal** 水平 and **vertical** 竖直, or along and across a surface. For a vector $\vec{v}$ at angle θ to the horizontal:

$$v_\text{H} = v\cos\theta, \qquad v_\text{V} = v\sin\theta.$$



Resolving a vector into horizontal and vertical components

Choose the directions that make the problem easiest. On a slope (an **inclined plane** 斜面), split the weight into one part along the slope and one part at right angles to it:

$$W_{\parallel} = W \sin\theta, \qquad W_{\perp} = W \cos\theta,$$

where θ is the angle of the slope to the horizontal.

You split a vector into components whenever you need to know how much of it acts in one direction. For example: the part of a force that acts along a slope, or the horizontal and vertical parts of a ball's velocity after it is thrown.

Exam tips

- Give every answer a **unit**, and check **homogeneity** — both sides of an equation must have the same base units.
- Distinguish **random error** (reduce by repeating and averaging) from **systematic error** (a zero or calibration error that repeats do not remove).
- Combine uncertainties: **add absolute** uncertainties when adding/subtracting, **add percentage** uncertainties when multiplying/dividing (and multiply the % by any power).
- Distinguish **precision** (small spread) from **accuracy** (close to the true value).
- Resolve a vector into perpendicular components ($F \cos\theta$, $F \sin\theta$); add vectors tip-to-tail or by components.

Kinematics

A-Level Physics

Key definitions

A speedometer shows speed: the distance travelled per unit time.

These five quantities come up in almost every **kinematics** 运动学 question. Learn the exact words — the examiner gives marks for precise wording.

- **distance** 距离 — the total length of the path travelled. A **scalar** 标量.
- **displacement** 位移 — the straight-line distance from the start to the end, with a direction. A **vector** 矢量.
- **speed** 速率 — the rate of change of distance with time. A scalar.
- **velocity** 速度 — the rate of change of displacement with time. A vector.
- **acceleration** 加速度 — the rate of change of velocity with time. A vector.

The unit of speed and velocity is m s^{-1}; the unit of acceleration is m s^{-2}.

A common mistake: **deceleration** 减速度 just means acceleration in the opposite direction to the velocity. It is not a separate quantity.

Motion graphs

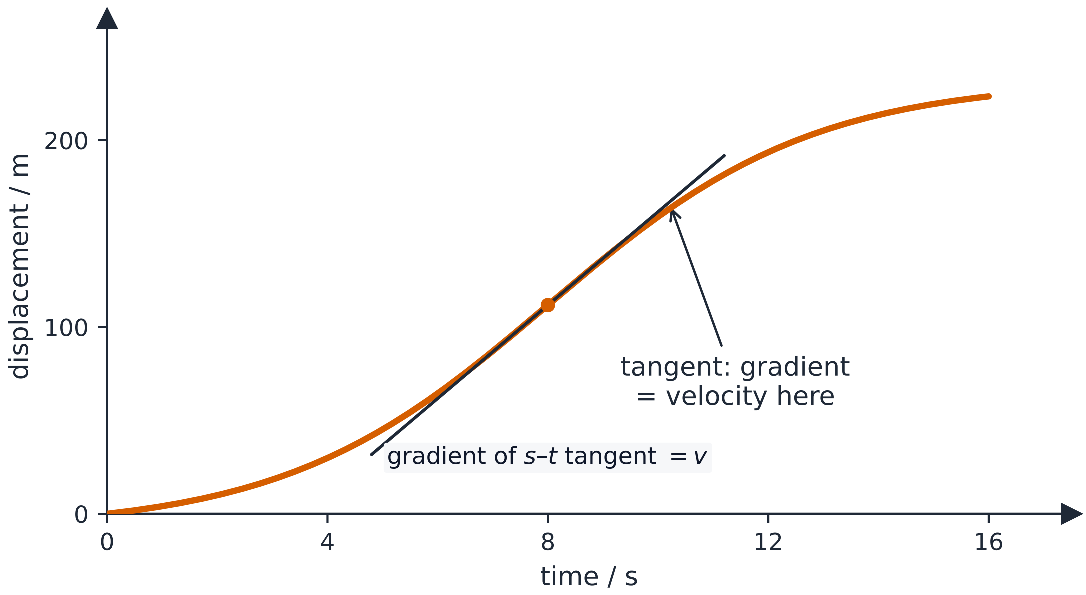
A high-speed train: its motion can be shown on a distance-time graph.

Many marks come from reading or drawing motion graphs. Two graphs matter.

Displacement–time graph

The **gradient** 斜率 (steepness) of a displacement–time graph at a point gives the velocity at that moment.

- flat line → the object is at rest.
- straight sloping line → constant velocity (gradient = velocity).
- curved line → changing velocity. Draw a **tangent** 切线 at the point and find its gradient.



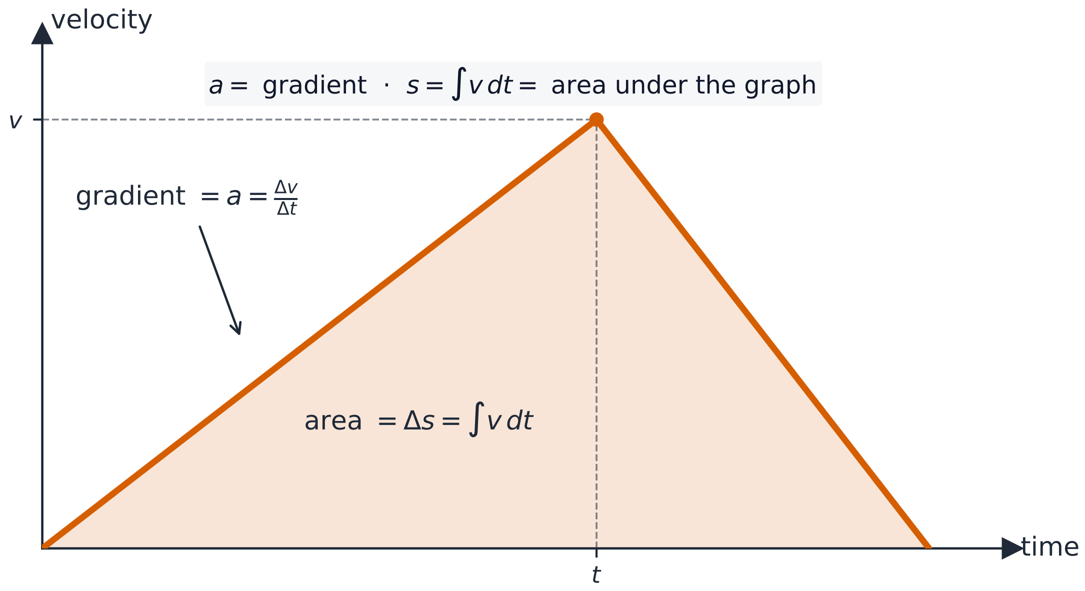
Displacement–time graph of a car on a test track

Velocity–time graph

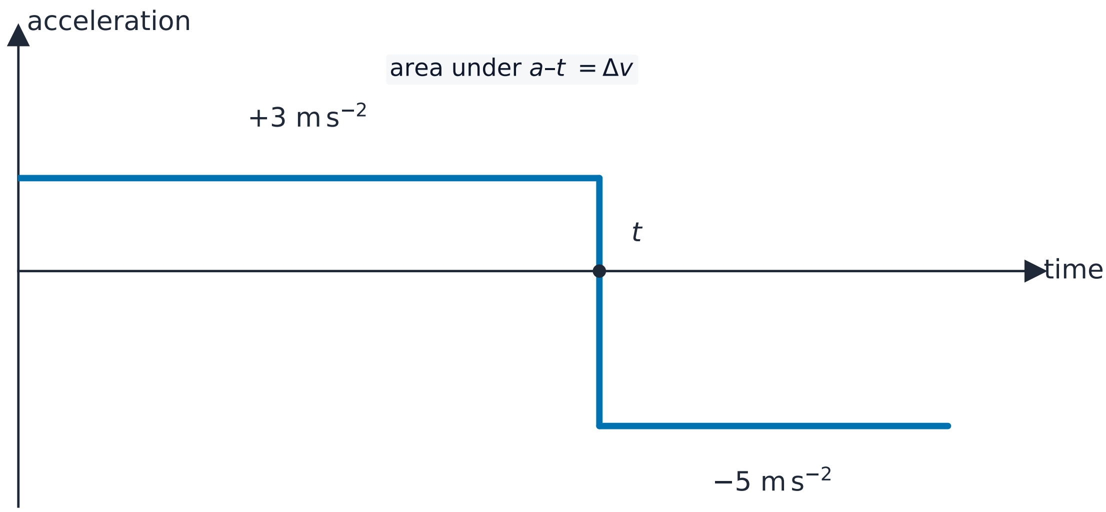
The gradient of a velocity–time graph gives the acceleration at that moment.

The **area** between the line and the time axis gives the displacement in that time.

- flat line → constant velocity (zero acceleration).
- straight sloping line → **uniform acceleration** 匀加速 (constant acceleration).
- curved line → changing acceleration.
- area above the time axis is positive displacement; area below is negative (the object moved backwards).



Velocity–time graph — gradient gives acceleration, area gives displacement



Acceleration–time graph derived from the same motion

To find the displacement, split the area into triangles and rectangles, or count grid squares. Area of a triangle is $\frac{1}{2}$ × base × height; area of a rectangle is base × height.

The four SUVAT equations

For motion in a straight line with uniform acceleration, we use five symbols: starting velocity u, final velocity v, acceleration a, displacement s, and time t. Four equations link them:

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = \tfrac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

Each equation uses four of the five symbols. To pick the right one: write down what you know and what you want, then choose the equation with exactly those four.

Worked example. A car accelerates uniformly from 8 m s^{-1} to 20 m s^{-1} over a distance of 56 m. Find its acceleration.

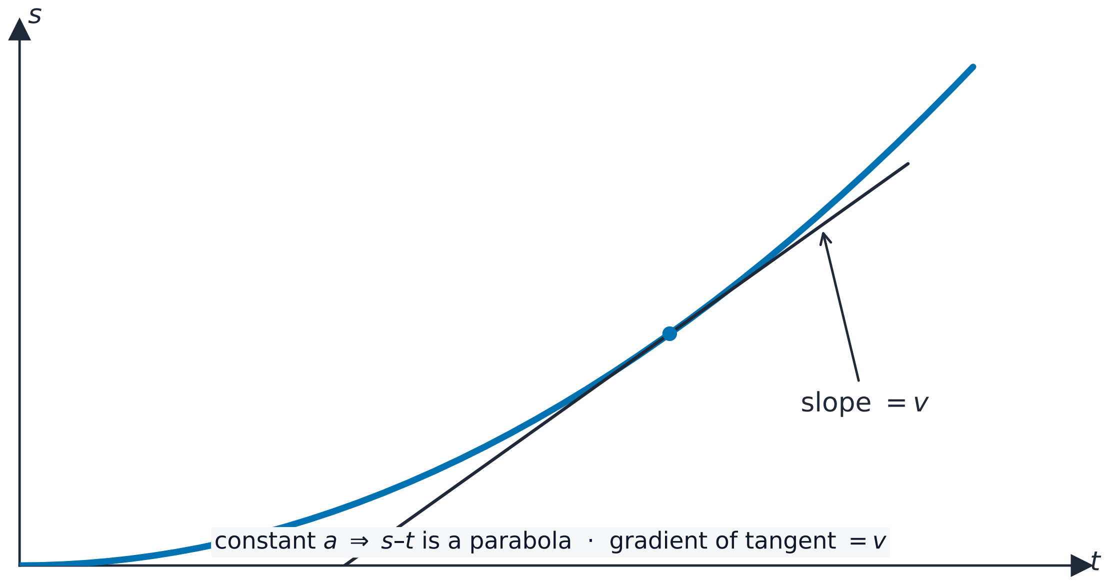
We know u, v and s and want a, so use $v^2 = u^2 + 2as$:

$$20^2 = 8^2 + 2a(56) \quad \Rightarrow \quad 336 = 112\,a \quad \Rightarrow \quad a = 3.0 \text{ m s}^{-2}.$$

Where the SUVAT equations come from

You should be able to get these from the definitions of velocity and acceleration:

- $v = u + at$ comes from $a = (v - u)/t$, the gradient of the line.
- $s = \frac{1}{2}(u + v)t$ is the area under the line — a **trapezium 梯形** with parallel sides u and v and width t.
- $s = ut + \frac{1}{2}at^2$ comes from putting $v = u + at$ into the area.
- $v^2 = u^2 + 2as$ comes from removing t from the first two.



Displacement–time graph for uniform acceleration — the slope at any point equals the instantaneous velocity

If a question asks ”which equation can be found using only the gradient of a velocity–time graph?”, the answer is $v = u + at$ (the gradient is the acceleration).

Choosing a positive direction

Pick a positive direction at the start and keep it. Anything pointing the other way gets a minus sign. For a ball thrown straight up, if ”up” is positive: u is positive, $a = -g$ (**gravity 重力** pulls down), and at the highest point the displacement is positive but the velocity is zero.

Free fall under gravity

When **air resistance** 空气阻力 can be ignored, an object in **free fall** 自由落体 has a constant acceleration $g \approx 9.81$ m s^{-2} downwards. This is the same for every mass.

For a ball dropped from rest and falling a distance h:

$$h = \tfrac{1}{2}gt^2, \qquad v = gt, \qquad v^2 = 2gh.$$

For a ball thrown straight up with speed u:

- greatest height: put $v = 0$ in $v^2 = u^2 - 2gh$, giving $h = u^2/(2g)$.
- time to reach the top: put $v = 0$ in $v = u - gt$, giving $t = u/g$.
- total time to fall back to the start height: $2u/g$ (the motion is **symmetric** 对称).

Worked example. A ball is thrown straight up at 20 m s^{-1}. Find the greatest height it reaches (take $g = 9.81$ m s^{-2}).

At the highest point $v = 0$, so from $h = u^2/(2g)$:

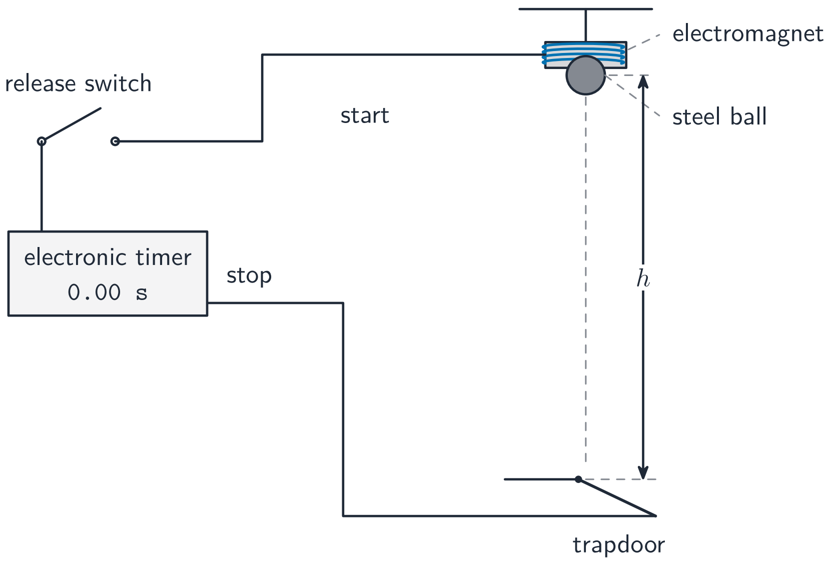
$$h = \frac{20^2}{2 \times 9.81} = \frac{400}{19.62} \approx 20.4 \text{ m}.$$

Experiment to find g

A common method: drop an object from rest, then measure the distance h it falls and the time t it takes. Then

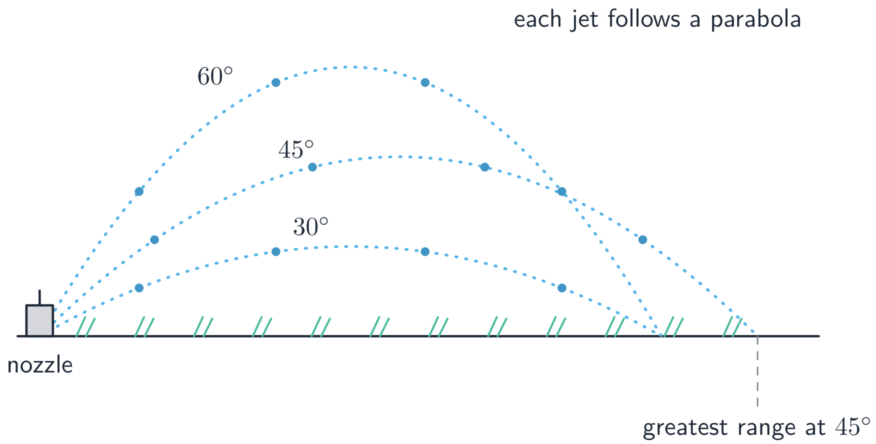
$$g = \frac{2h}{t^2}.$$

Repeat for several heights and plot h against t^2. The gradient of the best straight line is $g/2$, so g is twice the gradient. Repeating reduces **random error** 随机误差. An electronic timer — using **light gates** 光电门, or a switch the ball hits — removes **reaction-time** 反应时间 error.



Experimental set-up for measuring the acceleration due to free fall

Motion in two directions



Water jets from a sprinkler trace parabola paths — a real example of projectile motion

When an object moves at constant velocity in one direction (say **horizontal** 水平) and speeds up in a direction at right angles to it (say **vertical** 竖直, under gravity), the two motions do not affect each other. Treat each direction on its own, with its own SUVAT equation.

Horizontal throw

An object thrown horizontally with speed u_{H} from height h, with air resistance ignored:

- **horizontal**: constant velocity u_{H}. After time t, the horizontal distance is $x = u_{\mathrm{H}}t$.
- **vertical**: starts from rest and speeds up downwards at g. After time t, it has fallen $y = \frac{1}{2}gt^2$ and has vertical velocity $v_{\mathrm{V}} = gt$.

The time to reach the ground depends **only** on the height h, not on u_{H}. Solve $h = \frac{1}{2}gt^2$ for t; then the horizontal **range** 射程 is $u_{\mathrm{H}}t$.

Worked example. A ball is thrown horizontally at 15 m s^{-1} from the top of a cliff 20 m high. Find the time it takes to land and how far from the base it lands (take $g = 9.81$ m s^{-2}).

Vertical motion gives the time: from $h = \frac{1}{2}gt^2$,

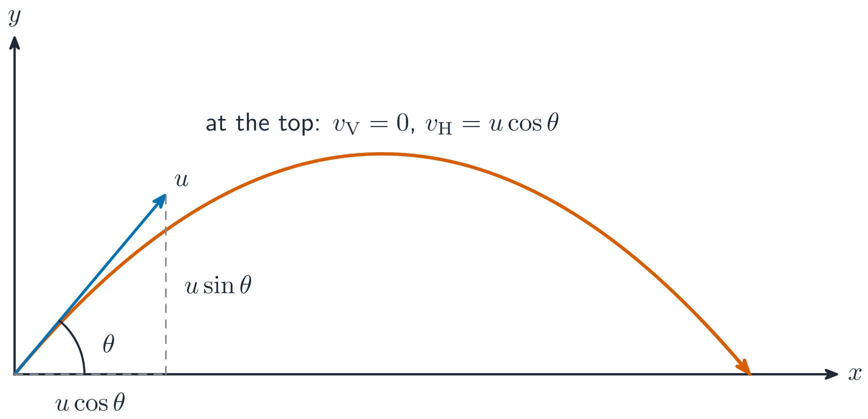
$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 20}{9.81}} \approx 2.0 \text{ s}.$$

Horizontal motion then gives the range: $x = u_{\mathrm{H}}t = 15 \times 2.0 \approx 30$ m.

The horizontal-velocity graph is a flat line at u_H. The vertical-velocity graph is a straight line from the origin with gradient g.

Projectile at an angle

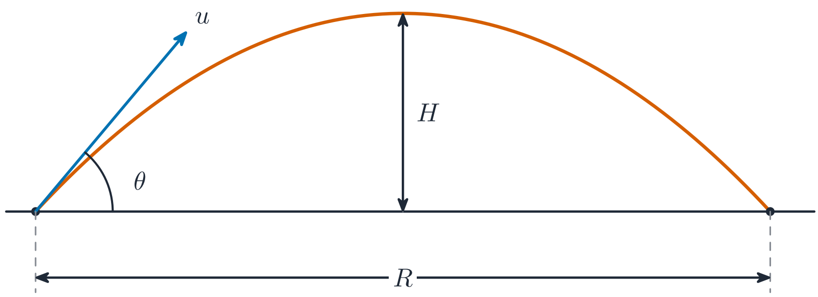
A **projectile** 抛体 thrown at speed u at angle θ above the horizontal:



Projectile launched at angle θ — horizontal and vertical motions are independent

- horizontal **component** 分量 of the starting velocity: $u_H = u\cos\theta$ (stays constant during the flight).
- vertical component of the starting velocity: $u_V = u\sin\theta$ (gets smaller, becomes zero at the top, then grows downwards).

At the highest point, $v_V = 0$, but v_H is still $u\cos\theta$. The time to the top is $t_{up} = u\sin\theta/g$; the total flight time (back to the start height) is $2t_{up}$.



Range R of a projectile launched from and landing on level ground

Bouncing ball

When a ball bounces, its velocity–time graph is a set of straight sloping lines (constant g) with a sudden jump at each bounce (the velocity flips direction, and gets smaller if some **energy** 能量 is lost). Add up the times and the distances across the bounces.

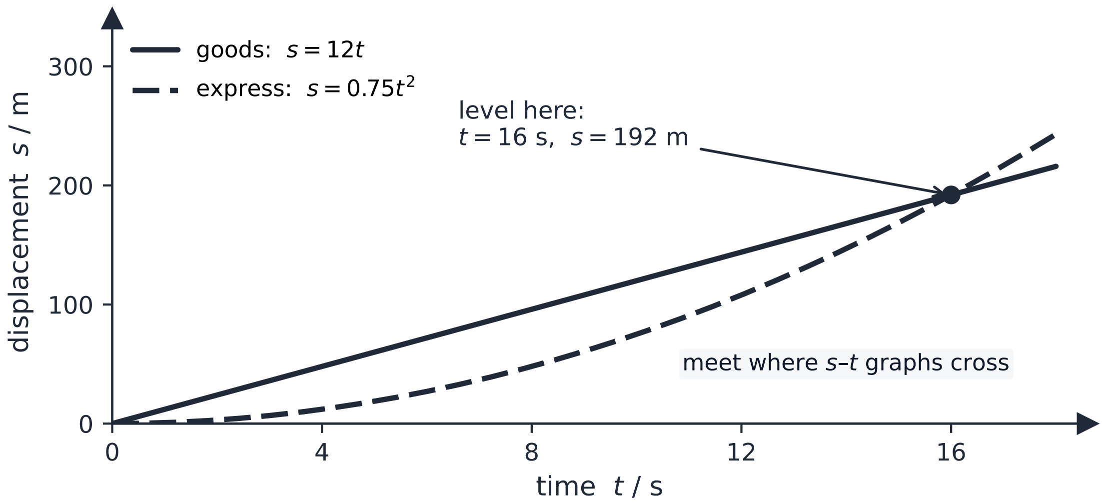
Two objects meeting

When two objects move along the same line in different ways, write a displacement equation for each. Use the same start time and the same positive direction. Then set the two displacements equal (or set their difference to a given gap).

For a goods train at constant velocity u_G and an express train starting from rest with acceleration a, both passing the same point at $t = 0$:

$$s_G = u_G t, \qquad s_E = \frac{1}{2}at^2.$$

They are level again when $s_G = s_E$, giving $t = 2u_G/a$.



Meet where the displacement–time graphs cross (same s at same t)

Tips for solving problems

1. **Draw a diagram** and mark the positive direction.
2. **List the SUVAT symbols** with their known and unknown values, including signs.

3. **Choose the SUVAT equation** with exactly the four symbols you have, plus the one you want.
4. For projectile motion, **split into horizontal and vertical** SUVAT problems, linked only by the time t.
5. Always **check the units** of your answer, and that its size is sensible.

Exam tips

- Use the **SUVAT** equations only for **constant acceleration**; list s, u, v, a, t and pick the equation missing your unknown.
- Choose one direction as **positive** and keep signs consistent (usually $g = -9.81\ \mathrm{m\ s^{-2}}$).
- On a **velocity-time graph**, gradient = acceleration and area = displacement.
- For projectiles, treat **horizontal (constant velocity) and vertical ($a = g$) motion separately**, linked by the same time.

Dynamics

A-Level Physics

Mass, momentum and force

Mass

Mass 质量 tells you how hard it is to change an object's motion. The larger the mass, the larger the **force** 力 needed to give it a certain **acceleration** 加速度. Mass is measured in kilograms (kg) and is a **scalar** 标量.

Momentum

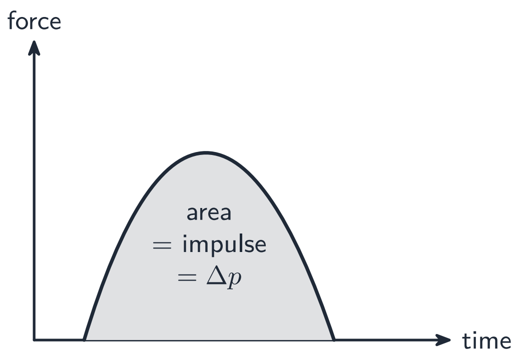
Linear momentum 动量 is the product of mass and velocity:

$$p = mv.$$

Momentum is a **vector** 矢量 — it points the same way as the **velocity** 速度. Its unit is kg m s^{-1}, which is the same as N s.

Force as the rate of change of momentum

Newton's second law, in its general form: the **resultant force** 合力 on an object equals the rate of change of its momentum.



Impulse is the area under a force-time graph, equal to the change in momentum

$$F = \frac{\Delta p}{\Delta t}.$$

When the mass is constant this becomes $F = ma$, because $\Delta p = m\,\Delta v$ and $\Delta v/\Delta t = a$. Cambridge questions often want you to use $F = \Delta p/\Delta t$ directly for a **collision** 碰撞 or an **impulse** 冲量: the average force equals the change in momentum divided by the contact time.

A ball hits a wall with momentum p_1 and bounces back with momentum p_2 in the opposite direction. The change in momentum is $\Delta p = p_2 - p_1$ (give each direction the correct sign). The average force is $\Delta p/\Delta t$, where Δt is the contact time.

Worked example. A 0.20 kg ball hits a wall at 8.0 m s^{-1} and bounces straight back at 6.0 m s^{-1}. The contact lasts 0.050 s. Find the average force on the ball.

Take the rebound direction as positive, so $u = -8.0$ m s^{-1} and $v = +6.0$ m s^{-1}:

$$\Delta p = m(v-u) = 0.20 \times \big(6.0 - (-8.0)\big) = 2.8\ \text{kg m s}^{-1},$$

$$F = \frac{\Delta p}{\Delta t} = \frac{2.8}{0.050} = 56\ \text{N}.$$

When you know the momentum but not the speed, the change in **kinetic energy** 动能 is

$$\Delta E_\mathrm{k} = \frac{p_2^2 - p_1^2}{2m}.$$

This comes from $E_\mathrm{k} = \frac{1}{2}mv^2 = p^2/(2m)$.

Newton's three laws of motion

A rocket pushes gas down; by Newton's third law the gas pushes the rocket up.

First law

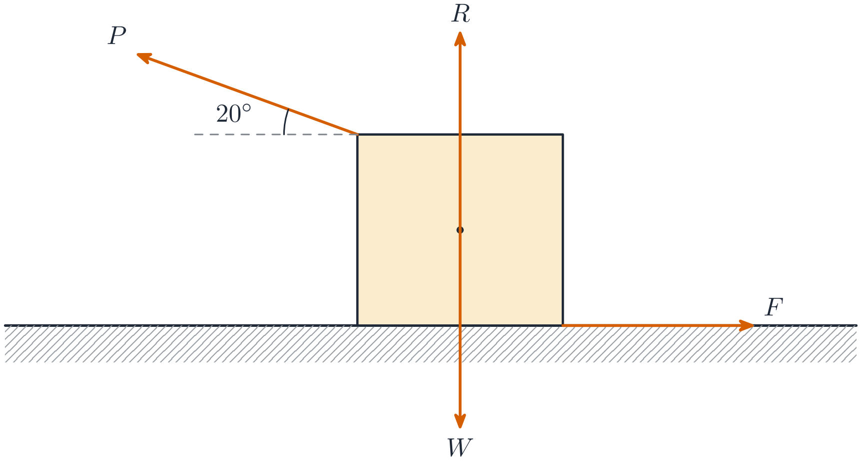
An object stays at rest, or keeps moving at constant velocity in a straight line, unless a resultant **external force** 外力 acts on it. In short: zero resultant force means zero acceleration.

Second law

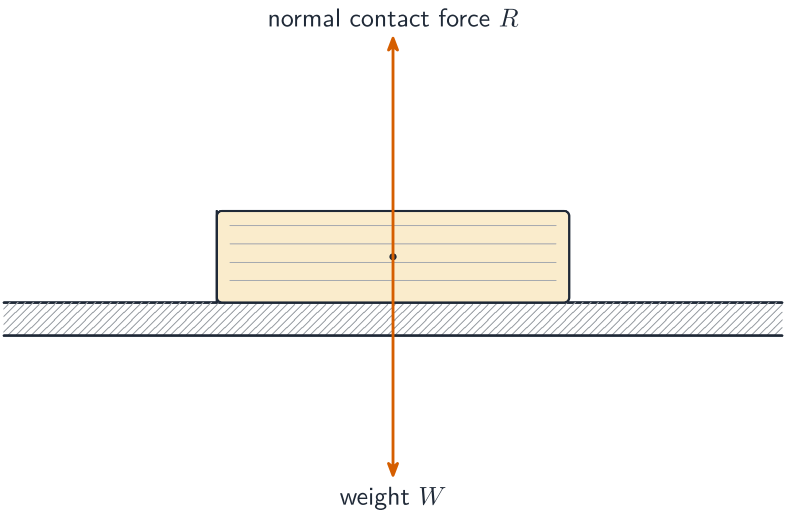
The resultant force on an object equals its rate of change of momentum, and acts in the same direction as that change. In SI units,

$$F = \frac{\Delta p}{\Delta t} = ma \quad \text{(for constant mass).}$$

Acceleration and resultant force always point the **same way**.



Free-body diagram showing all forces on a block being pulled at an angle



Weight and normal contact force on a book resting on a table

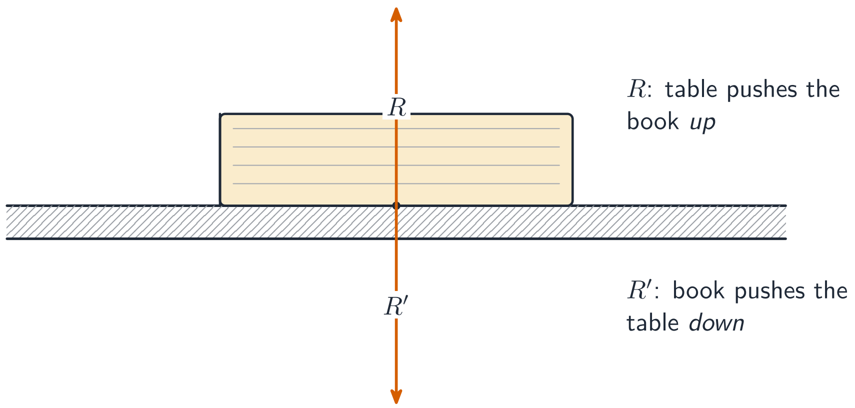
Third law

When body A pushes on body B, body B pushes back on body A with an **equal and opposite** force. The two forces:

- act on **different objects**,
- are of the **same type** (both gravitational, both contact, both **electrostatic** 静电, and so on),
- have the same size and opposite direction.

A common trap: the **weight** 重力 of a block on a table and the **normal contact force** 支持力 from the table are **not** a third-law pair (they act on the same object and are different types). The third-law partner of the block's weight is the pull the block makes on the Earth. The third-law partner of the table's contact force is the push the block makes on the table.

For a rocket: the **thrust** 推力 on the rocket and the force on the gases are a third-law pair (the engine pushes the gas down, the gas pushes the engine up). Weight and air resistance are not part of this pair.



Newton's third-law pair: R on the book (up) and R' on the table (down)

Weight

Weight is the force on an object from a **gravitational field** 重力场. Near the Earth's surface,

$$W = mg,$$

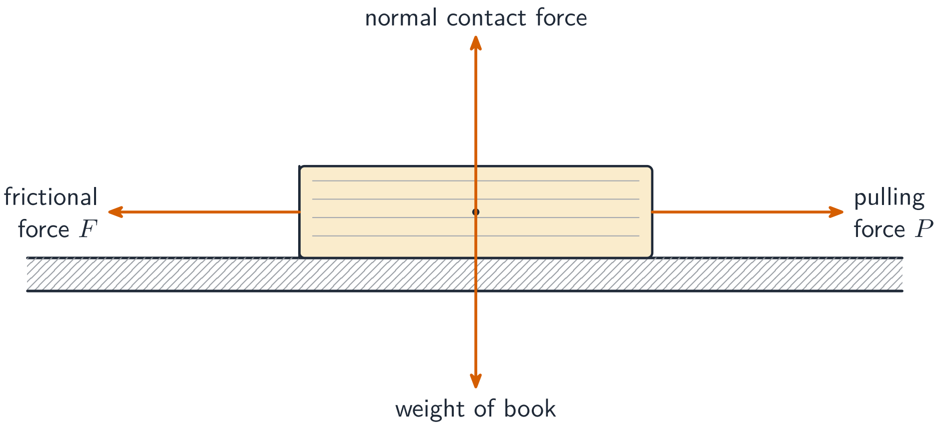
where $g \approx 9.81\ \mathrm{m\ s^{-2}}$ is the acceleration of free fall. Weight is a vector that points towards the centre of the Earth. Do not mix it up with mass: mass is the same everywhere, but weight changes with place.

Non-uniform motion: friction, drag and terminal velocity

Friction and drag forces

A **friction** 摩擦力 force between two solid surfaces acts along the surface and opposes the sliding. A **viscous** 黏性 or **drag** 阻力 force is the resistive force from a **fluid** 流体 (a liquid or gas) on an object moving through it; air resistance is the case for air. You do not need to use any **coefficient** 系数 of friction or viscosity.

A simple model: the drag gets bigger as the speed gets bigger. At zero speed, the drag is zero.



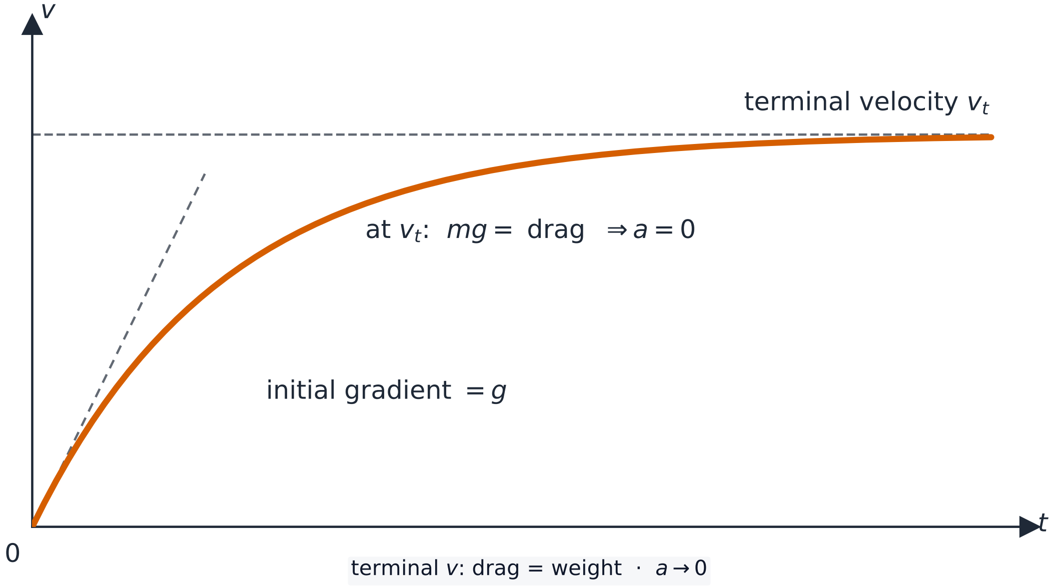
Free-body diagram of a book being pulled on a table

An object falling through air

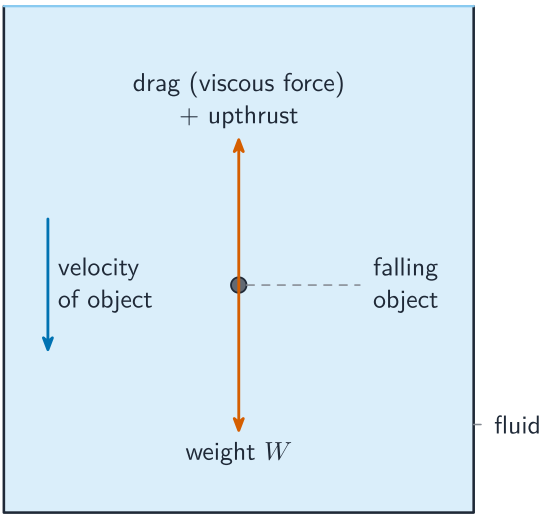
For an object dropped from rest and falling through air:

1. At first, only weight acts, so the object speeds up downwards at g.
2. As the speed grows, the upward drag grows. The resultant force gets smaller, so the acceleration gets smaller.
3. In the end, the drag equals the weight. The resultant force is zero, the acceleration is zero, and the speed stays constant — the **terminal velocity** 收尾速度.

On a velocity–time graph, the line starts straight with gradient g, then bends and flattens at the terminal velocity. This shape (fast start, then slowing acceleration, then constant speed) is how ”falling with air resistance” differs from ”free fall in a vacuum”.



Velocity–time graph for an object falling through air



Forces on a falling object in a fluid

Why the spread-eagle pose? Spreading out gives the largest area, so the most drag. The bigger the drag, the sooner drag balances weight — and the lower the steady terminal velocity they fall at

Energy during a terminal-velocity fall

At terminal velocity, a **parachutist** 跳伞者 has constant kinetic energy. But the **gravitational potential energy** 重力势能 keeps falling as they go down. Where does it go? Almost all of it turns into **thermal energy** 热能 of the air around them. It does **not** become kinetic energy of the parachutist — that stays constant.

Cyclist or car at constant speed

A vehicle at constant speed on a flat road has zero resultant force. The forward driving force is equal and opposite to the total resistive force (friction, air resistance, and rolling resistance). At higher speed the drag is larger, so the driving force must be larger too — and so the power must be larger.

Conservation of linear momentum

In a crash, a large force acts over a very short time to change momentum.

The principle

For a system with no resultant external force, the total momentum stays constant. This is **conservation of momentum** 动量守恒.

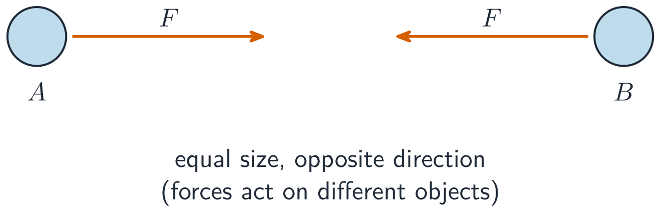
It always holds when there is no outside resultant force — in collisions, **explosions** 爆炸, and **recoil** 反冲. In two dimensions, momentum is conserved along each direction on its own.

Worked example. A 2.0 kg trolley and a 3.0 kg trolley are held together against a compressed spring, then released from rest. The 2.0 kg trolley flies off at 6.0 m s^{-1}. Find the speed of the other trolley.

The total momentum stays zero (it started at rest), so

$$0 = (2.0)(6.0) + (3.0)(-v) \quad \Rightarrow \quad v = \frac{12}{3.0} = 4.0 \text{ m s}^{-1}$$

in the opposite direction.



Newton's third law in an isolated two-particle system: equal and opposite forces

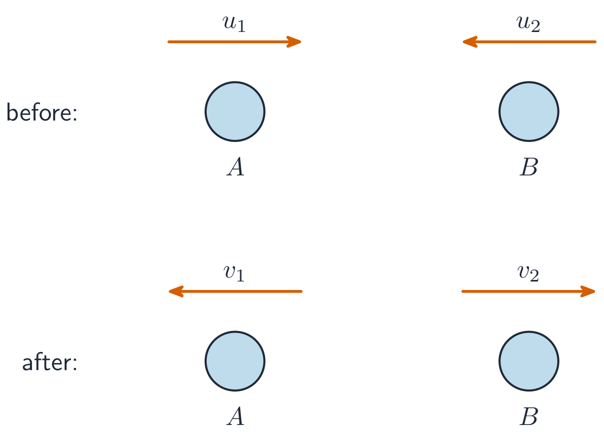
Elastic and inelastic collisions

In **every** collision, momentum is conserved (if there is no outside resultant force).

An **elastic collision** 弹性碰撞 is one where the total kinetic energy is **also** conserved. A quick test: in an elastic collision, the **relative speed** 相对速率 of approach equals the relative speed of separation.

In an **inelastic collision** 非弹性碰撞, momentum is conserved but kinetic energy goes down — some becomes thermal, sound, or **deformation** 形变 energy. If the two objects stick together, the collision is perfectly inelastic.

Solving collision problems (one dimension)



Head-on collision: velocities before and after

For two objects with masses m_1, m_2 and starting velocities u_1, u_2 that hit **head-on** 正面, write

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$

Use **signed** velocities (positive in one chosen direction). If the collision is elastic, add the relative-speed equation

$$u_1 - u_2 = -(v_1 - v_2),$$

or, the same thing, $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$. That gives two equations for two unknowns.

Worked example. A 1500 kg car moving at 12 m s^{-1} runs into a stationary 1000 kg car and they lock together. Find their common velocity just after the collision.

Momentum is conserved (the cars stick, so $v_1 = v_2 = v$):

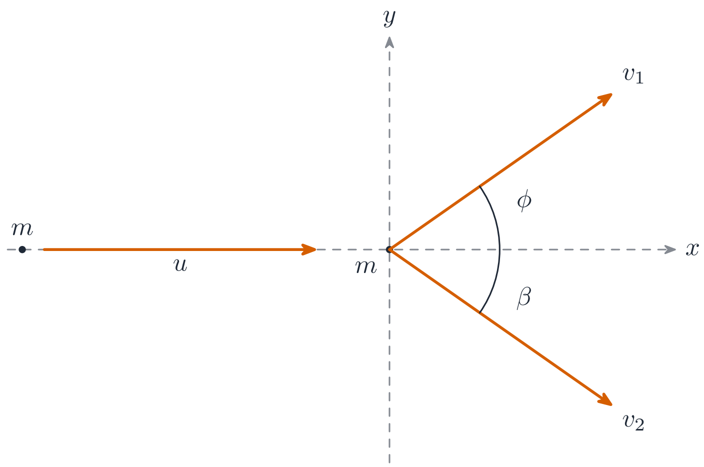
$$1500 \times 12 + 1000 \times 0 = (1500 + 1000)\,v \quad \Rightarrow \quad v = \frac{18\,000}{2500} = 7.2 \text{ m s}^{-1}.$$

A useful result for a head-on elastic collision of mass m with a **stationary** 静止 mass M:

$$v_m = \frac{m - M}{m + M}u, \qquad v_M = \frac{2m}{m + M}u.$$

Collisions in two dimensions

If the objects move in two dimensions, split the velocities into **perpendicular** 垂直 **components** 分量 and apply conservation of momentum along each direction on its own. For a collision where the objects hit at an angle, choose one axis along the first object's motion and one across it. The total momentum is conserved along each axis.



A glancing collision resolved along two perpendicular axes

Rocket / pushing out mass

A rocket pushes out gas at velocity u (relative to itself) at a **mass-flow rate 质量流率** $\dot{m}$ (kg per second). It feels a thrust

$$F = \dot{m} \cdot u,$$

which comes from $F = \Delta p/\Delta t$. The momentum given to the gas each second equals the thrust on the rocket (Newton's third law: the rocket pushes the gas one way, the gas pushes the rocket the other way).

Exam tips

- **Newton's second law is** $F = \Delta p/\Delta t$ (rate of change of momentum); $F = ma$ is the special case for constant mass.
- Identify **third-law pairs** correctly: the same type of force, acting on **two different bodies** — not the balanced forces on one body.
- **Momentum is conserved** in every collision; kinetic energy is conserved only in an **elastic** collision.
- For **terminal velocity**, explain that drag rises with speed until drag = weight, so the acceleration becomes zero.

Forces, density and pressure

A-Level Physics

Turning effects of forces

Centre of gravity

The **weight** 重力 of a large object can be treated as acting at one single point, called the **centre of gravity** 重心 (the same as the **centre of mass** 质心 in a uniform gravitational field). For a uniform, regular shape — a rectangle, a sphere, a uniform rod — the centre of gravity is at the middle.

When you draw a **free-body diagram** 受力图, always put the weight arrow at the centre of gravity.

Moment of a force

The **moment** 力矩 of a force about a point is

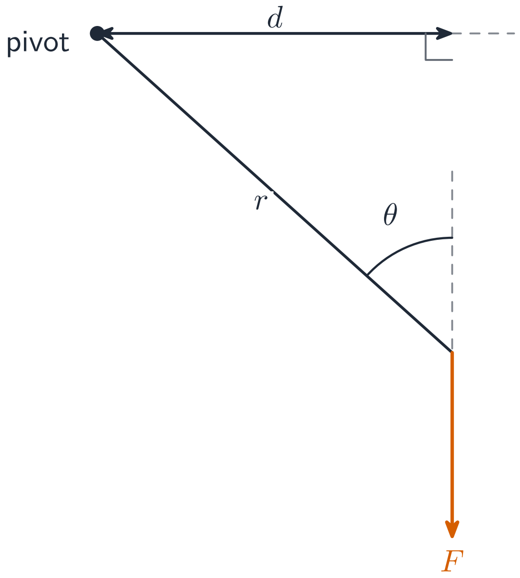
$$M = F \cdot d,$$

where F is the size of the **force** 力 and d is the **perpendicular distance** 垂直距离 from the point to the **line of action** 作用线 of the force. Unit: N m.

If the force acts at angle θ to a **lever arm** 力臂 of length r from the **pivot** 支点, then $d = r \sin \theta$, so $M = Fr \sin \theta$. Only the part of the force **perpendicular** 垂直 to the lever arm makes it turn.

Worked example. A force of 50 N is applied to the end of a spanner 0.40 m long, at 30° to the spanner. Find the moment of the force about the nut.

$$M = Fr \sin \theta = 50 \times 0.40 \times \sin 30° = 50 \times 0.40 \times 0.50 = 10 \text{ N m}.$$



The moment depends on the perpendicular distance d from the pivot to the line of action of the force

A moment is either **clockwise** 顺时针 or **anticlockwise** 逆时针 about the chosen point.

Couple and torque

A **couple** 力偶 is a pair of forces that are:

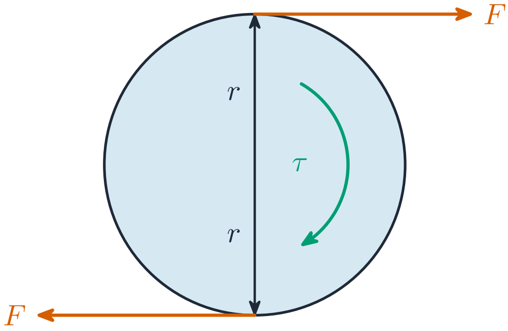
- **equal in size**,
- **opposite in direction**,
- with their lines of action a perpendicular distance apart.

A couple makes the body **turn only** — its resultant force is zero, so it gives no straight-line acceleration.

The **torque** 力偶矩 of a couple is the turning effect it makes:

$$\tau = F \cdot d,$$

where F is the size of one force and d is the perpendicular distance between the two lines of action. Unit: N m. The torque is the same about any point — a special property of couples.



A couple: two equal and opposite forces, a distance apart, producing a torque

A common multiple-choice trap: two equal forces in the **same** direction are not a couple (they have a resultant force and cause **translation** 平动). A couple needs equal size **and** opposite direction.

Equilibrium of forces

Conditions for equilibrium

A body is in **equilibrium** 平衡 when:

1. the **resultant force** 合力 is zero (no straight-line acceleration), AND

2. the resultant moment about any point is zero (no **angular acceleration** 角加速度).

Both must hold. A body with no resultant force can still be turning; a body with no resultant moment can still be moving in a straight line.

Principle of moments

For a body that is not turning, **the total clockwise moment about any point equals the total anticlockwise moment about the same point**. This is the **principle of moments** 力矩原理.

To solve a balance problem:

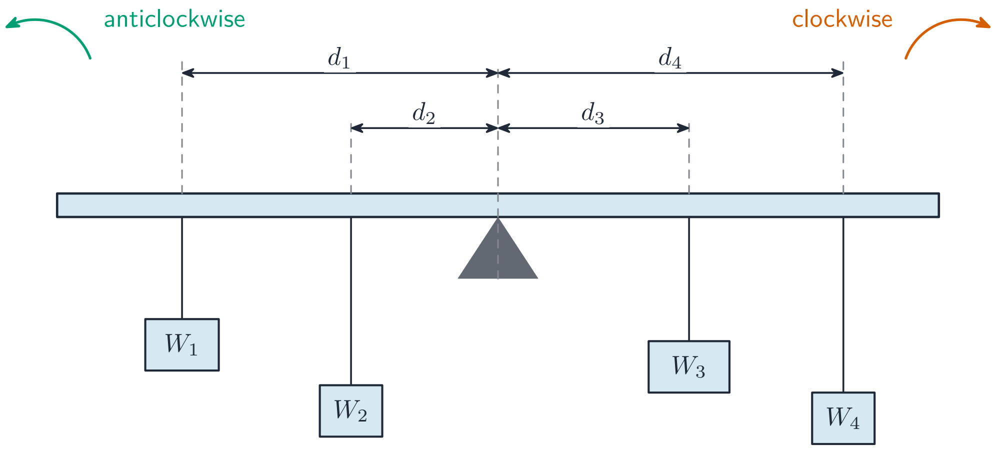
1. Choose a pivot — usually where an unknown force acts, so that force drops out (its distance is zero).
2. List every force and its perpendicular distance from the pivot.
3. Set $\sum M_{\text{clockwise}} = \sum M_{\text{anticlockwise}}$.
4. Use $\sum F = 0$ if you need a second equation.

A ruler balanced on a pivot with masses on each side is solved this way. For a heavy uniform rod, remember to include its weight acting at its centre of gravity.

Worked example. A uniform beam of length 4.0 m is pivoted at its centre. A 40 N weight hangs 1.5 m from the pivot on one side. How far from the pivot, on the other side, must a 30 N weight hang to balance the beam?

The beam's own weight acts at the centre (the pivot), so it has no moment. Setting clockwise = anticlockwise moments:

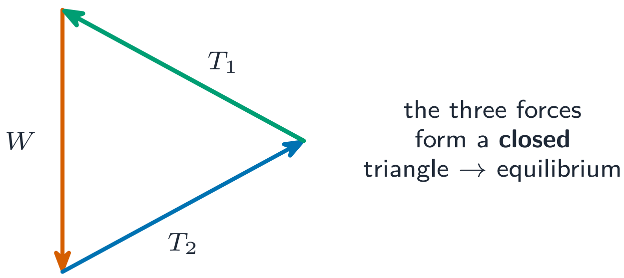
$$40 \times 1.5 = 30 \times d \quad \Rightarrow \quad d = \frac{60}{30} = 2.0 \text{ m}.$$



Weights on a rule balanced at a pivot — used to test the principle of moments

Vector triangle

Three forces in the same plane that are in equilibrium can be drawn as a **closed vector triangle** 矢量三角形 — drawn tip-to-tail, the three arrows come back to the start. This is a drawing method instead of splitting into **components** 分量.



Three forces in equilibrium form a closed vector triangle

Use the **sine rule** 正弦定理 or the **cosine rule** 余弦定理 on the triangle to find unknown sizes or directions, or draw the triangle to scale on graph paper.

You can also split each force into **horizontal** 水平 and **vertical** 竖直 components and set $\sum F_x = 0$ and $\sum F_y = 0$.

Density

An iceberg floats with most of its volume hidden: ice is slightly less dense than water.

Density 密度 is the mass per unit volume:

$$\rho = \frac{m}{V}.$$

Unit: kg m^{-3} (or g cm^{-3}; 1 g cm^{-3} = 1000 kg m^{-3}). Density is a **scalar** 标量.

Some useful densities to know:

- water: 1000 kg m^{-3}
- air at room conditions: $\sim$ 1.2 kg m^{-3}
- iron / steel: $\sim$ 7800 kg m^{-3}

Pressure

Pressure 压强 is the force per unit area, where the force acts at right angles to the surface:

$$p = \frac{F}{A}.$$

Unit: Pa = N m^{-2}. Pressure is a scalar.

A precision aneroid barometer measures atmospheric pressure

Hydrostatic pressure

Take a column of **fluid** 流体 with density ρ, **cross-sectional area** 横截面积 A and height Δh. Its weight is

$$W = mg = (\rho \cdot A \cdot \Delta h) \cdot g.$$

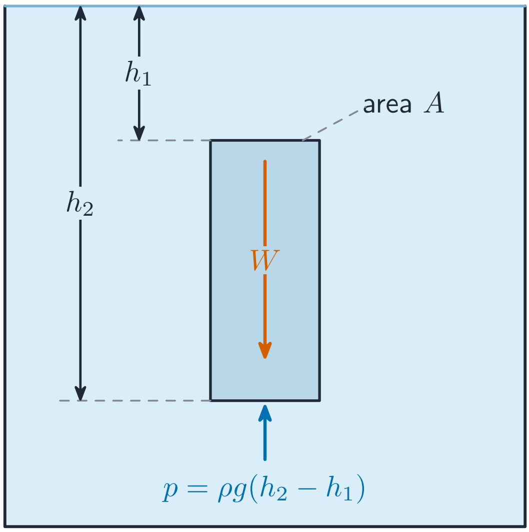
This weight presses down on the area A at the bottom, so the extra pressure at the bottom compared with the top is

$$\Delta p = \frac{W}{A} = \rho g \Delta h.$$

This is the **hydrostatic pressure** 流体静压强 equation. It depends only on the density of the fluid and the **depth** 深度 — the shape of the container does not matter.

Worked example. Find the extra pressure due to the water at the bottom of a swimming pool 2.5 m deep. (Water density 1000 kg m^{-3}, $g = 9.81$ m s^{-2}.)

$$\Delta p = \rho g \Delta h = 1000 \times 9.81 \times 2.5 \approx 2.5 \times 10^4 \text{ Pa}.$$

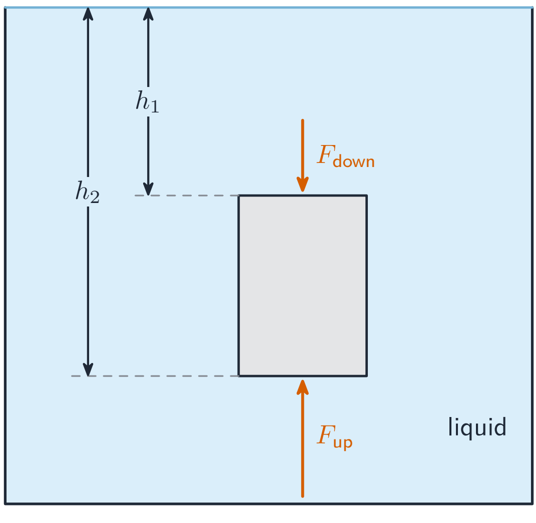


A column of liquid of area A: its weight sets the extra pressure at the depth below

For a submarine at depth h below the surface, the pressure from the water is $\rho_{\text{seawater}}\, g\, h$. For the total pressure, add the **atmospheric pressure** 大气压强 at the surface (about 1.0×10^5 Pa).

Upthrust and Archimedes' principle

When an object is **submerged** 浸没 in a fluid, the pressure at the bottom of the object is greater than the pressure at the top (by $\rho g \Delta h$, where Δh is the object's height). This difference gives a net upward force called the **upthrust** 浮力.



Upthrust arises because the pressure on the bottom of the object is greater than on the top

For an object of volume V (the volume of fluid **displaced** 排开), the upthrust is

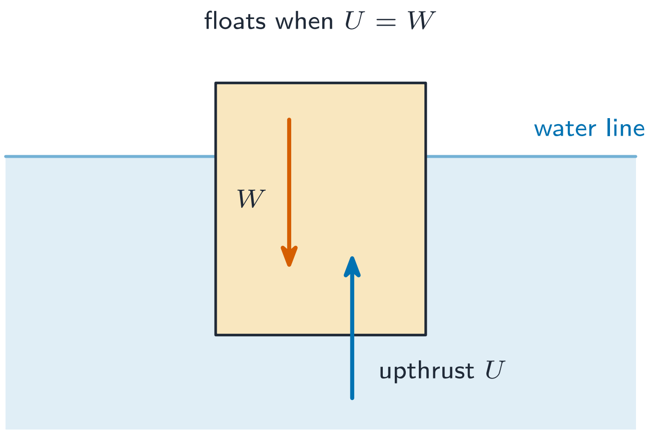
$$F_{\text{upthrust}} = \rho_{\text{fluid}}\, g\, V.$$

This is **Archimedes' principle** 阿基米德原理: the upthrust on a body in a fluid equals the weight of the fluid it pushes aside.

Worked example. A metal block of volume 2.0×10^{-3} m^3 is fully submerged in water. Find the upthrust on it. (Water density 1000 kg m^{-3}, $g = 9.81$ m s^{-2}.)

$$F_{\text{upthrust}} = \rho_{\text{fluid}}\, g\, V = 1000 \times 9.81 \times 2.0 \times 10^{-3} \approx 20 \text{ N}.$$

For a fully submerged object, V is its full volume. For a **floating** 漂浮 object, V is only the volume below the surface — the object floats when the upthrust on the part below the surface equals its weight.



A floating object sinks until the upthrust on the submerged part equals its weight

Force balance with upthrust

A block held under water by a string tied to the bottom of the container is in equilibrium under three vertical forces: weight (down), **tension** 张力 (down), upthrust (up). Set $F_{\text{upthrust}} = W + T$ to find the tension.

A submerged block hanging from a **newton meter** 弹簧测力计 reads less than its weight in air, because of the upthrust: reading $= W - F_{\text{upthrust}}$.

The upthrust depends on the fluid density and the displaced volume, not on the object's material or depth (for an **incompressible** 不可压缩 fluid). On a planet with smaller g, the upthrust is smaller in the same ratio as the weight, so a floating object still floats with the same fraction below the surface.

Exam tips

- Take **moments** about a chosen pivot; for equilibrium, total clockwise = total anticlockwise moments **and** the resultant force is zero.
- A body in equilibrium under three forces gives a **closed triangle** of forces.
- Fluid pressure = $\rho g h$; **upthrust** = **weight of fluid displaced** (Archimedes).
- Distinguish mass, weight and density, and give the base unit each time.

Work, energy and power

A-Level Physics

Work, energy and power

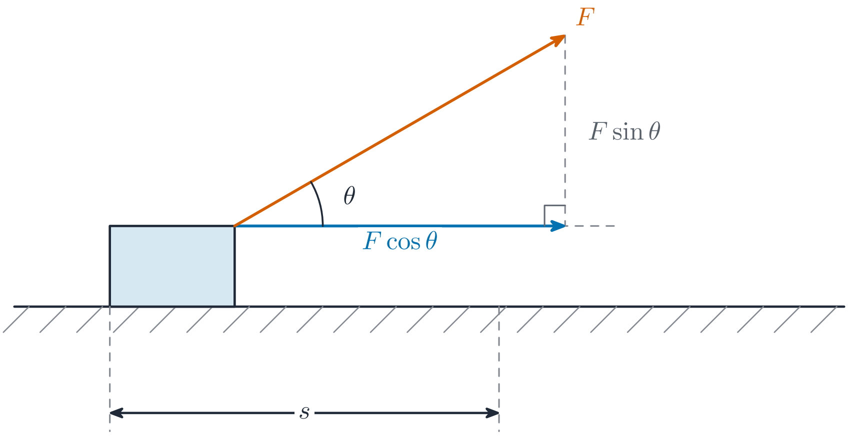
Wind turbines transfer the kinetic energy of the wind into electrical energy.

Work done by a force

Work 功 is done when a force moves its point of contact along the line of the force. The work done by a constant force F that causes a **displacement 位移** s is

$$W = F \cdot s \cdot \cos\theta,$$

where θ is the angle between the force and the displacement. Only the **component 分量** of the force along the displacement does work.



Only the component of the force along the displacement ($F\cos\theta$) does work

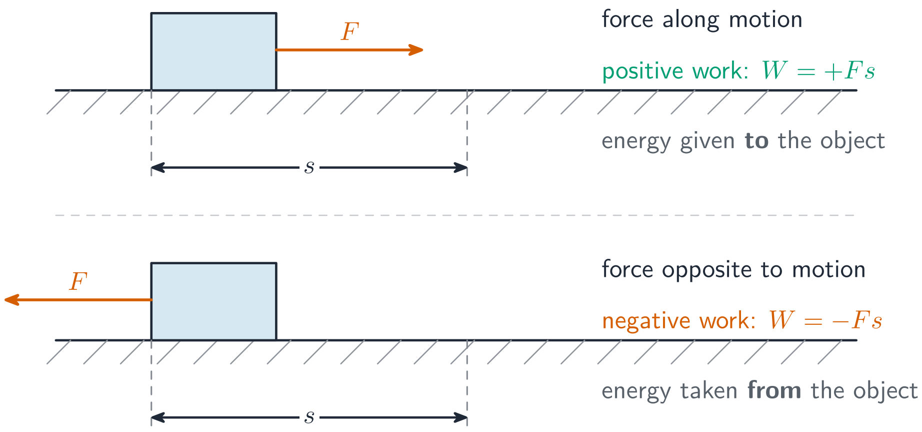
Worked example. A child pulls a sledge 5.0 m across the snow with a rope, using a force of 20 N at 60° to the ground. Find the work done by the rope.

$$W = Fs\cos\theta = 20 \times 5.0 \times \cos 60^\circ = 20 \times 5.0 \times 0.50 = 50 \text{ J}.$$

Unit: J = N m. Work is a **scalar 标量**.

Special cases:

- force in the same direction as the motion ($\theta = 0$): $W = Fs$, positive work, **energy 能量** given **to** the object.
- force at right angles to the motion ($\theta = 90^\circ$): $W = 0$. The **normal contact force 支持力** on a car on a flat road does no work.
- force opposite to the motion ($\theta = 180^\circ$): $W = -Fs$, negative work, energy taken **from** the object (for example **friction 摩擦力**).



Positive work ($W = +Fs$): force along the motion (top). Negative work ($W = -Fs$): force opposite to the motion, e.g. friction (bottom)

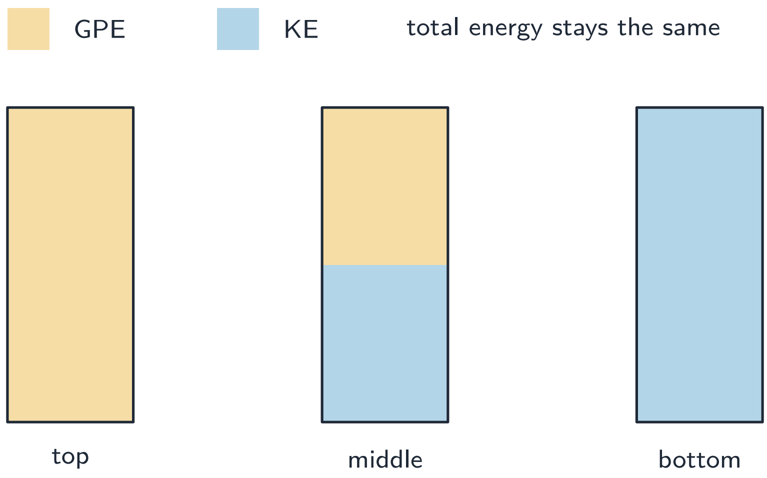
For an object moving up a slope at angle α to the **horizontal** 水平, the work done against gravity in rising a height h is mgh, while the work done by a horizontal push over the slope length L uses $\cos\alpha$.

Conservation of energy

Energy is **never made or destroyed** — it only changes from one form to another, or moves from one object to another. In a **closed system** 封闭系统, the total energy stays constant. This is **conservation of energy** 能量守恒.

When you write an energy equation, list every form the energy starts as and ends as. Common forms in this syllabus: kinetic, gravitational potential, **elastic potential energy** 弹性势能, **electrical** 电能, thermal, sound, **chemical energy** 化学能.

A ball rolling down a **frictionless** 无摩擦 **ramp** 斜坡 turns **gravitational potential energy** 重力势能 into **kinetic energy** 动能: $mgh = \frac{1}{2}mv^2$, so $v = \sqrt{2gh}$. With friction, some of this energy becomes **thermal energy** 热能 of the ramp and the air.



On a frictionless ramp, GPE turns into KE while the total energy stays constant

Worked example. A ball is released from rest at the top of a smooth ramp 1.2 m high. Find its speed at the bottom (take $g = 9.81$ m s^{-2}).

All the gravitational potential energy becomes kinetic energy, so $v = \sqrt{2gh}$ (the mass cancels):

$$v = \sqrt{2 \times 9.81 \times 1.2} \approx 4.9 \text{ m s}^{-1}.$$

Efficiency

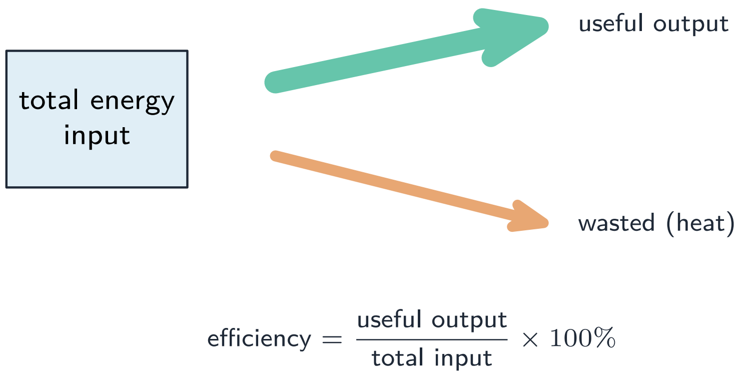
The **efficiency** 效率 of a system is

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%.$$

The same idea with power:

$$\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}} \times 100\%.$$

Efficiency is always less than 100% in a real system, because some input energy becomes "useless" forms — usually thermal energy.



Efficiency: only part of the energy input leaves as useful output; the rest is wasted, usually as heat

For an electric motor lifting a load with efficiency η at **voltage** 电压 V and **current** 电流 I, the useful output power is ηVI. From this you can find a force, a lifting speed, or a **tension** 张力.

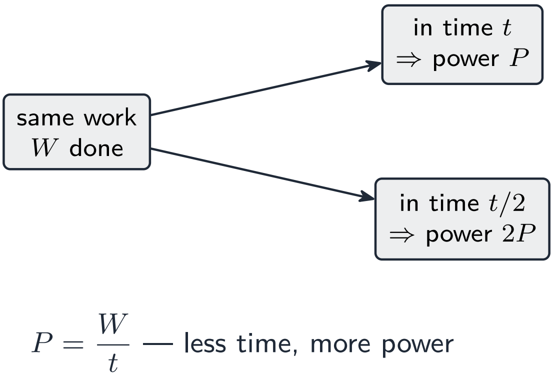
Worked example. An electric motor lifts a 50 kg load at a steady 0.40 m s^{-1} while drawing 250 W of electrical power. Find its efficiency (take $g = 9.81$ m s^{-2}).

The useful output power is $P = mgv = 50 \times 9.81 \times 0.40 = 196$ W, so

$$\text{efficiency} = \frac{196}{250} \times 100\% \approx 78\%.$$

Power

Power 功率 is the rate of doing work, or the rate of transferring energy:



The same work done in less time means more power

$$P = \frac{W}{t} = \frac{\Delta E}{\Delta t}.$$

Unit: W = J s^{-1}. Power is a scalar.

Power, force and velocity

For an object moving at **velocity 速度** v with a force F along the direction of motion, in a short time Δt the displacement is $v\,\Delta t$ and the work done is $Fv\,\Delta t$. Dividing by Δt:

$$P = Fv.$$

This is one of the most useful results in mechanics.

Worked example. A car travels at a steady 25 m s^{-1} against a total resistive force of 600 N. Find the output power of its engine.

At constant speed the driving force equals the resistive force, so

$$P = Fv = 600 \times 25 = 15\,000 \text{ W} = 15 \text{ kW}.$$

- For a car at constant velocity v on a flat road, the engine power must balance the total resistive force: $P = F_{\text{resist}} \cdot v$. If the **drag 阻力** grows with v^2, doubling the speed roughly **quadruples** the power needed.

- For lifting a **weight** 重力 mg straight up at constant speed v, the useful output power is $P = mg \cdot v$.
- For an aircraft hovering at a fixed height, the lift force equals the weight, and a large power is needed because air must be pushed downwards all the time.

Gravitational potential energy

In a **uniform** gravitational field (close to a planet's surface), the change in gravitational potential energy of **mass** 质量 m rising or falling through a height Δh is

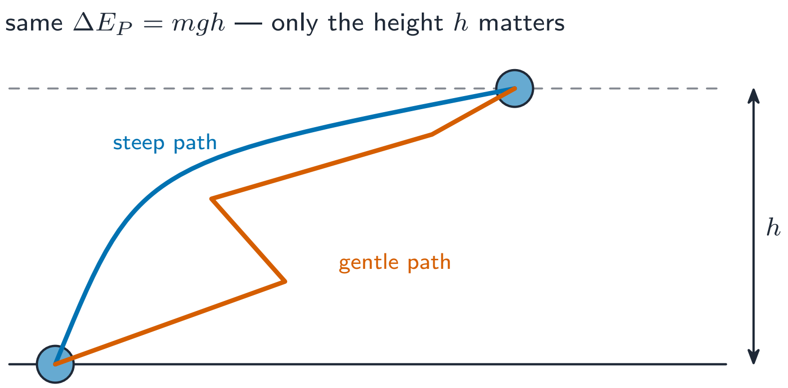
$$\Delta E_{\mathrm{P}} = mg\Delta h.$$

Where it comes from

The work done **against** gravity to raise a mass m slowly (no change in kinetic energy) through height Δh equals the gravitational potential energy gained:

- the gravitational force on the mass is mg downwards,
- the force needed to lift it slowly is mg upwards,
- the work done by this force is $W = F \cdot s = mg \cdot \Delta h$,
- this work becomes ΔE_{P}.

So $\Delta E_{\mathrm{P}} = mg\Delta h$. To use it you need m and Δh (and g). You do not need speed or time.



Gravitational PE depends only on the height risen, not the path taken

Kinetic energy

The kinetic energy of an object of mass m moving at speed v is

$$E_\mathrm{K} = \tfrac{1}{2}mv^2.$$

Where it comes from

Apply a resultant force F to a mass m that starts at rest. It speeds up evenly from 0 to v over a displacement s. From $v^2 = u^2 + 2as$ with $u = 0$,

$$s = \frac{v^2}{2a}.$$

The work done on the mass is

$$W = F \cdot s = ma \cdot \frac{v^2}{2a} = \tfrac{1}{2}mv^2.$$

All this work becomes kinetic energy, so $E_\mathrm{K} = \tfrac{1}{2}mv^2$.

Kinetic energy and momentum

Combining $p = mv$ and $E_\mathrm{K} = \tfrac{1}{2}mv^2$:

$$E_\mathrm{K} = \frac{p^2}{2m}.$$

This is handy when the **momentum 动量** is given but not the velocity. For a momentum change from p_1 to p_2 at constant mass, the change in kinetic energy is $(p_2^2 - p_1^2)/(2m)$.

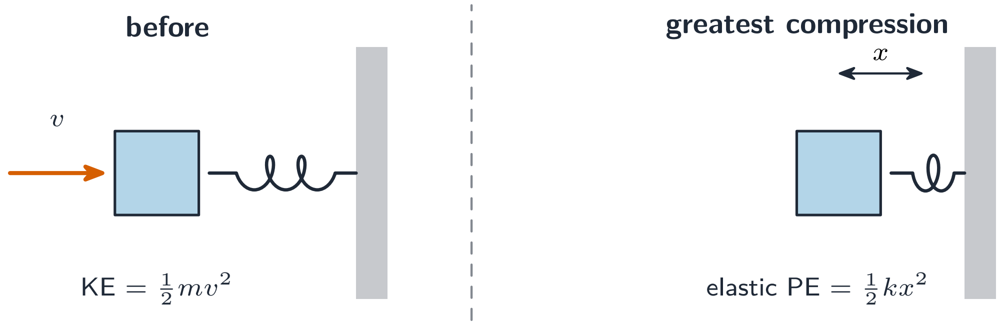
Using energy methods

A useful plan for problems that mix forces and energy:

1. **Find the start and end states.** Write the kinetic and potential energies in each.
2. **List any work done by outside forces** (friction, a push). Friction usually takes energy out; a push can add it.
3. **Conservation of energy**: $E_\text{start} + W_\text{in} = E_\text{end} + W_\text{lost as heat etc.}$.

Examples:

- A box pushed at **constant velocity** up a ramp of length L rising by h: E_K does not change, so the work done by the push goes into ΔE_P plus the work done against friction.
- A block sliding into a **spring** 弹簧 with kinetic energy E_K on a frictionless surface: at greatest **compression** 压缩 x, all the kinetic energy has become elastic potential energy $\frac{1}{2}kx^2$ (where k is the **spring constant** 劲度系数).
- A ball dropped from height h_1 that bounces to height h_2: the ratio h_2/h_1 is the fraction of mechanical energy kept, $h_2/h_1 = (v_{\text{up}}/v_{\text{down}})^2$.
- A **projectile** 抛体 thrown to the same height at different angles: the final speed is the same (only the height matters); use components to get its direction.



A block's kinetic energy becomes elastic potential energy as it compresses the spring

Exam tips

- **Work** = **force** × **distance moved in the direction of the force** — use $Fs\cos\theta$ when they are at an angle.
- Use **conservation of energy**: loss in GPE = gain in KE (+ work done against resistance).
- **Power** = **work** / **time** = Fv; efficiency = useful output ÷ total input.
- GPE uses the **vertical** height gained, not the distance along a slope.

Deformation of solids

A-Level Physics

Forces that cause deformation

When a force acts on a solid along its length, the object changes shape (**deformation** 形变). Two cases (treated as one-dimensional here):

- a **tensile** 拉伸 force stretches the object — it makes an **extension** 伸长量 x,
- a **compressive** force squeezes the object — it makes a **compression** 压缩, treated as a negative extension.

The applied force is the **load** 负载. The change from the natural length is the extension (or compression).

Hooke's law and the spring constant

A spring obeys Hooke's law: extension is proportional to the force applied.

For many materials at small extensions, **the extension is proportional to the load** — this is **Hooke's law** 胡克定律. The constant that links them is the **spring constant** 劲度系数 k:

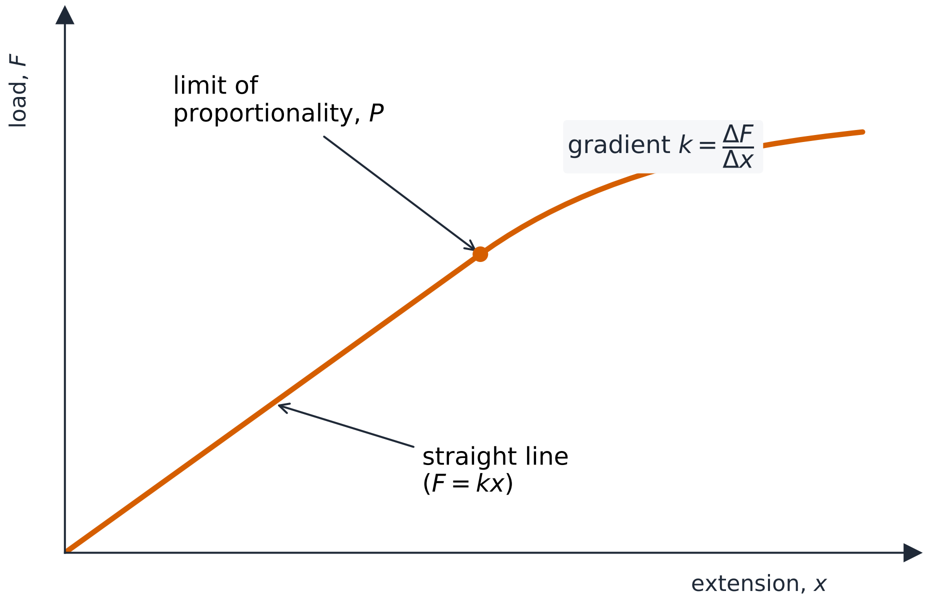
$$F = kx \qquad \Longleftrightarrow \qquad k = \frac{F}{x}.$$

Unit of k: N m^{-1}.

Worked example. A spring stretches by 4.0 cm when a 2.0 N load is hung from it. Find its spring constant.

Converting the extension to metres (4.0 cm = 0.040 m):

$$k = \frac{F}{x} = \frac{2.0}{0.040} = 50 \text{ N m}^{-1}.$$



A load–extension graph: straight up to the limit of proportionality P, then it curves

Reading a graph:

- A force–extension (F against x) graph has gradient k in the Hooke's-law region.
- An extension–force (x against F) graph has gradient $1/k$ in the Hooke's-law region.

A common trap: if a graph plots **length** L against force, you can still find the spring constant from the gradient (since $L = L_0 + F/k$, the gradient is $1/k$ — read it off carefully).

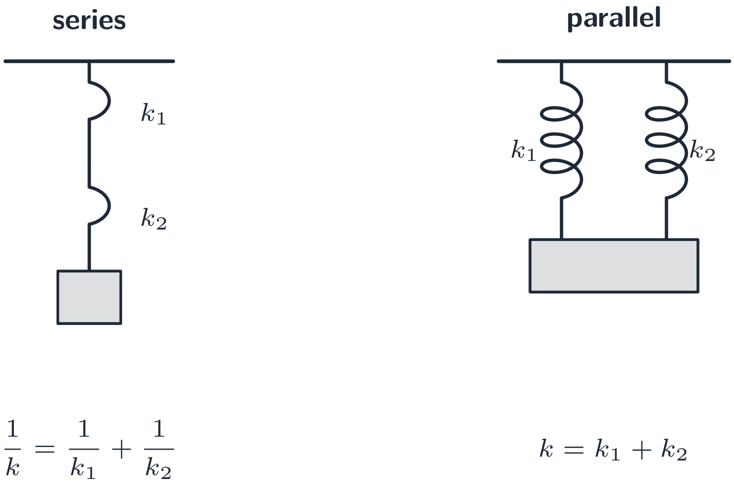
Limit of proportionality

Hooke's law only holds up to the **limit of proportionality** 比例极限. Past this point the F against x line curves and is no longer straight. The material may still be **elastic** 弹性 (it returns to its first length when you remove the load) a little further, then it becomes **plastic** 塑性.

Springs in series and parallel

You may need to combine spring constants:

- **Series** 串联 (one spring hangs from another): the same load passes through both, the total extension is the sum, so $\dfrac{1}{k_{\text{total}}} = \dfrac{1}{k_1} + \dfrac{1}{k_2}$.
- **Parallel** 并联 (two springs side by side holding the same load): each takes half the load (if they are identical), the extensions are equal, so $k_{\text{total}} = k_1 + k_2$.

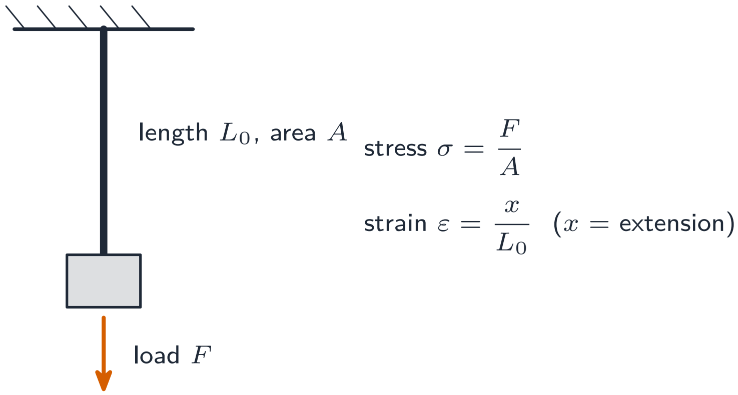


Combining spring constants: series gives a softer spring, parallel a stiffer one

Stress, strain and the Young modulus

For a wire of uniform cross-section under a tensile load:

- **Stress** 应力 $\sigma = \dfrac{F}{A}$, where F is the load and A is the **cross-sectional area** 横截面积. Unit: Pa.
- **Strain** 应变 $\varepsilon = \dfrac{x}{L_0}$, where x is the extension and L_0 is the original length. Strain has no unit (it is a ratio of lengths).



Stress is the load per cross-sectional area; strain is the extension per original length

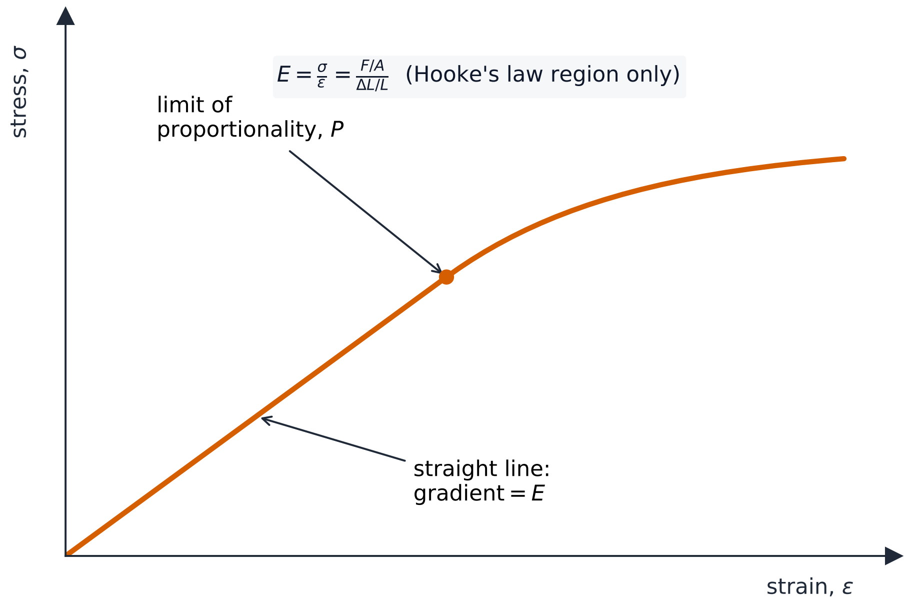
The **Young modulus** 杨氏模量 is the ratio of stress to strain in the Hooke's-law region:

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{x/L_0} = \frac{FL_0}{Ax}.$$

Unit: Pa (about 10^{11} for metals; e.g. steel $\approx 2.0 \times 10^{11}$ Pa).

Worked example. A steel wire of length 2.0 m and cross-sectional area 1.5×10^{-7} m^2 stretches by 1.0 mm under a load of 15 N. Find the Young modulus.

$$E = \frac{FL_0}{Ax} = \frac{15 \times 2.0}{(1.5 \times 10^{-7})(1.0 \times 10^{-3})} = 2.0 \times 10^{11} \text{ Pa}.$$



A stress–strain graph, straight up to the limit of proportionality P

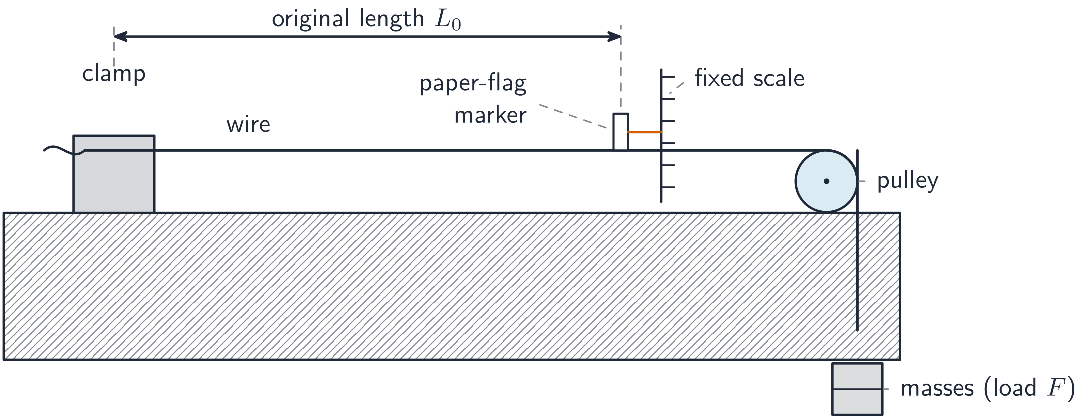
The Young modulus is a property of the **material** — it does not depend on the wire's shape or size. The spring constant k depends on both the material and the size: $k = EA/L_0$.

Experiment to find the Young modulus of a metal wire

A standard setup:

1. Clamp one end of a long, thin wire to a fixed support. Pass the wire over a **pulley** 滑轮 at the edge of the bench so it hangs straight down.
2. Measure the original length L_0 between the clamp and a marker near the pulley, using a metre rule.
3. Measure the **diameter** 直径 d of the wire at several places with a **micrometer** 螺旋测微器 and take the average. Work out $A = \pi d^2/4$.
4. Hang weights one at a time. Record the load F and the extension x (how far the marker moves against a fixed scale).
5. Plot F against x. In the straight region the gradient is EA/L_0, so $E = \text{gradient} \times L_0/A$.

Why a **long, thin** wire? To make the extension big enough to measure well. Why repeat readings and measure d at several places? To reduce **random error** 随机误差 and check the wire is uniform.



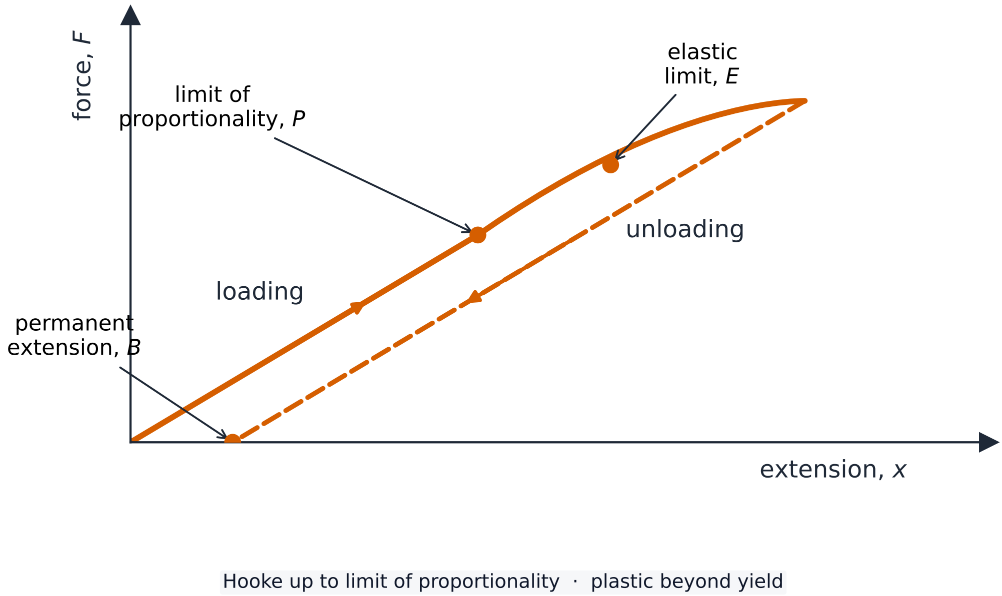
Apparatus for measuring the Young modulus of a wire

Elastic and plastic behaviour

As the load grows:

1. **Elastic and straight (Hooke obeyed)** — up to the limit of proportionality. Removing the load returns the object to its first length.
2. **Elastic but curved** — between the limit of proportionality and the **elastic limit** 弹性极限. The extension is no longer straight in the load, but on unloading the object still returns to its first length.
3. **Plastic** — past the elastic limit. On unloading, the object does **not** return to its first length; a permanent extension stays.

Hooke's law only holds in the straight, elastic region.



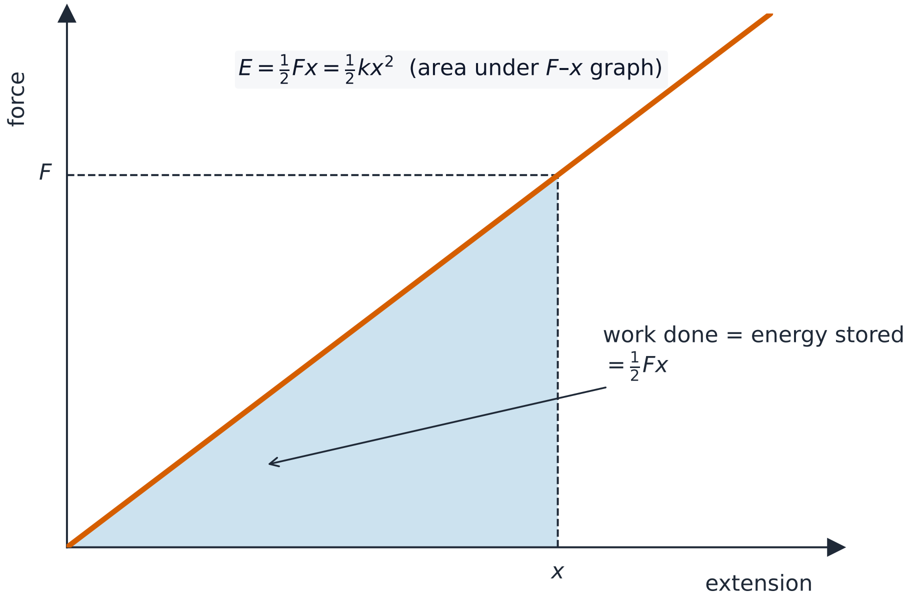
Force–extension past the elastic limit: P and E marked, with a permanent extension B left after unloading

A modern universal (tensile) testing machine stretches a sample and records the force and extension

On a force–extension graph for a material taken into the plastic region and then unloaded, the loading line and the unloading line are different. The unloading line is parallel to the first Hooke line but shifted to the right (the permanent extension left when the load reaches zero). The area between the loading and unloading lines is the energy turned into **thermal energy** 热能 in the material.

Energy stored in a stretched material

The **work done** in stretching a material from 0 to extension x, as the load grows from 0 to F, is the **area under the force–extension graph**.



The work done stretching a material is the area under the force–extension graph

Hooke's-law material

When Hooke's law holds, the F against x graph is a straight line through the origin. The area under it from 0 to x is a triangle:

$$E_P = \tfrac{1}{2}Fx = \tfrac{1}{2}kx^2.$$

This is the **elastic potential energy** 弹性势能 stored in a spring or wire stretched within its limit of proportionality. An equal form:

$$E_P = \frac{F^2}{2k}.$$

Worked example. A spring of spring constant 50 N m^{-1} is stretched by 0.20 m, within its limit of proportionality. Find the elastic potential energy stored.

$$E_P = \tfrac{1}{2}kx^2 = \tfrac{1}{2} \times 50 \times 0.20^2 = 1.0 \text{ J}.$$

Non-Hooke material

For a graph that is not a straight line (a stretched rubber band, or a spring past its limit of proportionality), find the area by counting grid squares or by using **trapezia** 梯形. The same idea holds: **the area under the force–extension graph is the work done on the material**.

Comparing stored energy

A common multiple-choice case: two materials are stretched by the same force, or by the same extension. Using $E_{\mathrm{P}} = \frac{1}{2}Fx$:

- same F, smaller k (less stiff) → larger x → more energy stored.
- same x, larger k (stiffer) → larger F → more energy stored.

When a stretched spring is released onto a mass, the elastic potential energy becomes **kinetic energy** 动能 (and gravitational potential energy if the mass rises). Set $\frac{1}{2}kx^2$ equal to $\frac{1}{2}mv^2$ (plus any mgh) to find the speed or height.

Exam tips

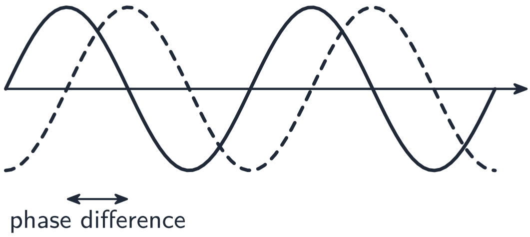
- **Hooke's law ($F = kx$)** holds only up to the limit of proportionality.
- **Stress** $= F/A$, **strain** $= x/L$, **Young modulus** $=$ **stress/strain** (gradient of the straight part of the stress-strain graph) — watch the units (Pa).
- Energy stored $=$ **area under the force-extension graph** $= \frac{1}{2}Fx$ in the elastic region.
- Distinguish **elastic** (returns to shape) from **plastic** (permanent) deformation.

Waves

A-Level Physics

Progressive waves

Ripples spreading on water are progressive waves that carry energy outward.



Two waves of the same frequency, shifted by a phase difference

A **wave** 波 carries **energy** 能量 from one place to another without moving matter overall. The particles of the **medium** 介质 **oscillate** 振动 about fixed rest positions; only the disturbance (and its energy) **propagates** 传播. Examples: a **transverse wave** 横波 on a rope, a **longitudinal wave** 纵波 on a slinky spring, ripples on water, and sound in air. A wave that travels and carries energy is a **progressive wave** 行波.

Key terms

- **displacement** 位移 y — how far a particle has moved from its rest position at a moment. A **vector** 矢量.
- **amplitude** 振幅 A — the largest displacement from the rest position.
- **wavelength** 波长 λ — the shortest distance along the wave between two points that move **in phase** 同相 (for example, two next-door **crests** 波峰).
- **period** 周期 T — the time for one full oscillation of a particle.
- **frequency** 频率 f — the number of full oscillations per second; $f = 1/T$. Unit: **hertz** 赫兹, Hz.
- **speed** 速率 v — how fast a crest travels along the medium.
- **phase difference** 相位差 — the fraction of a cycle by which one oscillation leads or lags another. Given in **radians** 弧度 (a full cycle is 2π) or degrees (a full cycle is 360°).

Two points one wavelength apart are in phase (phase difference 0 or 2π). Two points half a wavelength apart are exactly out of phase (phase difference π).

The wave equation

In one period T, the wave moves forward by one wavelength λ. So speed = distance/time = $\lambda/T = \lambda f$:

$$v = f\lambda.$$

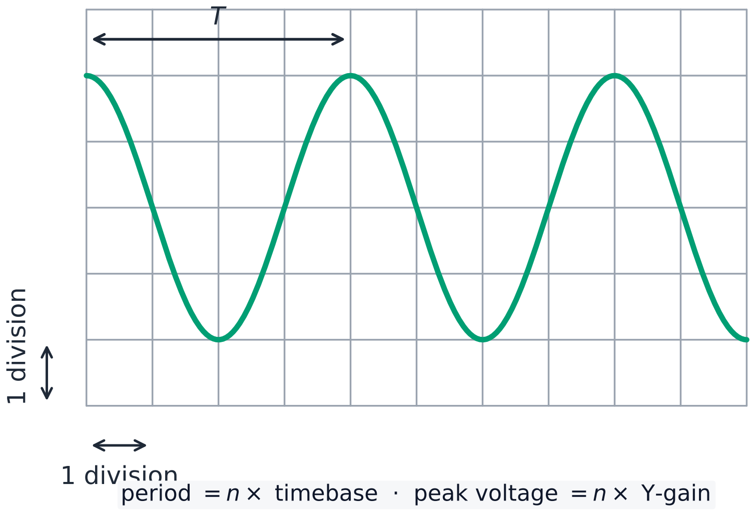
This comes straight from the definitions of speed, frequency and wavelength, and works for every progressive wave.

Reading a CRO trace

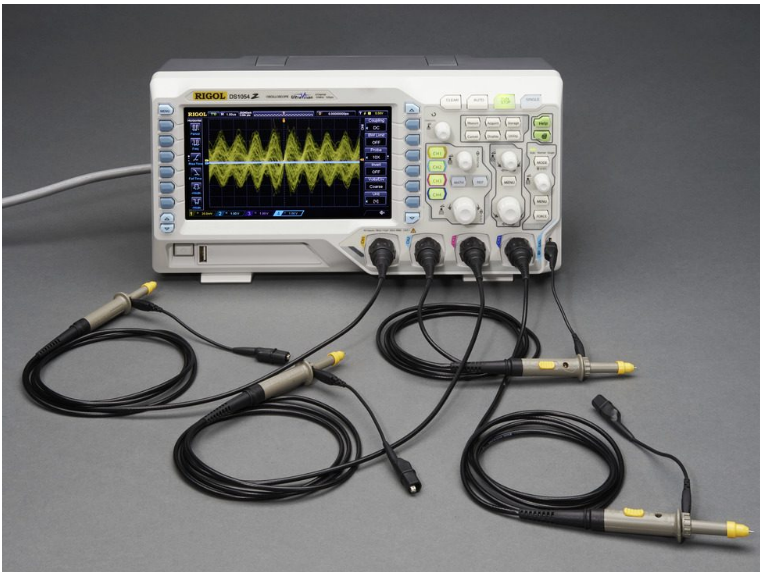
A **cathode-ray oscilloscope** 示波器 (CRO) draws a voltage signal — for sound, the output of a microphone — against time. Two controls matter:

- **time-base** 时基 (seconds per division across): turns horizontal distance on the screen into time. Read the period T as the distance between two next-door peaks, then $f = 1/T$.
- **y-gain** 垂直增益 (volts per division up): turns vertical distance into voltage. The amplitude in volts is the peak height from the centre line.

If the time-base is 5 ms/div and one full cycle takes 4 divisions, then $T = 4 \times 5$ ms = 20 ms and $f = 50$ Hz.



Reading the period T from a CRO trace using the grid and time-base



A real oscilloscope: the grid lets you read off the period and the amplitude

Intensity of a wave

A wave carries energy. The **intensity** 强度 at a point is the **power** 功率 passing through unit area at right angles to the direction of travel:

$$I = \frac{P}{A}.$$

Unit: $\mathrm{W\ m^{-2}}$.

Intensity is proportional to the square of the amplitude:

$$I \propto A^2.$$

For a **point source** 点源 sending out energy equally in all directions, the **wavefronts** 波前 are spheres; the surface area at distance r is $4\pi r^2$, so

$$I = \frac{P}{4\pi r^2}, \qquad I \propto \frac{1}{r^2}.$$

Doubling the distance cuts the intensity to a quarter, which means the amplitude is halved (since $I \propto A^2$).

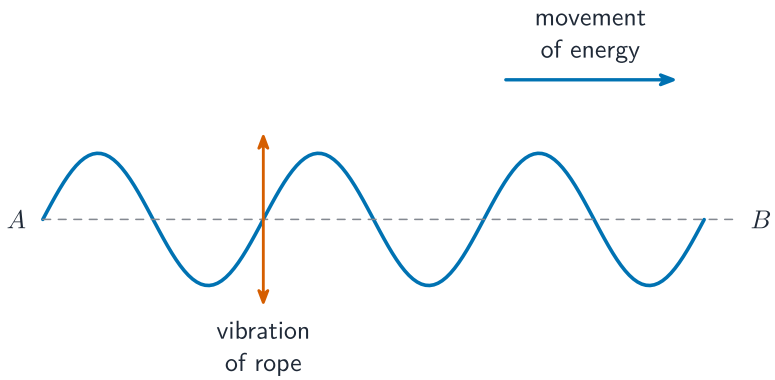
Worked example. A lamp emits 60 W of light equally in all directions. Find the intensity of the light 2.0 m away.

$$I = \frac{P}{4\pi r^2} = \frac{60}{4\pi(2.0)^2} \approx 1.2\ \mathrm{W\ m^{-2}}.$$

Transverse and longitudinal waves

Transverse waves

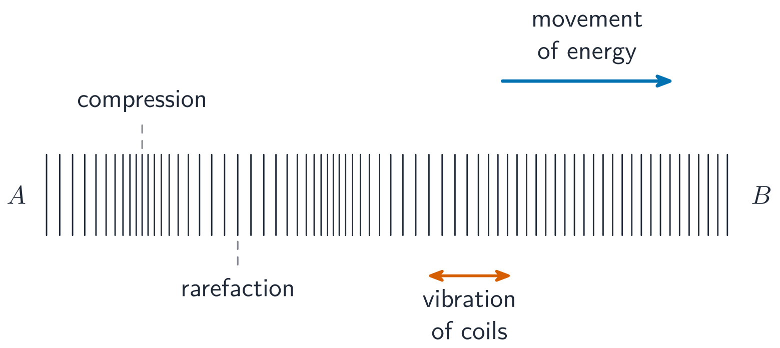
The particles oscillate **perpendicular** 垂直 to the direction the energy travels. A wave on a rope, all electromagnetic waves, and S-waves in the Earth are transverse.



Transverse wave on a rope

Longitudinal waves

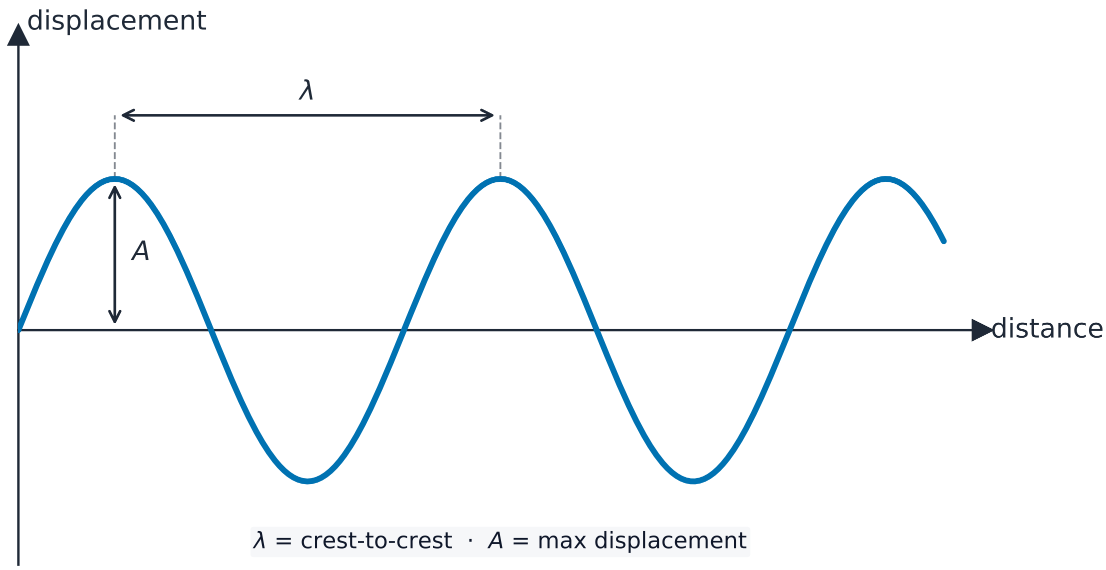
The particles oscillate **parallel** to the direction the energy travels. Sound in any medium, P-waves in the Earth, and the squashes on a slinky are longitudinal. The wave is made of **compressions** 压缩 (higher pressure, particles close together) and **rarefactions** 稀疏 (lower pressure, particles spread out).



Longitudinal wave on a slinky spring

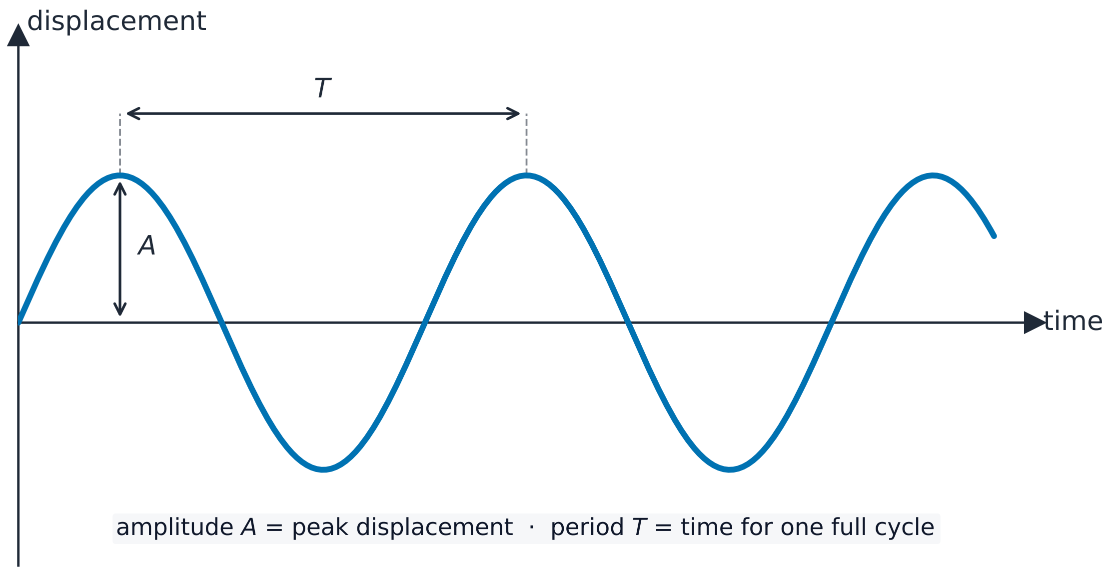
Graphs of waves

A graph of **particle displacement against position** at one moment looks like a **sine curve** 正弦曲线 for both kinds of wave. The difference: for a transverse wave the displacement axis is the real sideways displacement; for a longitudinal wave it is the small back-and-forth displacement along the direction of travel (positive one way, negative the other).



A displacement–distance graph shows the wave's amplitude and wavelength

A graph of **particle displacement against time** at one point in space is also a sine curve for both kinds. Read the period T from this graph.

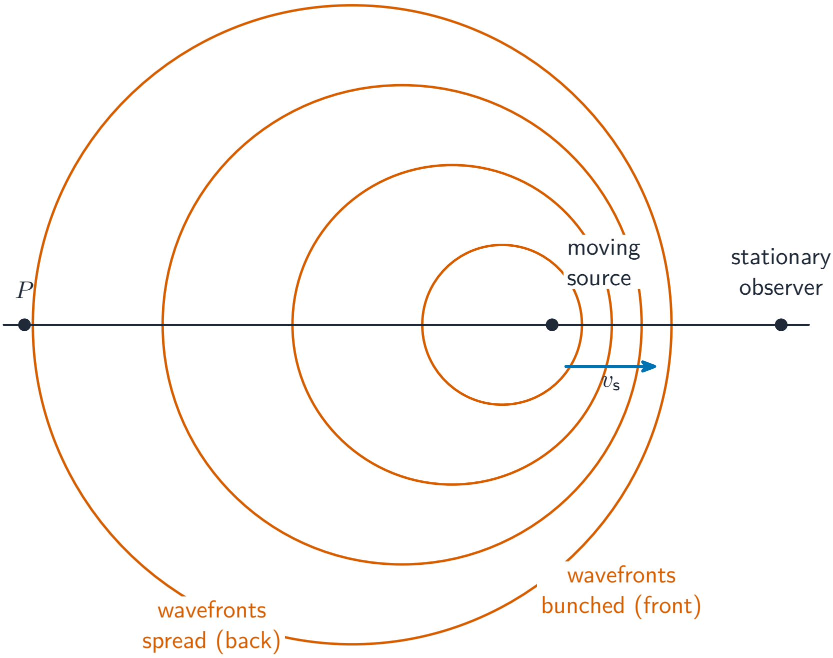


A displacement–time graph shows the wave's amplitude and period

Doppler effect (moving source, stationary observer)

When the **source** 波源 of a sound moves relative to a **stationary** 静止 **observer** 观察者, the heard frequency is different from the source frequency. This is the **Doppler effect** 多普勒效应.

- source moving towards the observer: the wavefronts in front are squashed, so the wavelength is shorter and the heard frequency is **higher**.
- source moving away from the observer: the wavefronts behind are spread out, so the wavelength is longer and the heard frequency is **lower**.



A moving source squashes the wavefronts ahead of it, raising the observed frequency

The formula (source moving at speed v_s along the line to the observer; wave speed v, source frequency f_s, heard frequency f_o):

$$f_o = \frac{v \cdot f_s}{v \pm v_s}.$$

Choose the sign to match the physics:

- minus sign on the bottom when the source moves **towards** the observer ($f_o > f_s$),
- plus sign when the source moves **away** ($f_o < f_s$).

You only need the case of a stationary observer.

Worked example. A car horn at $f_s = 800$ Hz moves at 30 m s^{-1} towards a still listener. Speed of sound $v = 340$ m s^{-1}:

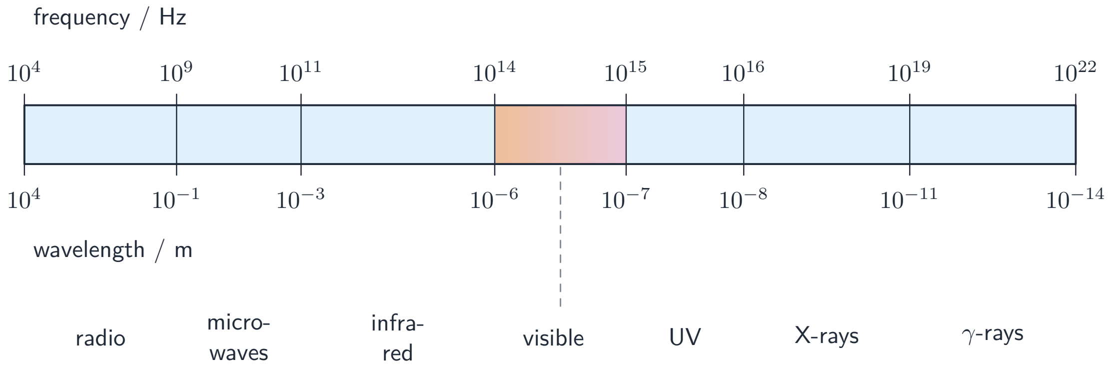
$$f_o = \frac{340 \times 800}{340 - 30} = \frac{272\,000}{310} \approx 877 \text{ Hz}.$$

Electromagnetic spectrum

All **electromagnetic waves** 电磁波 (EM waves) are **transverse** and travel in a **vacuum** 真空 at the same speed:

$$c = 3.00 \times 10^8 \text{ m s}^{-1}.$$

The **electromagnetic spectrum** 电磁波谱 includes radio waves, **microwaves** 微波, **infrared** 红外线, visible light, **ultraviolet** 紫外线, **X-rays** X 射线 and **γ-rays** γ 射线.



The electromagnetic spectrum

Approximate wavelength ranges in free space (learn the orders of magnitude):

- radio waves: $> 10^{-1}$ m (up to many km).
- microwaves: 10^{-3} m to 10^{-1} m.
- infrared: $\sim 7 \times 10^{-7}$ m to 10^{-3} m.
- visible light: 400 nm (violet) to 700 nm (red), i.e. 4×10^{-7} m to 7×10^{-7} m.
- ultraviolet: $\sim 10^{-8}$ m to 4×10^{-7} m.
- X-rays: $\sim 10^{-11}$ m to 10^{-8} m.
- γ-rays: $< 10^{-11}$ m.

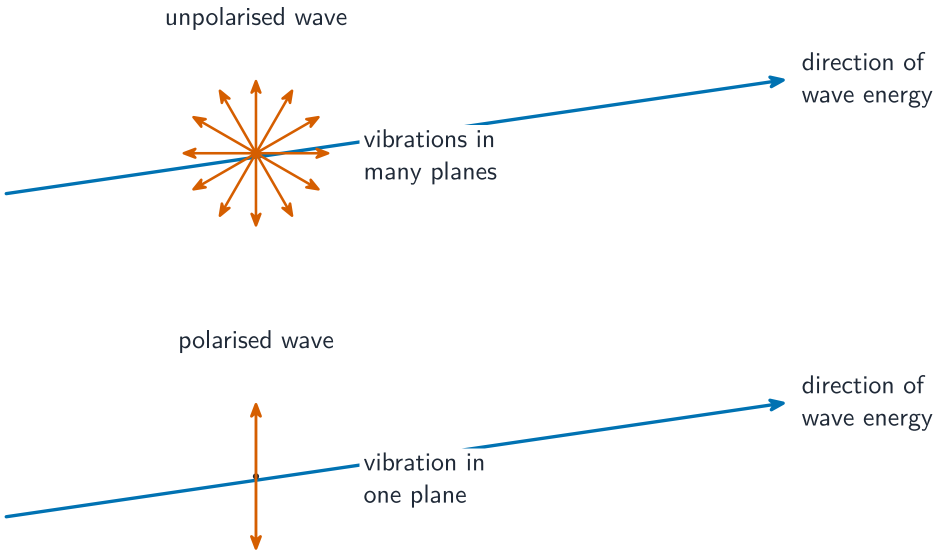
The boundaries between regions are not sharp. Use $c = f\lambda$ to change between wavelength and frequency. Only light with wavelengths 400–700 nm can be seen.

Polarisation

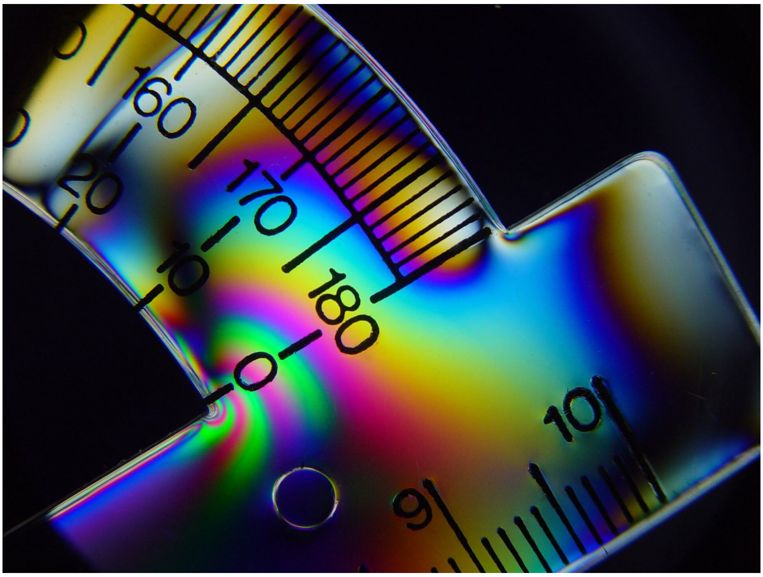
Polarisation 偏振 means making a transverse wave oscillate in one plane only.

- Only **transverse** waves can be polarised — the oscillation is perpendicular to the direction of travel, so different perpendicular planes are real choices.
- **Longitudinal** waves (sound) cannot be polarised — the oscillation is along the direction of travel, so there is no other plane.

So polarisation is a test: if a wave can be polarised, it must be transverse.



Unpolarised waves vibrate in many planes; a polarised wave vibrates in one plane



A see-through plastic protractor placed between two crossed polarising filters. Light only reaches your eye because the stressed plastic rotates its plane of polarisation — the colours map where the plastic is squeezed most. With ordinary light it would just look clear

Malus's law

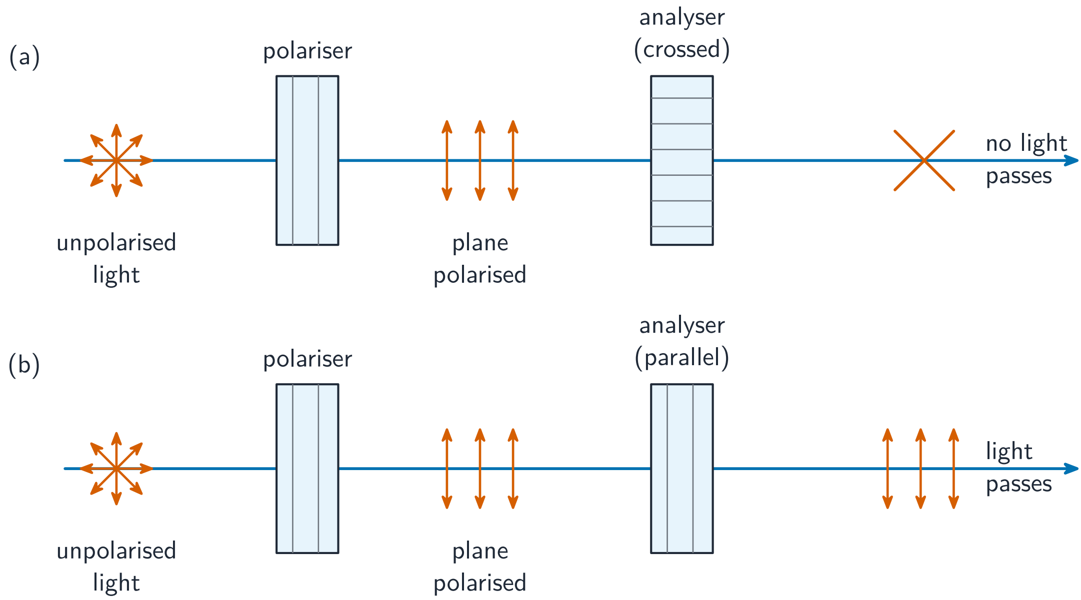
Plane-polarised 平面偏振 light of intensity I_0 passes through a **polarising filter** 偏振片 whose **transmission axis** 透光轴 is at angle θ to the plane of polarisation. The transmitted intensity is given by **Malus's law** 马吕斯定律:

$$I = I_0 \cos^2 \theta.$$

- $\theta = 0°$: filter lined up with the polarisation, $I = I_0$, all passes through.
- $\theta = 90°$: filter at right angles, $I = 0$, all blocked.
- $\theta = 60°$: $I = I_0 \cos^2 60° = I_0 \cdot 0.25 = I_0/4$.

Worked example. Plane-polarised light of intensity 12 W m^{-2} meets a polarising filter whose axis is at 30° to the plane of polarisation. Find the transmitted intensity.

$$I = I_0 \cos^2 \theta = 12 \times \cos^2 30° = 12 \times 0.75 = 9.0 \text{ W m}^{-2}.$$



Crossed filters (a) block the light; parallel filters (b) let it pass

For two filters in a row, use Malus's law twice with the angle between each pair. Be careful with the angle each time — after the first filter the polarisation is along that filter's axis, and the second filter's angle is measured from there.

(You do not need to work out the effect of a polarising filter on an unpolarised wave.)

Exam tips

- Define terms precisely (displacement, amplitude, wavelength, period, frequency) and use $v = f\lambda$.
- Distinguish **transverse** (vibration perpendicular to travel) from **longitudinal** (parallel); only transverse waves can be **polarised**.
- For the **Doppler effect**, the observed frequency **rises** as the source approaches and falls as it recedes.
- Learn the **electromagnetic spectrum** order; all its waves travel at c in a vacuum.

Superposition

A-Level Physics

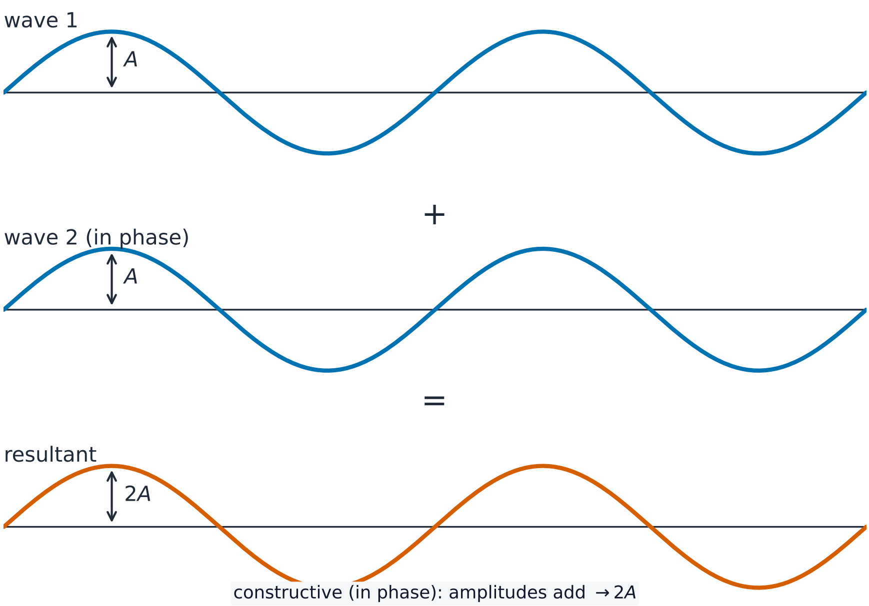
Principle of superposition

When two or more **waves** 波 overlap at a point, the **displacement** 位移 there is the **vector sum** 矢量和 of the displacements each wave would make on its own. This is the **principle of superposition** 叠加.

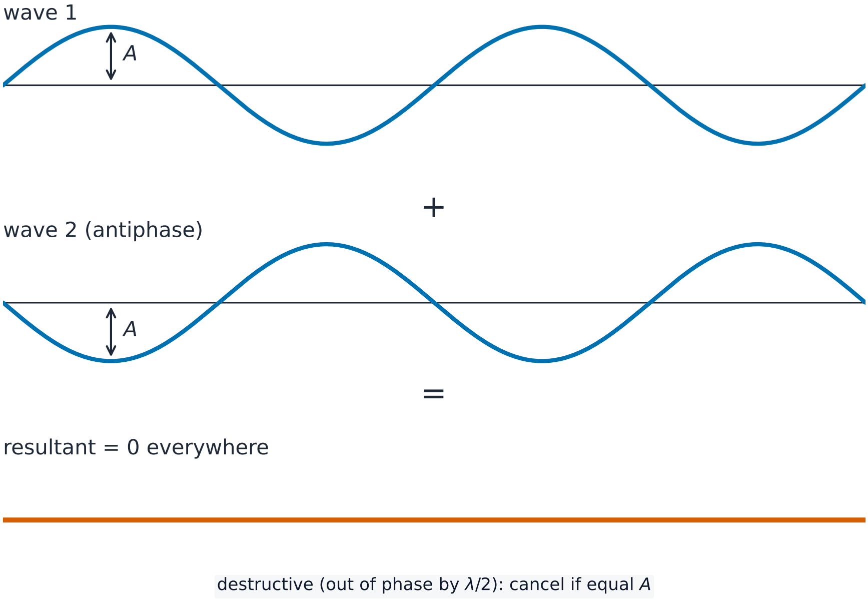
The waves pass through each other and come out unchanged. Superposition is the base of everything in this topic.

If two waves of **amplitude** 振幅 A_1 and A_2 meet:

- **in phase** 同相 (**crest** 波峰 meets crest): the amplitude is $A_1 + A_2$ (**constructive interference** 相长干涉).
- exactly out of phase (crest meets **trough** 波谷, **phase difference** 相位差 π): the amplitude is $|A_1 - A_2|$ (**destructive interference** 相消干涉).
- any other phase difference ϕ: the amplitude is somewhere between these two.



Two waves arriving in phase add to give double the amplitude (constructive)

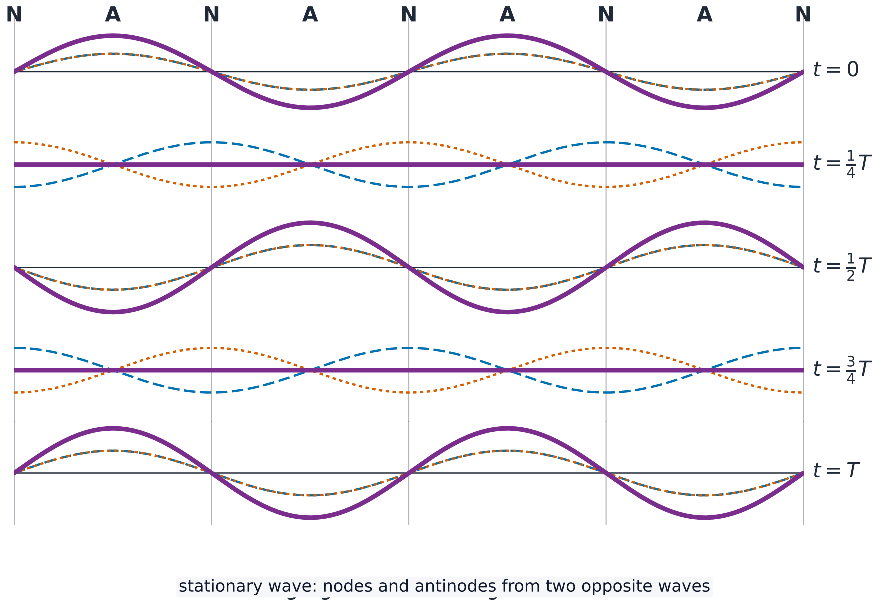


Two waves arriving exactly out of phase cancel to zero (destructive)

For **intensity** 强度, $I \propto A^2$. Two equal waves meeting in phase give intensity $(2A)^2 = 4A^2$ — **four times** the intensity of one wave alone.

Stationary (standing) waves

When two identical **progressive waves** 行波 travel in **opposite directions** and overlap, they make a **stationary wave** 驻波. Examples: a wave on a string **reflected** 反射 from a fixed end overlapping the incoming wave; sound in an air column reflected from a closed end; microwaves between an emitter and a metal sheet.



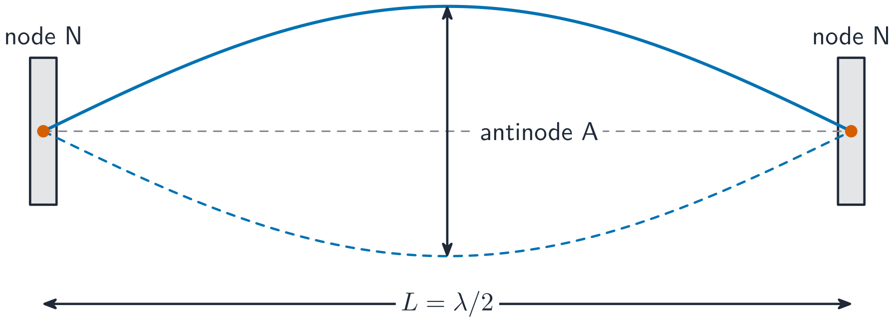
A stationary wave forms where two opposite waves overlap (N marks a node, A an antinode)

Nodes and antinodes

In a stationary wave:

- **node** 波节 — a point that is always at zero displacement (the two waves always cancel). The distance between next-door nodes is $\lambda/2$.
- **antinode** 波腹 — a point of largest amplitude (the two waves always add). The distance between next-door antinodes is $\lambda/2$.
- a node and the next antinode are $\lambda/4$ apart.

Particles between two nodes oscillate **in phase** with each other, but with different amplitudes (largest at the antinode, zero at the nodes). Particles on opposite sides of a node oscillate in **antiphase** 反相 (phase difference π).



Fundamental mode on a stretched string — one loop, with L equal to half a wavelength

Worked example. A string of length 0.80 m is fixed at both ends and vibrates in its fundamental mode, where the wave speed is 240 m s^{-1}. Find the fundamental frequency.

One loop fits the string, so $\lambda = 2L = 1.60$ m. Then

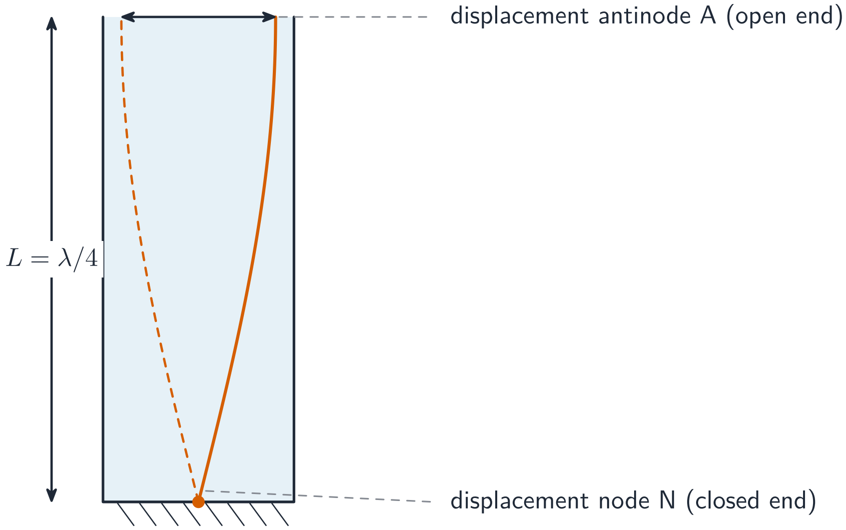
$$f = \frac{v}{\lambda} = \frac{240}{1.60} = 150 \text{ Hz}.$$

How a stationary wave differs from a progressive wave: it does not carry **energy** 能量 along its length, the pattern does not move along, and the nodes stay fixed; a progressive wave has the same amplitude everywhere and carries energy.

Measuring wavelength from node spacing

Drive a string with a vibrator at frequency f until a stationary pattern appears. Measure the distance between two well-separated nodes and divide by the number of half-**wavelengths** 波长 between them. Then λ is known, and $v = f\lambda$ gives the wave speed.

For a tube closed at one end and open at the other (a **resonance tube** 共鸣管), the closed end is a displacement node and the open end is a displacement antinode. The **fundamental** 基频 has $L = \lambda/4$; the next resonance is at $L = 3\lambda/4$; and so on. For a tube open at both ends, both ends are antinodes; the fundamental is at $L = \lambda/2$.



Fundamental mode in a closed pipe — a node at the closed end, an antinode at the open end

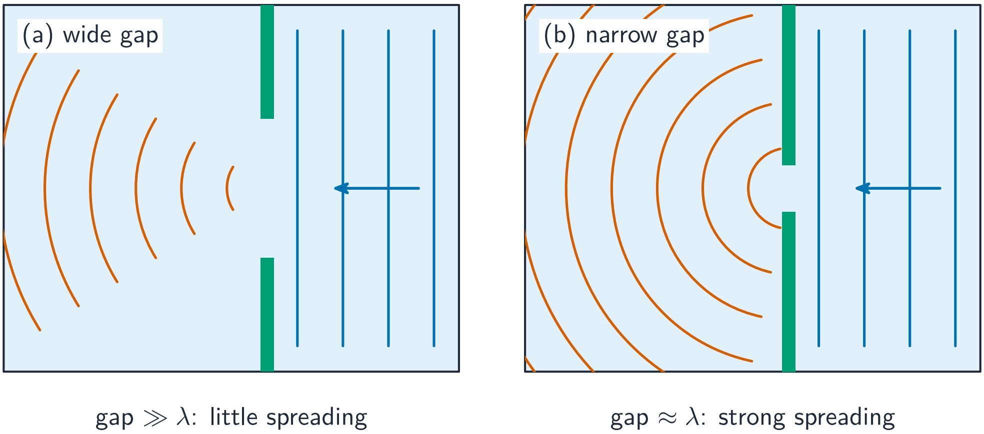
Diffraction

Diffraction 衍射 is the spreading of a wave after it passes through a gap or around an **obstacle** 障碍物. All waves diffract — water, sound, light, microwaves.

The amount of spreading depends on the ratio of **wavelength to gap width**:

- gap **much wider** than λ: very little spreading; the wave goes nearly straight through.
- gap **about the size of** λ: a lot of spreading; the wave fans out.
- gap **smaller than** λ: very strong spreading; the gap acts almost like a point source.

Show this with water waves in a **ripple tank** 水波槽: straight waves meet a barrier with a gap, and the waves curve more as the gap is made narrower. The same idea is why you can hear someone around a corner (speech has λ near 1 m, close to the gap size) but cannot see them (visible light has $\lambda \sim 500$ nm, far smaller than the gap).

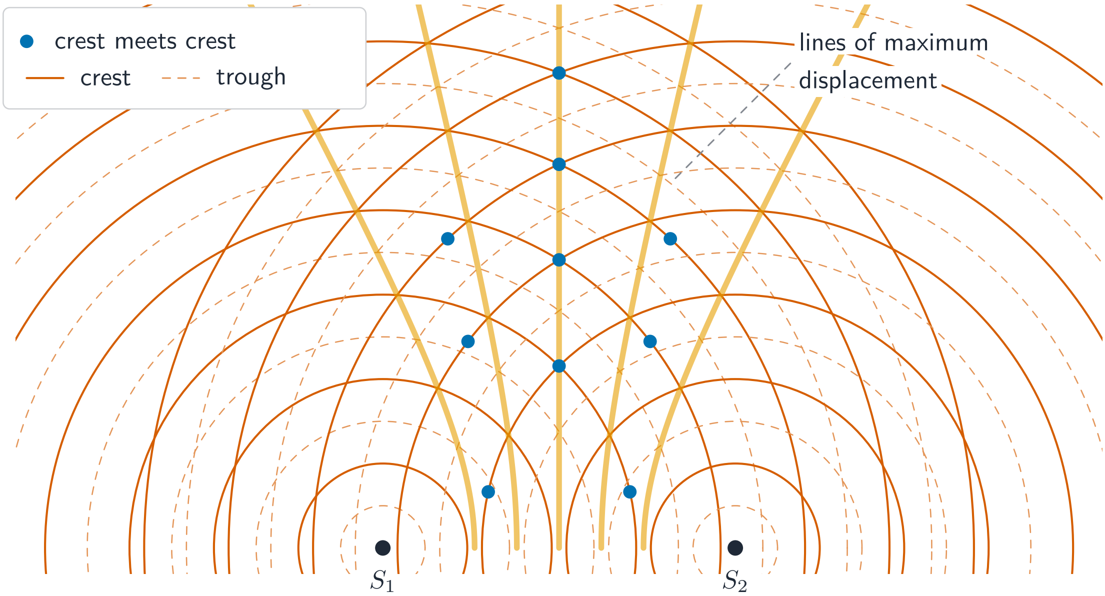


Diffraction in a ripple tank — a wide gap (a) spreads the waves little, a narrow gap (b) much more

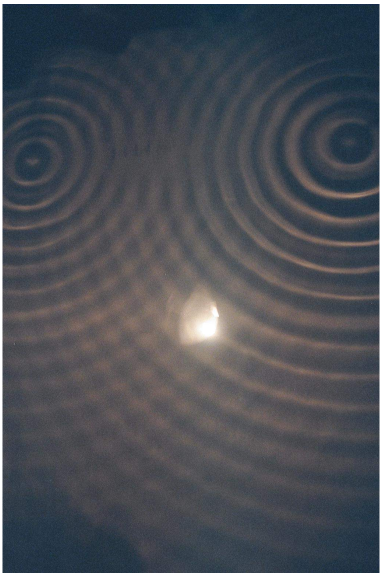
Interference

The shifting colours on a soap bubble come from the interference of light.

Interference 干涉 is the superposition of two **coherent** 相干 waves to give a steady pattern of high-amplitude regions (constructive) and low-amplitude regions (destructive).



Two coherent sources give lines of maximum displacement where crests meet crests



The same effect in a real ripple tank — two coherent sources give a steady interference pattern

Coherence

Two sources are **coherent** when they emit waves with a **constant phase difference** (which also needs the same frequency). Two separate lamps are not coherent — their phase changes randomly, so any pattern flickers too fast to see and you get only an average.

To make coherent light from one source, pass it through two **slits** 狭缝 in a **double-slit** 双缝 setup. Both slits are lit by the same wavefront, so the two beams keep a fixed phase relationship.

Conditions for a clear pattern

To see two-source fringes you need:

1. two **coherent** sources (constant phase difference).
2. roughly equal amplitudes (or the dark regions are not very dark).
3. the waves overlap where you look.
4. for light (a transverse wave), the same plane of **polarisation** 偏振.

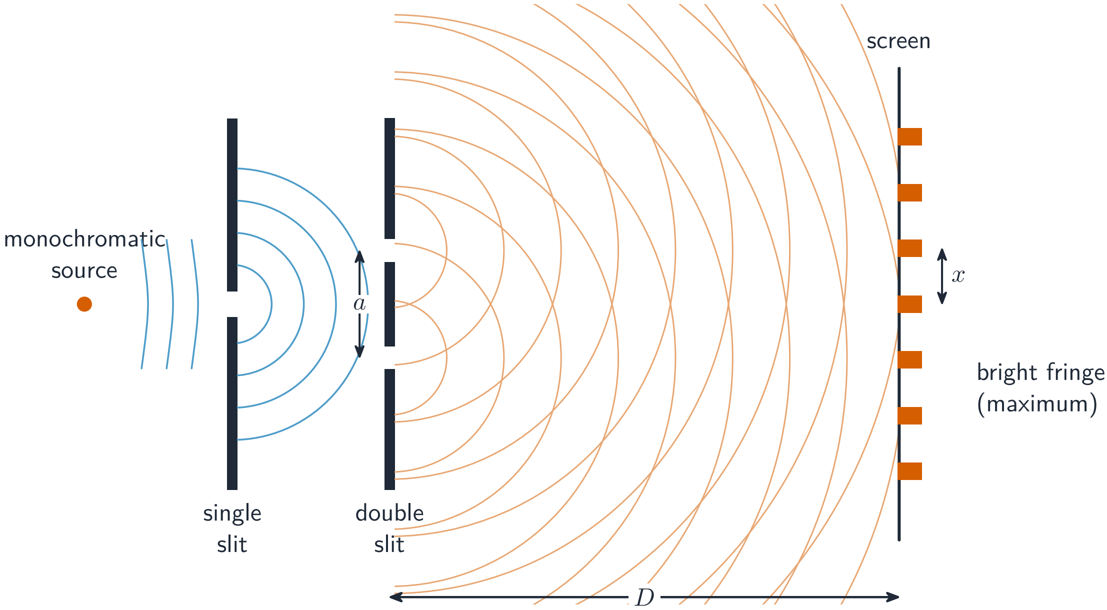
Path difference

For two coherent sources, what happens at a point depends on the **path difference** 路程差 Δx between the two waves arriving there:

- constructive: $\Delta x = n\lambda$ (for whole numbers $n = 0, 1, 2, \ldots$).
- destructive: $\Delta x = (n + \frac{1}{2})\lambda$.

Double-slit (Young's) experiment

For two slits a distance a apart, with a screen a distance D away (assume $D \gg a$), light of wavelength λ makes fringes on the screen.



Young's double-slit experiment — the single slit makes the two slits coherent sources

The **fringe spacing** 条纹间距 x (one **fringe** 条纹 to the next) is

$$\lambda = \frac{ax}{D}, \qquad x = \frac{\lambda D}{a}.$$

Bright fringes (**maximum** 极大) are where the path difference is a whole number of λ; dark fringes (**minimum** 极小) where it is $(n+\frac{1}{2})\lambda$. The fringes are equally spaced.

To make the fringe spacing smaller: increase a (slits further apart), reduce D (screen closer), or use a shorter λ (bluer light).

Worked example. In a double-slit experiment the slits are 0.50 mm apart and lit by light of wavelength 600 nm. The screen is 2.0 m away. Find the fringe spacing.

$$x = \frac{\lambda D}{a} = \frac{(600 \times 10^{-9})(2.0)}{0.50 \times 10^{-3}} = 2.4 \times 10^{-3}\ \text{m} = 2.4\ \text{mm}.$$

Diffraction grating

A **diffraction grating** 衍射光栅 has many equally spaced slits — often hundreds or thousands per millimetre. Each slit is a coherent source. A maximum is seen at angle θ from the **normal** 法线 to the grating when

$$d\sin\theta = n\lambda,$$

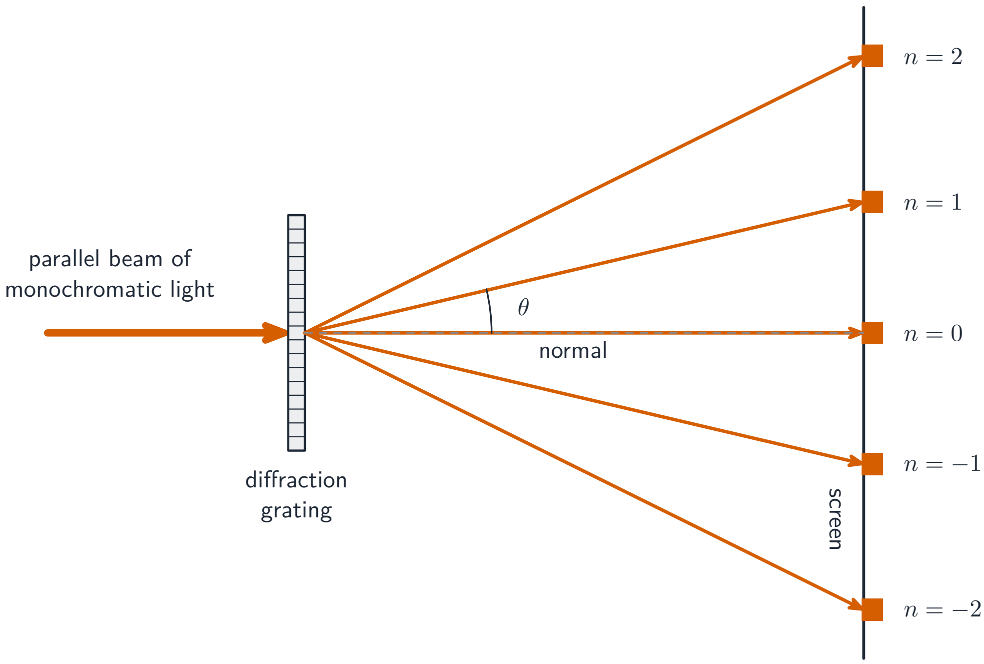
where d is the **slit spacing** 缝间距 (centre to centre), $n = 0, \pm1, \pm2, \ldots$ is the **order** 级次, and λ is the wavelength.

Worked example. A diffraction grating has 500 lines per mm. Light of wavelength 600 nm is shone normally on it. Find the angle of the first-order ($n = 1$) maximum.

The slit spacing is $d = \dfrac{1}{500}$ mm $= 2.0 \times 10^{-6}$ m, so

$$\sin\theta = \frac{n\lambda}{d} = \frac{600 \times 10^{-9}}{2.0 \times 10^{-6}} = 0.30 \quad\Rightarrow\quad \theta \approx 17^\circ.$$

Compared with the double slit, a grating gives much **sharper** maxima, because more slits add together — every other direction is cancelled by many slits.



A diffraction grating splits monochromatic light into sharp maxima on a screen

Slit spacing from "lines per mm"

If a grating has N lines per millimetre, then $d = 1/N$ millimetres $= 10^{-3}/N$ metres. For 450 lines per mm, $d = 1/450$ mm ≈ 2.22 μm.

Highest order

For a given grating and wavelength, $\sin\theta = n\lambda/d$ cannot be more than 1, so the highest order seen is

$$n_{\max} = \left\lfloor \frac{d}{\lambda} \right\rfloor .$$

If $d/\lambda = 3.27$, orders up to $n = 3$ exist; $n = 4$ would need $\sin\theta > 1$ and is not seen.

Finding λ with a grating

Shine parallel light of unknown wavelength straight at the grating. Measure the angle θ_1 of the first-order maximum from the centre. Then $\lambda = d\sin\theta_1$. Repeating for higher orders and averaging reduces error.

Exam tips

- Two-source interference: **constructive** when path difference $= n\lambda$, **destructive** when $= (n + \frac{1}{2})\lambda$; the sources must be **coherent**.
- Double slit: $\lambda = ax/D$; diffraction grating: $d\sin\theta = n\lambda$ — know every symbol.
- On a **stationary wave** mark nodes and antinodes; adjacent nodes are $\lambda/2$ apart; it stores energy but does not transfer it.
- A stationary wave needs two waves of the **same frequency travelling in opposite directions**.

Electricity

A-Level Physics

Electric current

An **electric current** 电流 is a flow of **charge carriers** 载流子. In a metal the carriers are negative conduction **electrons** 电子; in an **electrolyte** 电解质 they are positive and negative **ions** 离子; in a **semiconductor** 半导体 they may be electrons or "**holes**" 空穴. The **conventional current** 常规电流 direction is the way **positive** charge would flow — opposite to the real flow of electrons in a wire.

Charge

Charge is **quantised** 量子化: the smallest free unit of charge is the **elementary charge** 基本电荷

$$e = 1.60 \times 10^{-19}\ \text{C}.$$

Every free charge in this syllabus is a whole-number multiple of e. The unit of charge is the **coulomb** 库仑, C.

Current as the rate of flow of charge

If charge Q passes a point in time t, the current is

$$I = \frac{Q}{t}, \qquad Q = It.$$

Unit of current: ampere, A ($= \text{C s}^{-1}$). For a changing current, the charge that has flowed in a time is the **area under** an I–t graph.

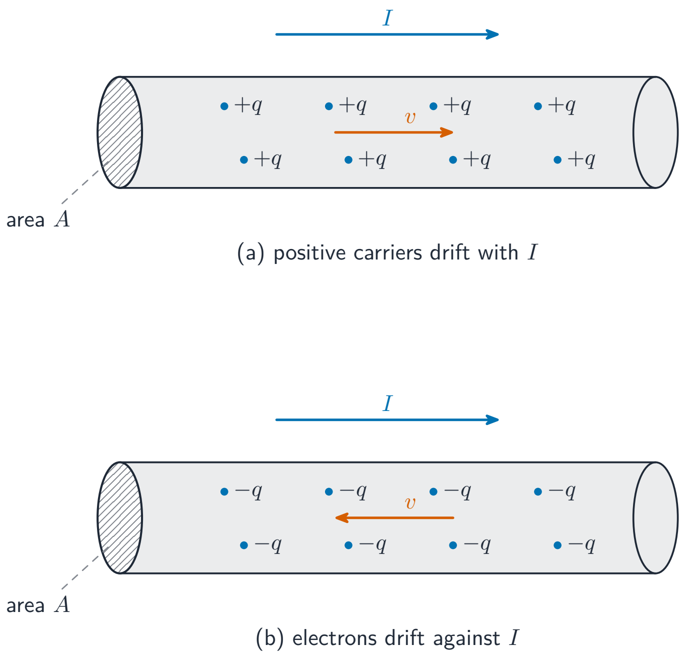
Worked example. A current of 0.50 A flows for 2.0 minutes. Find the charge that passes, and how many electrons this represents. ($e = 1.60 \times 10^{-19}$ C.)

$$Q = It = 0.50 \times 120 = 60\ \text{C}, \qquad N = \frac{Q}{e} = \frac{60}{1.60 \times 10^{-19}} \approx 3.8 \times 10^{20}.$$

Drift velocity equation

For a uniform conductor of cross-section area A, with n charge carriers per unit volume (the **number density** 数密度), each carrying charge q, moving with average **drift velocity** 漂移速度 v:

$$I = Anvq.$$



Charge carriers drifting inside a conductor: positive carriers drift with I, electrons against it

Worked example. A copper wire of cross-sectional area 1.0×10^{-6} m^2 carries a current of 5.0 A. Copper has $n = 8.5 \times 10^{28}$ free electrons per m^3. Find the drift velocity. ($e = 1.60 \times 10^{-19}$ C.)

Rearranging $I = Anvq$ gives $v = \dfrac{I}{Anq}$:

$$v = \frac{5.0}{(1.0 \times 10^{-6})(8.5 \times 10^{28})(1.60 \times 10^{-19})} \approx 3.7 \times 10^{-4} \text{ m s}^{-1}.$$

The electrons drift very slowly — less than a millimetre per second.

Use this to compare currents:

- a thinner wire (smaller A) at the same I needs a faster drift v.

- a semiconductor has far fewer free carriers than a metal (smaller n), so for the same I the drift velocity is much larger.
- in **series** 串联 components, I is the same everywhere, so if A stays the same but the material changes, nv changes the other way.

Potential difference

The **potential difference** 电势差 (p.d.) across a component is the **energy** 能量 transferred per unit charge as that charge passes through it:

$$V = \frac{W}{Q}.$$

Unit: **volt** 伏特, V (= J C^{-1}).

If 1 J of electrical energy changes into other forms (thermal, light, kinetic, …) when 1 C of charge passes through a component, the p.d. across it is 1 V.

The **electromotive force** 电动势 (e.m.f.) of a source is the energy given **per unit charge by the source**. The formula is the same as for p.d.; the difference is direction: e.m.f. is energy given **to** the charge by the source; p.d. is energy given **up** by the charge to the component.

Electrical power

High-voltage power lines carry electrical energy across the country.

Combining $V = W/Q$ and $I = Q/t$:

$$P = \frac{W}{t} = VI.$$

Using Ohm's law $V = IR$:

$$P = VI = I^2R = \frac{V^2}{R}.$$

Pick the form with the quantities you know. Examples:

- two heaters of equal resistance — the one with the larger current gives more **power** 功率 ($P = I^2R$).
- two resistors in **parallel** 并联 across the same **voltage** 电压 — the one with smaller R gives more power ($P = V^2/R$).
- a kettle marked "2.4 kW, 240 V" draws $I = P/V = 10$ A and has resistance $R = V^2/P = 24\ \Omega$.

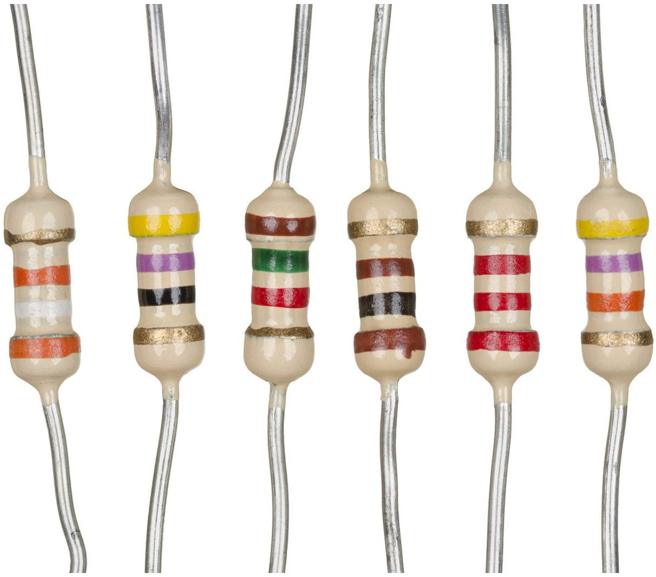
Energy transferred in time t is $E = Pt$.

Resistance and Ohm's law

The **resistance** 电阻 R of a component is

$$R = \frac{V}{I}.$$

Unit: **ohm** 欧姆, Ω ($=$ V A^{-1}). Resistance depends on the conditions (such as temperature) when it is measured.



Real fixed resistors — the coloured bands code the resistance in ohms

Ohm's law

A conductor obeys **Ohm's law** 欧姆定律 when the current through it is **proportional to the p.d.** across it, as long as the conditions (especially temperature) stay constant. For such a conductor R is constant and the I–V graph is a straight line through the origin.

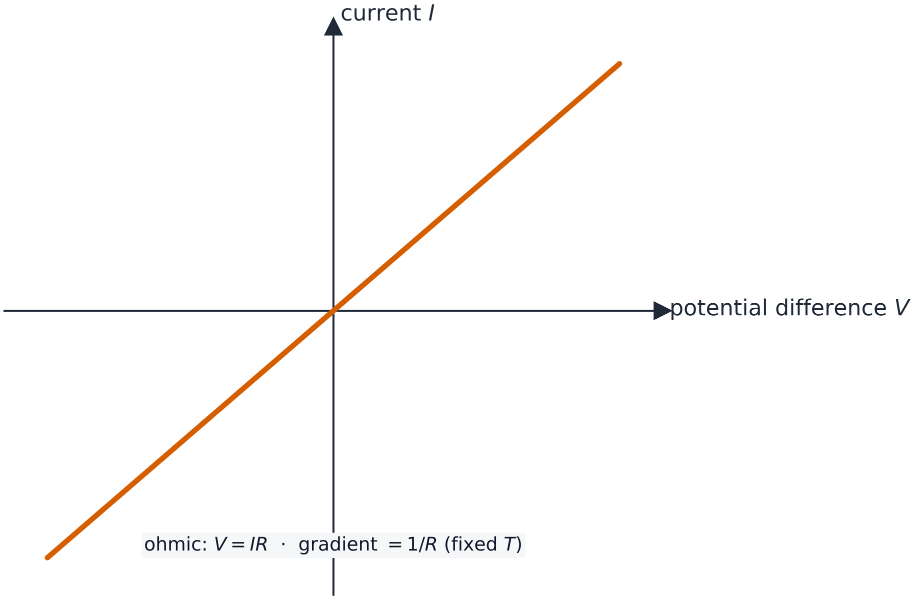
Ohm's law is an experimental result, not a definition. The definition $R = V/I$ works for **any** component; only ohmic ones have constant R.

I–V characteristics

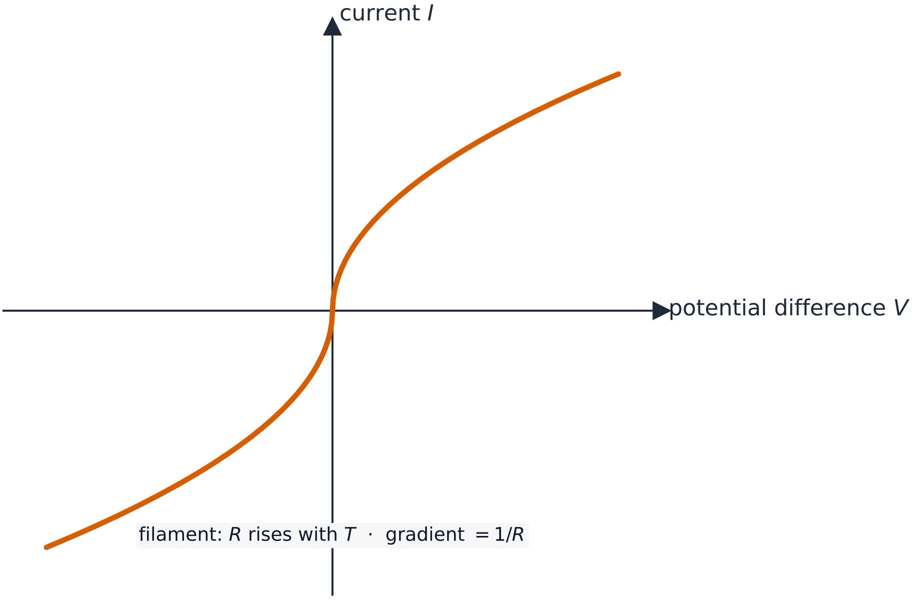
You should be able to sketch these:

- **metal wire at constant temperature** — a straight line through the origin (constant R). Reversing the p.d. drives the current the other way, giving a straight line in both directions.

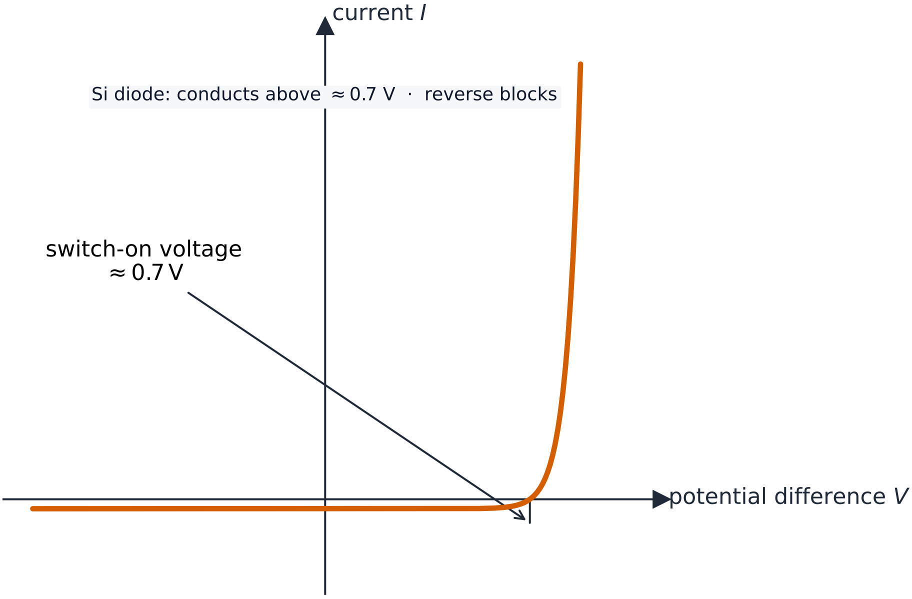
- **filament lamp** 灯丝灯泡 — through the origin, steep at first, then flatter as V (and I) grow. Reason: more current heats the filament, so its resistance rises and the gradient $1/R$ falls.
- **semiconductor diode** 二极管 — almost no current for negative V or small positive V. Above a "switch-on" voltage (about 0.7 V for silicon), the current rises sharply.



I–V characteristic of an ohmic conductor (metal wire at constant temperature)



I–V characteristic of a filament lamp



I–V characteristic of a semiconductor diode

Resistivity

For a uniform conductor of length L and cross-section area A,

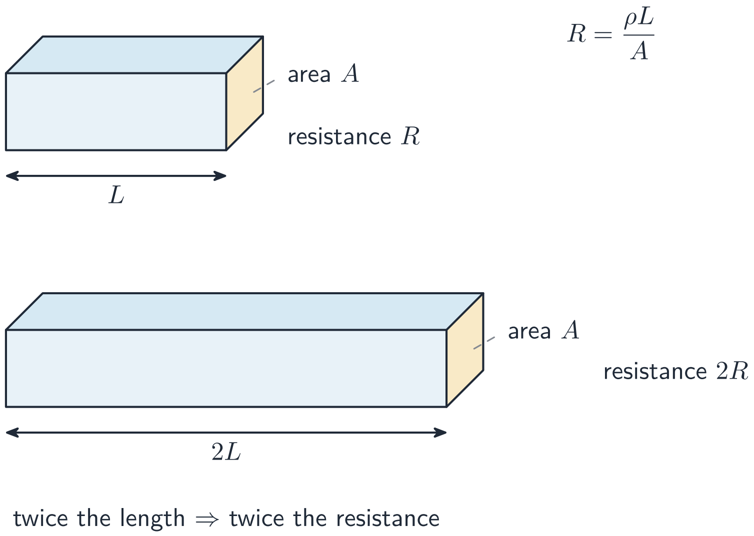
$$R = \frac{\rho L}{A}.$$

ρ is the **resistivity** 电阻率, a property of the material, with unit Ω m. Doubling the length doubles R; doubling the area halves it; halving the diameter quarters the area and so makes R four times bigger.

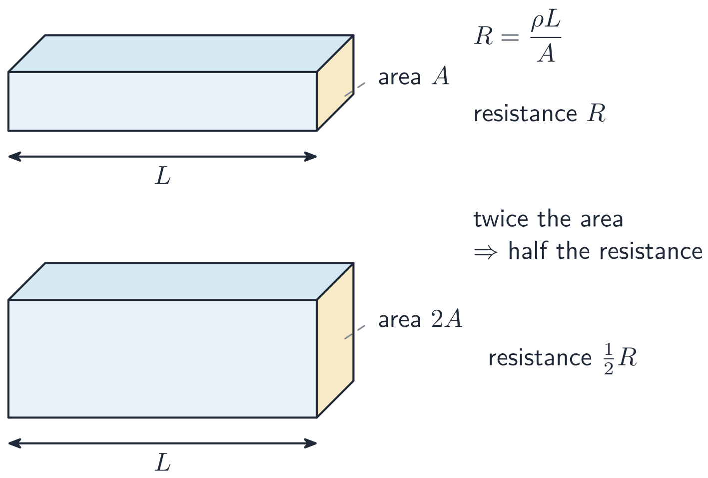
Worked example. A copper wire of length 2.0 m and cross-sectional area 1.7×10^{-7} m^2 has resistivity 1.7×10^{-8} Ω m. Find its resistance.

$$R = \frac{\rho L}{A} = \frac{(1.7 \times 10^{-8})(2.0)}{1.7 \times 10^{-7}} = 0.20\ \Omega.$$

Typical values: copper at room temperature $\rho \sim 1.7 \times 10^{-8}$ Ω m; an **insulator** 绝缘体 $\rho \sim 10^{15}$ Ω m or more.



A longer conductor has more resistance — doubling L doubles R

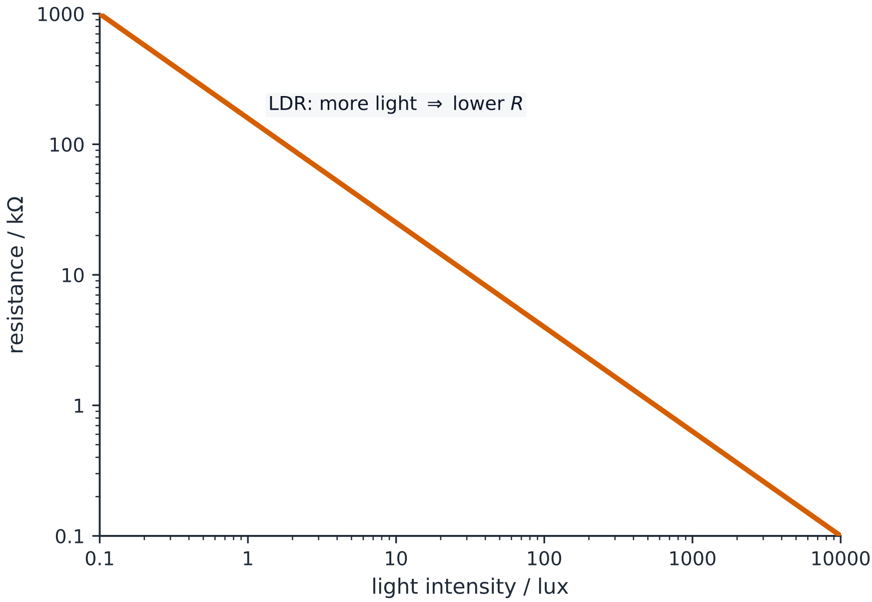


A wider conductor has less resistance — doubling A halves R

The resistivity of a metal **rises with temperature** (more **lattice vibration** 晶格振动 **scatters** 散射 the electrons), which is why the filament lamp's I–V line curves.

Light-dependent resistor (LDR)

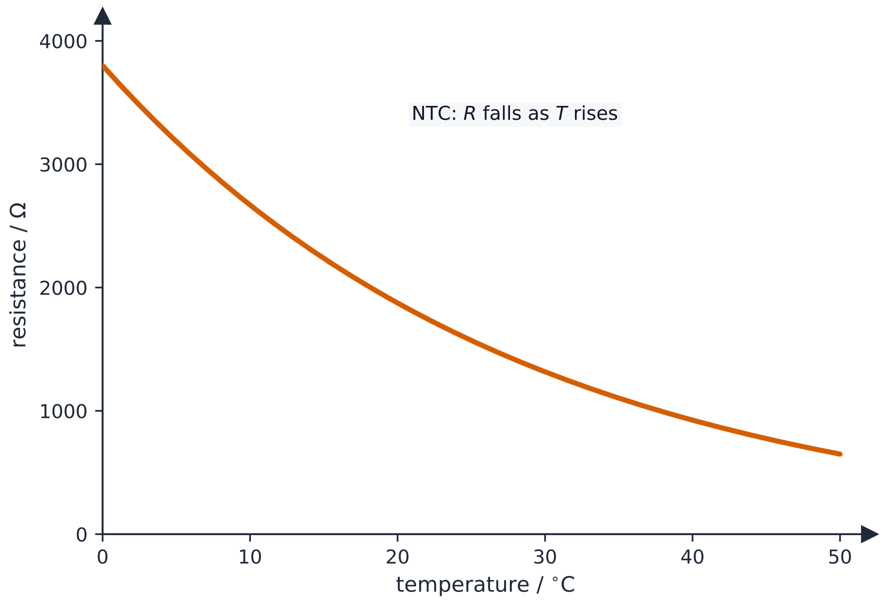
A **light-dependent resistor** 光敏电阻 (LDR) is a semiconductor whose **resistance falls as the light intensity rises**. In bright light R may be a few hundred Ω; in the dark it can be in the megaohms. LDRs are used in light-sensing circuits (street lamps, camera light meters). Here the **light intensity** 光强 controls the resistance.



Resistance of an LDR decreases as light intensity increases

Thermistor

In this syllabus a **thermistor** 热敏电阻 has a **negative temperature coefficient** 负温度系数: its **resistance falls as its temperature rises**. This is useful for sensing temperature — put it in a **potential divider** 分压器 and the output voltage changes with temperature.



Resistance of a thermistor falls as temperature rises

This is the opposite of a metal: in a semiconductor, more thermal energy frees more charge carriers, and this matters more than the extra scattering.

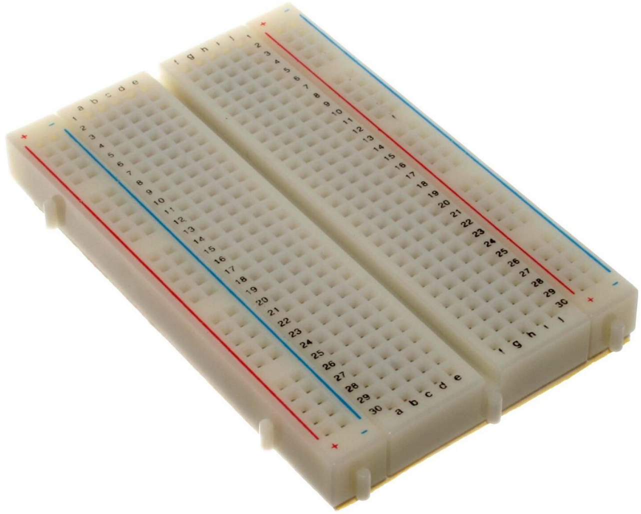
Exam tips

- Use $I = Q/t$, $V = W/Q$ (energy per unit charge) and $P = VI = I^2R = V^2/R$.
- **Ohm's law** ($V = IR$) applies only to an ohmic conductor at constant temperature — a filament lamp is non-ohmic.
- Sketch and interpret the I-V **characteristics** of a resistor, filament lamp and diode.
- An **LDR's** resistance falls with light; a **thermistor's** falls as temperature rises.

D.C. circuits

A-Level Physics

Practical circuits



A breadboard lets you build and test practical circuits without soldering.

e.m.f. and p.d.

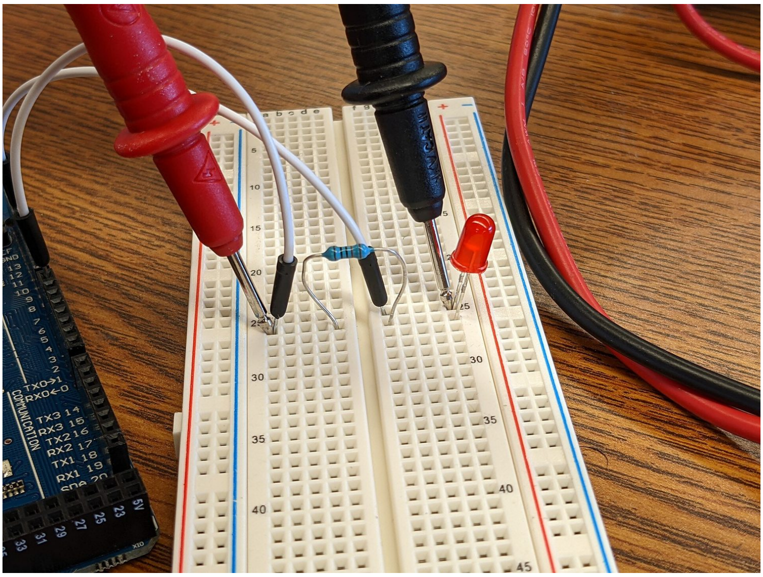
The **electromotive force** 电动势 (e.m.f.) ε of a source is the **energy** 能量 given to each unit of charge by the source as it drives the charge around a full circuit. Unit: volt.

The **potential difference** 电势差 (p.d.) across a component is the energy changed from electrical to other forms by each unit of charge as it passes through that component.

Both are in volts; they differ in energy direction:

- e.m.f. — energy put **into** the circuit by the source (chemical $\rightarrow$ electrical in a battery, mechanical $\rightarrow$ electrical in a generator).
- p.d. — energy taken **out** of the electrical form (electrical $\rightarrow$ thermal in a resistor, $\rightarrow$ light in a lamp, $\rightarrow$ kinetic in a motor).

In the lab you often build a circuit on a **breadboard** 面包板 (a board with rows of holes that connect components without soldering) and measure currents and p.d.s with a **multimeter** 万用表.



A real circuit on a breadboard, being measured with a multimeter

Internal resistance

A real source has some **internal resistance** 内阻 r — usually the resistance of the **electrolyte** 电解质 in a **cell** 电池, or the wire windings in a generator. When current I flows, an internal p.d. of Ir is "lost" inside the source, so the **terminal p.d.** 端电压 across the outside circuit is

$$V_{\text{terminal}} = \varepsilon - Ir.$$

So:

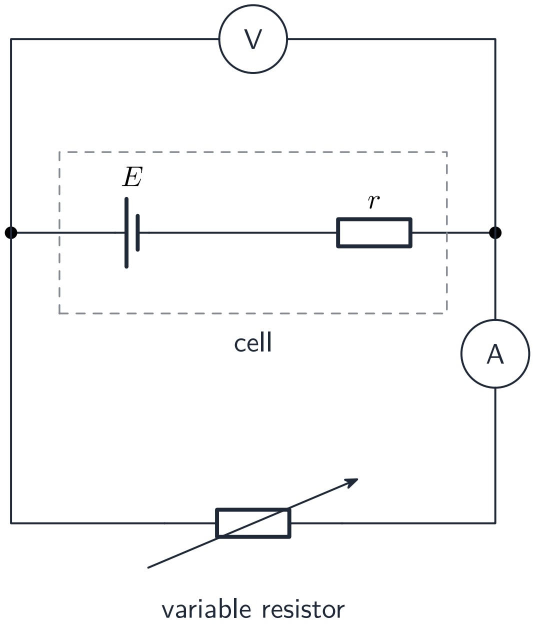
- **no current** (**open circuit** 开路, $I = 0$): the terminal p.d. equals the e.m.f.
- **larger current**: the terminal p.d. falls.
- **short circuit** 短路 ($R_{\text{external}} \to 0$): $I = \varepsilon / r$, a large current, with all the energy turned to heat inside the source.

To measure r, change the outside resistance and plot V_{terminal} against I: the line has y-intercept ε and gradient $-r$.

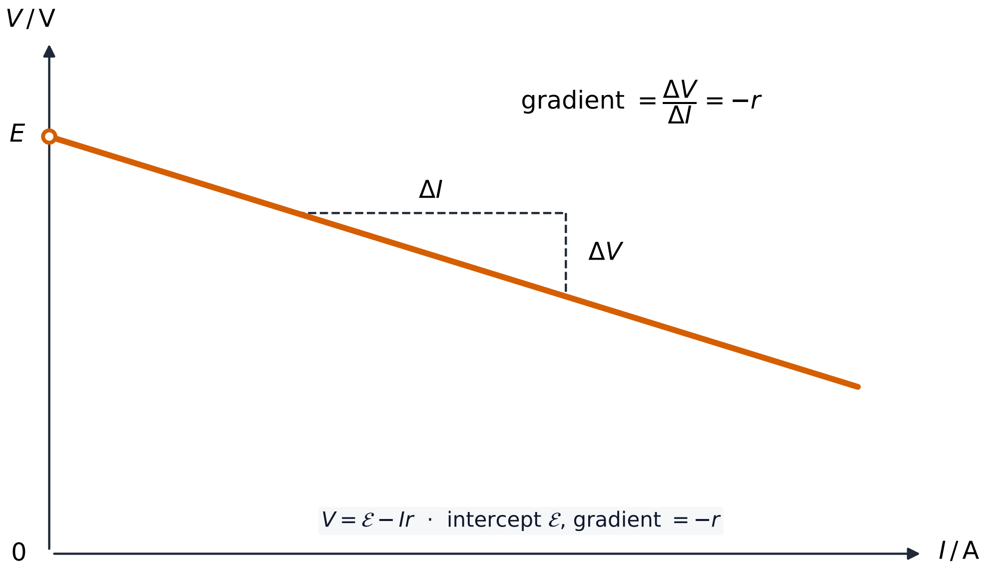
Worked example. A cell of e.m.f. 1.5 V and internal resistance 0.50 Ω is connected to a 2.5 Ω resistor. Find the current and the terminal p.d.

The e.m.f. drives the current through both resistances: $I = \dfrac{\varepsilon}{R+r} = \dfrac{1.5}{2.5+0.50} =$ 0.50 A. Then

$$V_{\text{terminal}} = \varepsilon - Ir = 1.5 - 0.50 \times 0.50 = 1.25 \text{ V}.$$



Circuit for measuring the e.m.f. and internal resistance of a cell

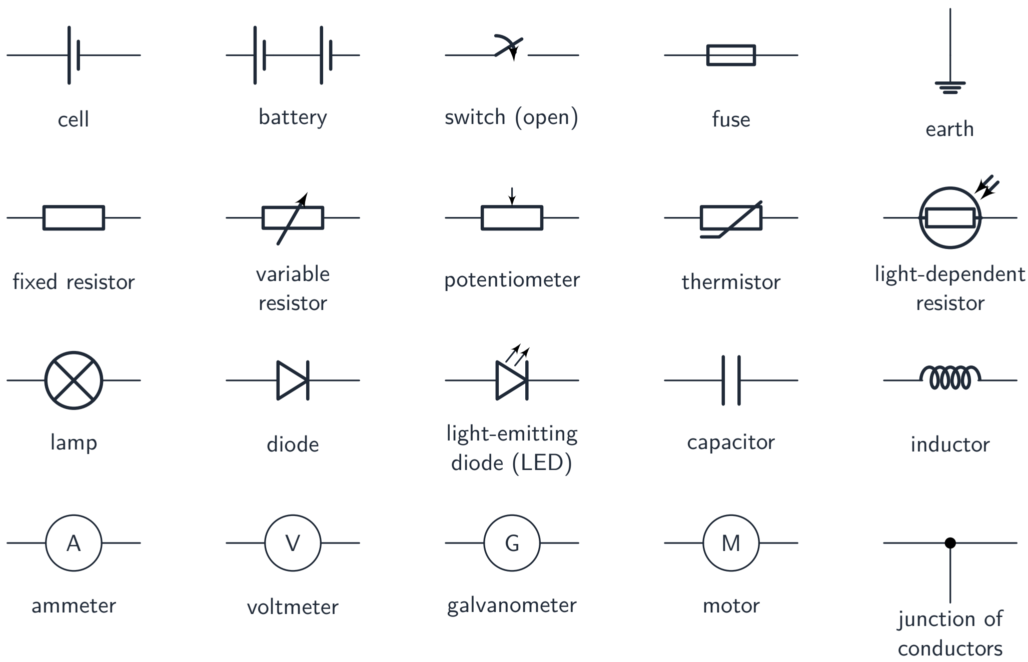


Terminal p.d. against current — the intercept is the e.m.f. and the gradient is minus the internal resistance

The **power** 功率 given to the outside load is $P_{\text{ext}} = (\varepsilon - Ir)I$; the power lost inside is $P_{\text{int}} = I^2 r$; the total power from the source is εI.

Circuit symbols

You must recognise and draw the standard symbols in the syllabus: cell, battery, switch, resistor, variable resistor, **ammeter** 电流表, **voltmeter** 电压表, lamp, **diode** (and **LED** 发光二极管), **capacitor** 电容器, inductor, thermistor, light-dependent resistor, **fuse** 保险丝, earth, junction. An ideal ammeter has zero **resistance** 电阻 and goes in **series** 串联. An ideal voltmeter has infinite resistance and goes in **parallel** 并联.



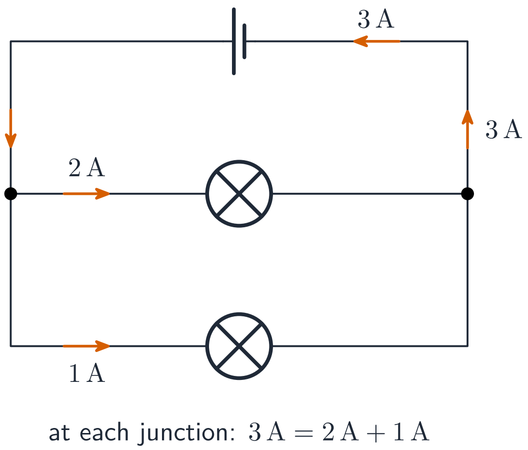
The standard circuit symbols you need to recognise and draw

Kirchhoff's laws

First law (junction rule)

At any **junction** 节点, **the total current flowing in equals the total current flowing out**. This follows from **conservation of charge** 电荷守恒 — charge cannot build up at a point in a steady circuit, so charge in per second equals charge out per second.

For a junction with three wires: $I_1 = I_2 + I_3$ if currents 2 and 3 flow out and current 1 flows in.



Current divides at a junction in a parallel circuit (3 A in equals 2 A plus 1 A)

Second law (loop rule)

Around **any closed loop** 回路, **the total e.m.f. equals the total p.d. across the components in that loop**. This follows from **conservation of energy** 能量守恒: as a unit of charge goes once round a loop, the energy it gains from sources equals the energy it gives up to components.

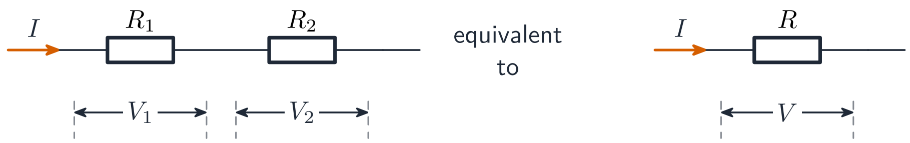
Pick a direction round the loop. Take an e.m.f. as positive when the loop direction goes from − to + of the source, and a p.d. as positive when the loop direction is the conventional current direction through the resistor.

Combining resistors

Resistors in series carry the same current; the total p.d. is the sum:

$$\varepsilon = IR_1 + IR_2 + \ldots = I(R_1 + R_2 + \ldots),$$

so $R_{\text{series}} = R_1 + R_2 + \ldots$.

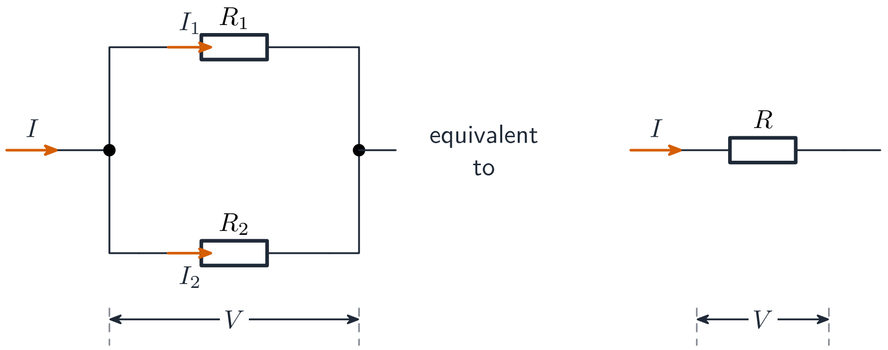


Two resistors in series and their single equivalent resistor

Resistors in parallel have the same p.d.; the total current is the sum:

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \ldots = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\right),$$

so $\dfrac{1}{R_{\text{parallel}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots$.



Two resistors in parallel and their single equivalent resistor

Two equal resistors R in parallel give $R/2$; N equal ones give R/N. A parallel combination is **always smaller** than any of its resistors; a series combination is always larger.

Worked example. A 4.0 Ω resistor and a 12 Ω resistor are connected in parallel. Find their combined resistance.

$$\frac{1}{R} = \frac{1}{4.0} + \frac{1}{12} = \frac{3}{12} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3} \quad \Rightarrow \quad R = 3.0\ \Omega.$$

Solving a circuit

1. **Label** every current with a symbol and a chosen direction.
2. **Use Kirchhoff's first law** 基尔霍夫第一定律 at each junction to link the currents.
3. **Use Kirchhoff's second law** 基尔霍夫第二定律 around each loop to get equations in the p.d.s.
4. **Use** $V = IR$ for each resistor.
5. Solve the equations together.

For symmetric resistor networks, use the symmetry to spot branches with equal currents — the branch with the most current gives the most power ($P = I^2R$).

Potential dividers

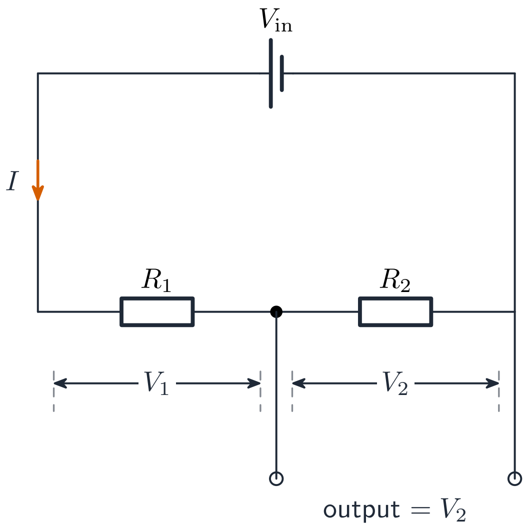
A **potential divider** 分压器 is two (or more) resistors in series across a source. The p.d. across each resistor is in **direct proportion** to its resistance:

$$V_1 = V_{\text{in}} \cdot \frac{R_1}{R_1 + R_2}, \qquad V_2 = V_{\text{in}} \cdot \frac{R_2}{R_1 + R_2}.$$

The output (tapped between R_1 and R_2) can be set to any **voltage** 电压 between 0 and V_{in} by choosing the resistances. A **rheostat** 变阻器 (a slider on a uniform-resistance wire) gives a smoothly variable divider.

Worked example. A 6.0 V supply is connected across a 2.0 kΩ resistor in series with a 4.0 kΩ resistor. Find the output voltage tapped across the 4.0 kΩ resistor.

$$V_2 = V_{\text{in}} \cdot \frac{R_2}{R_1 + R_2} = 6.0 \times \frac{4.0}{2.0 + 4.0} = 4.0 \text{ V}.$$



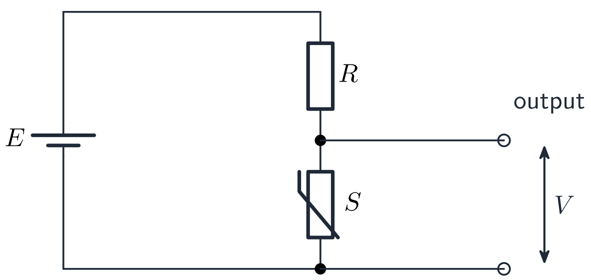
A potential divider — the p.d. splits between R1 and R2 in proportion to their resistances

Sensor circuits

Replace one fixed resistor with a **sensor** 传感器 whose resistance changes with a physical quantity:

- **thermistor** 热敏电阻 (NTC): R falls as temperature rises. In a divider, the output voltage changes with temperature in a fixed direction.
- **light-dependent resistor** 光敏电阻 (LDR): R falls as **light intensity** 光强 rises, giving a brightness-dependent output.

Connect the output to a **transistor** 晶体管 base or a **comparator** 比较器 to switch a load on or off when the temperature or light passes a **threshold** 阈值.



A thermistor in a potential divider gives an output voltage that changes with temperature

Potentiometer and the null method

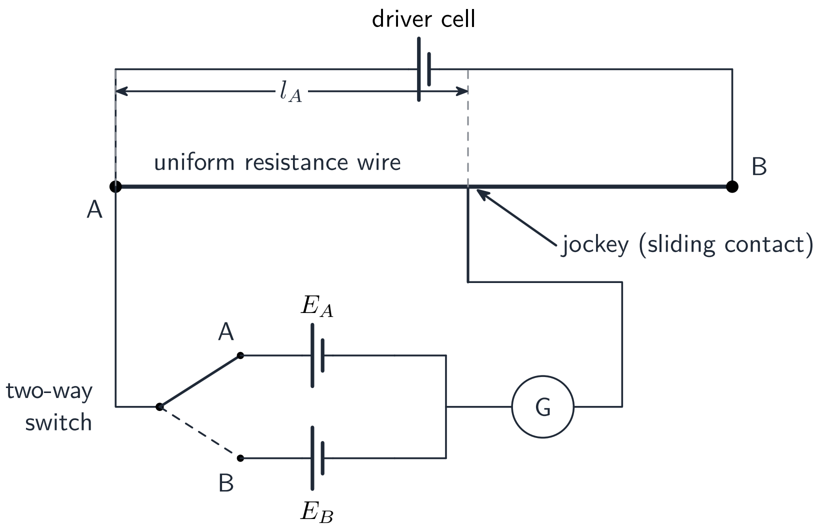
A **potentiometer** 电位差计 is a uniform resistance wire of length L_0 with a sliding contact (**jockey** 滑动触头). The resistance per unit length is uniform, so the p.d. from one end to the jockey is proportional to the length:

$$V_x = V_{\text{full}} \cdot \frac{x}{L_0}.$$

To **compare two e.m.f.s** (an unknown cell against a standard cell), connect each in turn with the jockey through a **galvanometer** 检流计. Slide the jockey until the galvanometer reads zero (a **null** — no current flows through the cell being measured, because the potentiometer's voltage there exactly opposes the cell's e.m.f.). The two balance lengths are in the ratio of the e.m.f.s:

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}.$$

This is a **null method** 零点法: you find the balance (zero current) instead of measuring a current's value. Its advantage is that at balance the unknown cell gives no current, so its internal resistance does not affect the result.



A potentiometer comparing two cell e.m.f.s by the null method

Exam tips

- Apply **Kirchhoff's laws**: current into a junction = current out (charge conserved); $\sum$ e.m.f. = $\sum$ p.d. round a loop (energy conserved).
- Combine resistors: **series** $R = R_1 + R_2$; parallel $1/R = 1/R_1 + 1/R_2$.
- A **potential divider** splits voltage in the ratio of the resistances.
- Include **internal resistance**: e.m.f. $= I(R + r)$ — the "lost volts" are Ir.

Particle physics

A-Level Physics

The nuclear atom

Geiger–Marsden α-particle scattering

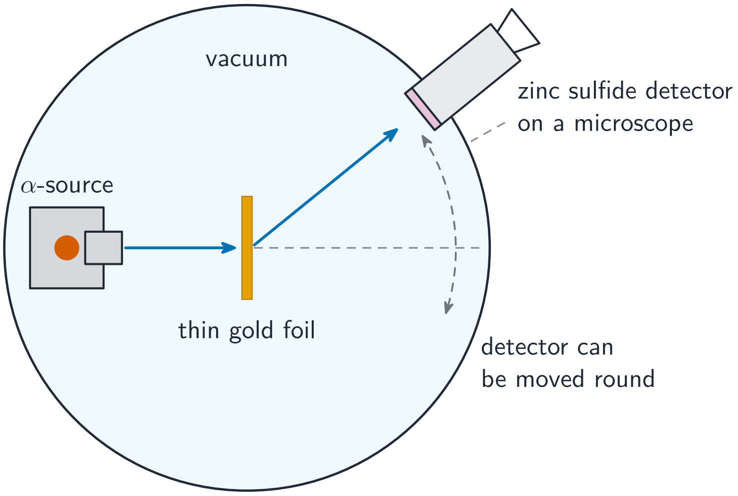
Alpha particles α 粒子 fired at a thin gold foil were seen to:

- mostly pass straight through, with very little **deflection** 偏转,
- sometimes deflect through small angles,
- rarely (about 1 in 8000) deflect through angles greater than 90°.

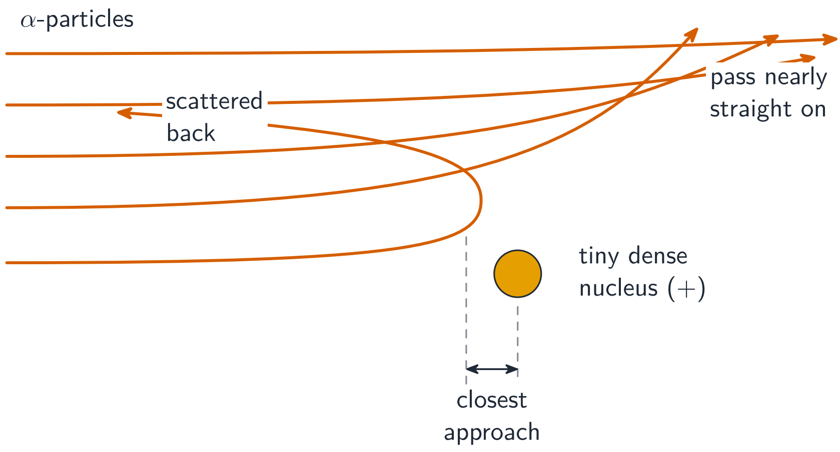
From this Rutherford worked out:

- the atom is **mostly empty space** (most α-particles pass straight through),
- there is a **tiny, dense, positively charged nucleus** 原子核 at the centre (the rare large deflections need a concentrated charge to push the α away),
- almost all of the atom's mass is in this nucleus.

Order of magnitude: atom diameter $\sim 10^{-10}$ m, nucleus diameter $\sim 10^{-15}$ m — the nucleus is about 10^5 times smaller than the atom.



The α-scattering experiment — α-particles strike a thin gold foil in a vacuum



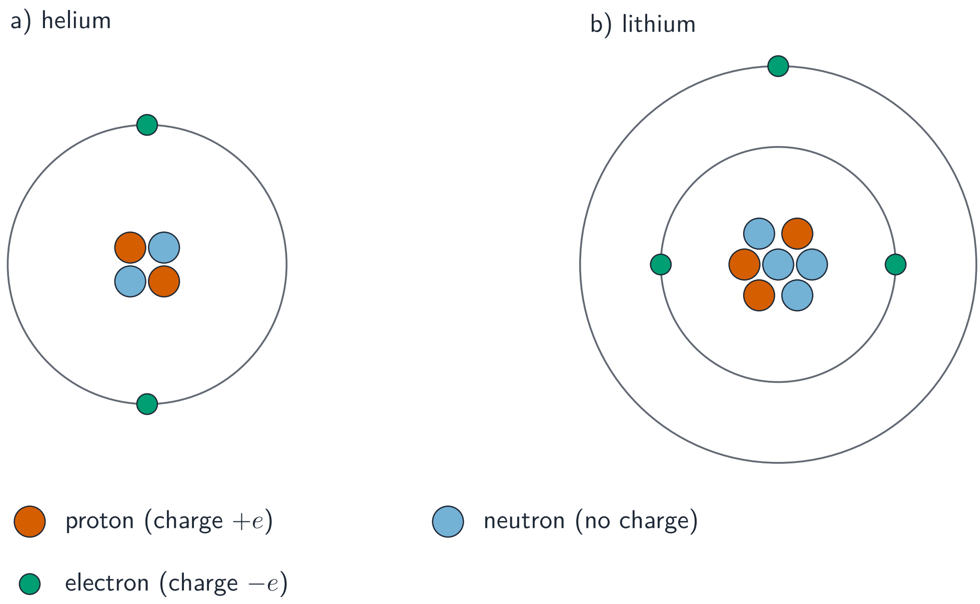
Most α-particles pass nearly straight through; a few are deflected sharply by the tiny nucleus

Simple nuclear model

An atom has:

- a central nucleus of **protons** 质子 (positive, charge $+e$) and **neutrons** 中子 (no charge),
- **electrons** 电子 (charge $-e$) around the nucleus.

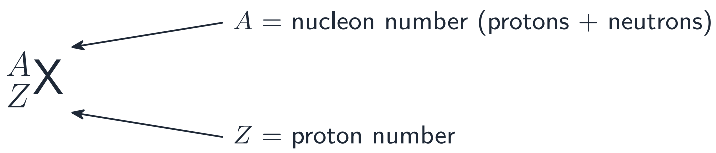
The proton and neutron have almost the same mass (≈ 1 u); the electron is about $\frac{1}{1836}$ of the proton's mass.



Simple models of a helium atom and a lithium atom (not to scale)

Notation and key numbers

For a **nuclide** 核素 written ${}^{A}_{Z}\mathrm{X}$:



Nuclide notation: nucleon number on top, proton number below

- **proton number** 质子数 Z (also the atomic number): the number of protons. It fixes the element.
- **nucleon number** 核子数 A (also the mass number): the total number of **nucleons** 核子 (protons + neutrons).
- number of neutrons $N = A - Z$.

A neutral atom has the same number of electrons as protons.

Isotopes

Isotopes 同位素 are atoms of the same element (same Z) with different numbers of neutrons (different A). They behave the same chemically but differently in the nucleus. Example: ${}^{12}_{6}\mathrm{C}$ and ${}^{14}_{6}\mathrm{C}$ are isotopes of carbon.

Conservation laws in nuclear processes

In any nuclear process:

- nucleon number A is conserved (total A before = total A after),
- **charge** is conserved (this is **conservation of charge** 电荷守恒).

These two rules let you balance decay and reaction equations.

Unified atomic mass unit

The **unified atomic mass unit** 统一原子质量单位, symbol u, is set so that an atom of ${}^{12}_{6}\mathrm{C}$ has mass exactly 12 u. Numerically,

$$1\ \mathrm{u} = 1.661 \times 10^{-27}\ \mathrm{kg}.$$

A proton has mass ≈ 1.007 u; a neutron ≈ 1.009 u; an electron $\approx 5.5 \times 10^{-4}$ u.

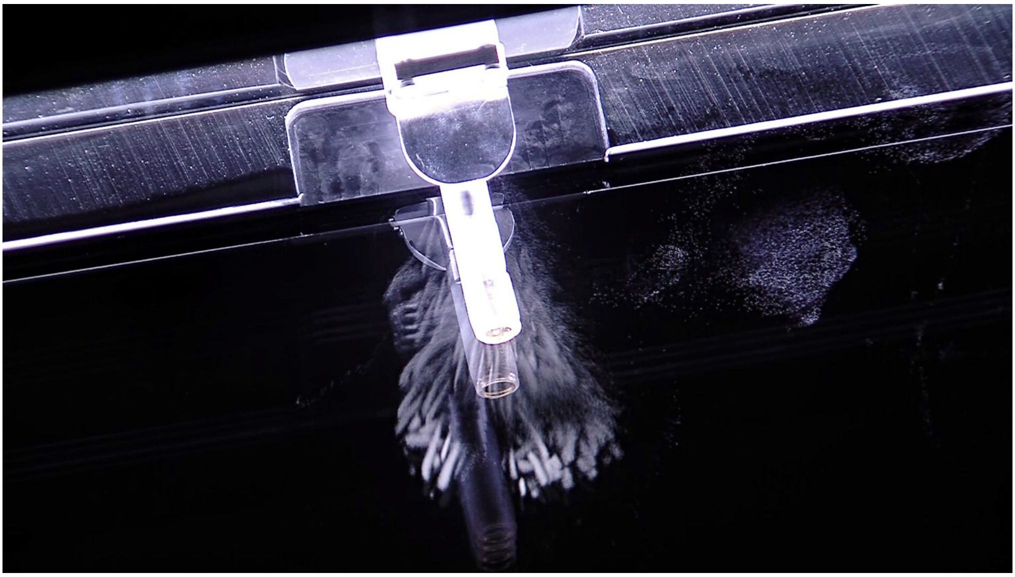
Radioactive emissions

An unstable nucleus rearranges itself and gives out one of three kinds of **radiation** 辐射. This is **radioactive** 放射性 decay. Each kind has its own properties.

α-radiation

- **Made of**: a helium-4 nucleus, ${}^{4}_{2}\alpha$ (two protons + two neutrons).
- **Mass**: ≈ 4 u.
- **Charge**: $+2e$.
- **Range in air**: a few cm. Stopped by a sheet of paper.
- **Ionising power**: strong — it is good at **ionising** 电离.
- **Energy spectrum**: **discrete** 分立 (one decay gives α-particles at one or a few sharp energies).

A **cloud chamber** 云室 makes the tracks visible: each α-particle leaves a short, straight, thick trail of tiny droplets as it ionises the air. The short equal lengths show the α-particles all carry about the same energy.



Alpha-particle tracks in a cloud chamber, fanning out from an americium-241 source

β-radiation

Two types of **beta particle** β 粒子:

- β^-: a fast electron, given out when a neutron turns into a proton.
- β^+: a **positron** 正电子 (the electron's antiparticle), given out when a proton in a proton-rich nucleus turns into a neutron.

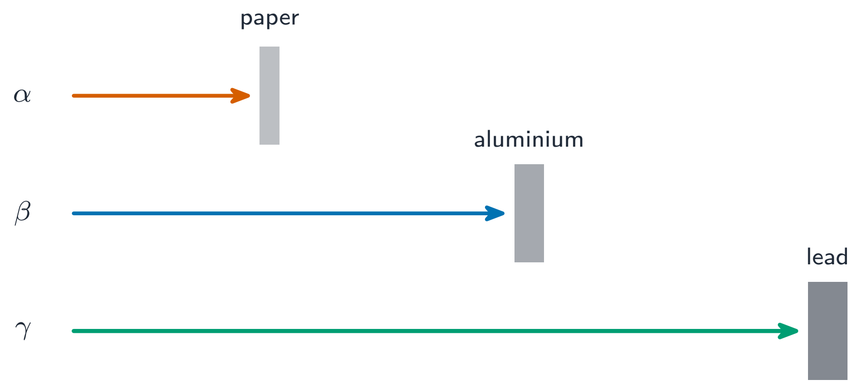
Properties (both types):

- **Mass**: $\approx 1/1836$ u (much less than α).
- **Charge**: $-e$ for β^-, $+e$ for β^+.
- **Range in air**: about 1 m. Stopped by a few mm of aluminium.
- **Energy spectrum**: **continuous** 连续 up to a maximum (see below).

γ-radiation

- **Made of**: a high-energy **photon** 光子 — part of the **electromagnetic spectrum** 电磁波谱.
- **Mass**: zero (rest mass).
- **Charge**: zero.
- **Range in air**: large (follows the inverse-square law). Strongly **attenuated** 衰减 by several cm of lead or about a metre of concrete.
- **Ionising power**: weakest.
- **Energy spectrum**: discrete (a **gamma ray** γ 射线 is given out as the nucleus drops between two nuclear energy levels).

A nucleus often gives out a γ-photon as a "tidy-up" step after an α or β decay leaves the **daughter nucleus 子核** in an **excited state 激发态**.



Penetrating power: α is stopped by paper, β by aluminium, γ only attenuated by lead

Antiparticles, neutrinos and antineutrinos

Every particle has an **antiparticle 反粒子** with the **same mass** but **opposite charge**. The positron is the antiparticle of the electron.

In β-decay, a third particle is always given out as well:

- β^- decay: an **antineutrino 反中微子** $\bar{\nu}_e$.
- β^+ decay: a **neutrino 中微子** ν_e.

Neutrinos and antineutrinos have zero charge, very small mass, and barely interact — they are very hard to detect, but they must be there to balance energy, **momentum 动量** and other conserved quantities in β-decay.

Why β has a continuous spectrum (and α does not)

In α-decay the **energy 能量** released is shared between just two particles (the daughter nucleus and the α). Conservation of momentum and energy then fixes the α's energy to one value (discrete).

In β-decay the energy is shared between **three** particles (the daughter nucleus, the β, and the (anti)neutrino). The β can take any share from zero up to a maximum, so its energy spectrum is continuous.

Writing decay equations

A general α-decay:

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\alpha$$

(check: $A = (A - 4) + 4$, $Z = (Z - 2) + 2$.)

A general β^- decay:

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + {}^{0}_{-1}\beta + \bar{\nu}_\mathrm{e}.$$

A general β^+ decay:

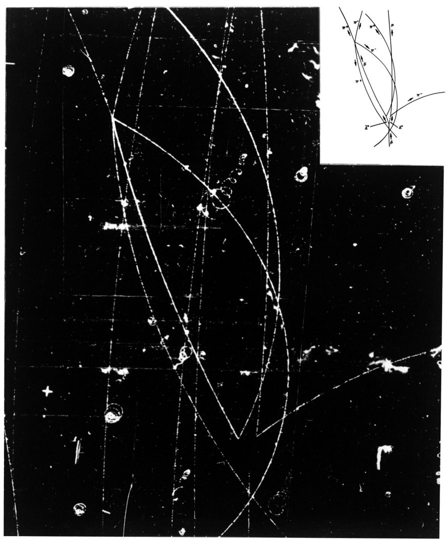
$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z-1}\mathrm{Y} + {}^{0}_{+1}\beta + \nu_\mathrm{e}.$$

Worked example. Uranium-238, $^{238}_{92}\mathrm{U}$, decays by α-emission; carbon-14, $^{14}_{6}\mathrm{C}$, decays by β^--emission. Find each daughter nuclide.

α-decay lowers A by 4 and Z by 2; β^--decay leaves A unchanged and raises Z by 1:

$$^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\alpha, \qquad {}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}\beta + \bar{\nu}_\mathrm{e}.$$

Fundamental particles



Production and decay of neutral lambda and anti-lambda hyperons

A bubble chamber reveals the curved tracks of charged particles.

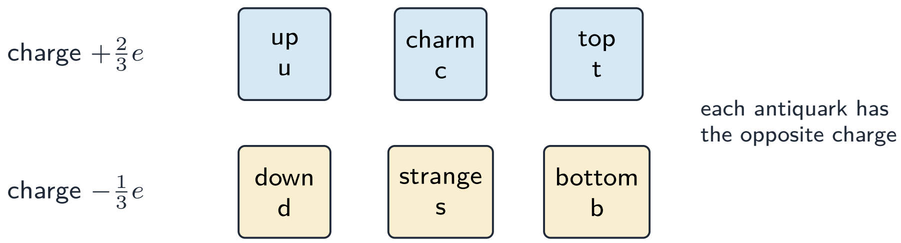
Some particles are **fundamental** 基本粒子 (point-like, with no smaller parts as far as we know); others are built from fundamental ones.

Quarks

A **quark** 夸克 is a fundamental particle. There are six **flavours** 味:

- **up** (u), charge $+\frac{2}{3}e$,
- **down** (d), charge $-\frac{1}{3}e$,
- **charm** (c), charge $+\frac{2}{3}e$,
- **strange** (s), charge $-\frac{1}{3}e$,
- **top** (t), charge $+\frac{2}{3}e$,
- **bottom** (b), charge $-\frac{1}{3}e$.

Each quark has an **antiquark** 反夸克 with the same size of charge but the opposite sign: $\bar{u}$ (charge $-\frac{2}{3}e$), $\bar{d}$ (charge $+\frac{1}{3}e$). No other quark property is tested.



The six quarks: up/charm/top carry $+\frac{2}{3}e$, down/strange/bottom carry $-\frac{1}{3}e$

Hadrons: baryons and mesons

Particles built from quarks are **hadrons** 强子. Two types:

- **baryons** 重子 — three quarks. Examples: proton (u u d), neutron (u d d). Charge check: $\frac{2}{3}+\frac{2}{3}-\frac{1}{3}=+1$ for the proton; $\frac{2}{3}-\frac{1}{3}-\frac{1}{3}=0$ for the neutron.
- **mesons** 介子 — one quark and one antiquark (for example π^+ is $\mathrm{u}\bar{\mathrm{d}}$).

Protons and neutrons are **not** fundamental — they are baryons made of quarks.

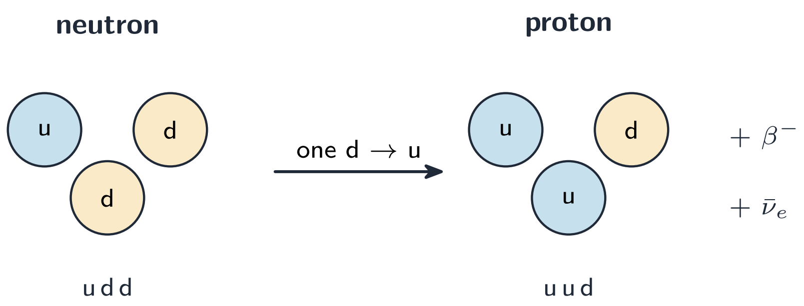
Quark changes in β-decay

In β^- decay a neutron turns into a proton; in quark terms, **one down quark turns into an up quark**:

$$\mathrm{d} \rightarrow \mathrm{u} + \beta^- + \bar{\nu}_\mathrm{e}.$$

In β^+ decay a proton turns into a neutron; **one up quark turns into a down quark**:

$$\mathrm{u} \rightarrow \mathrm{d} + \beta^+ + \nu_\mathrm{e}.$$



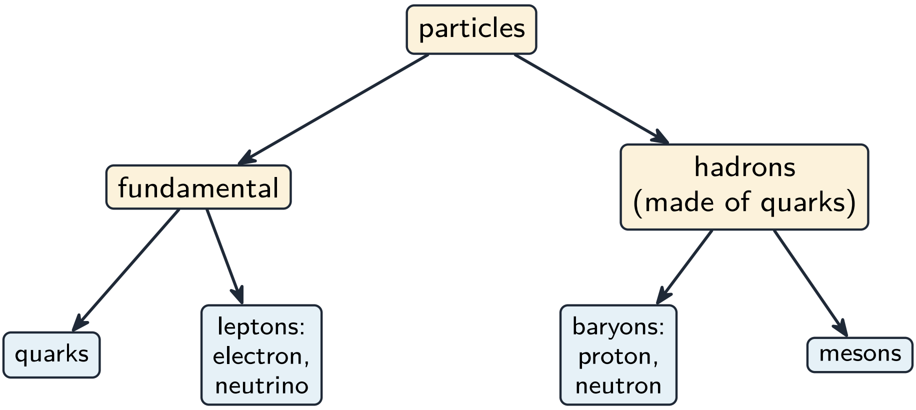
Beta-minus decay: one down quark becomes an up quark, turning a neutron into a proton

Leptons

Leptons 轻子 are fundamental particles that are not made of quarks. Electrons and neutrinos are leptons. (Heavier leptons — the muon and tau — are not needed for this syllabus.)

Classifying particles

To answer “which are fundamental?”: **quarks** and **leptons** (electrons, positrons, neutrinos, antineutrinos) are fundamental; protons, neutrons, baryons, mesons and hadrons are **not** — they are built from quarks.



Fundamental particles (quarks, leptons) versus hadrons (baryons, mesons)

Exam tips

- Describe the **nuclear atom** (a small, dense, positive nucleus) using the alpha-scattering evidence.
- Compare α, β and γ by **charge, mass, ionising power and penetration**.
- Use quark composition (proton uud, neutron udd) and check that **charge and nucleon number balance** in an equation.
- In β^- decay a neutron becomes a proton, an electron and an antineutrino.

Motion in a circle

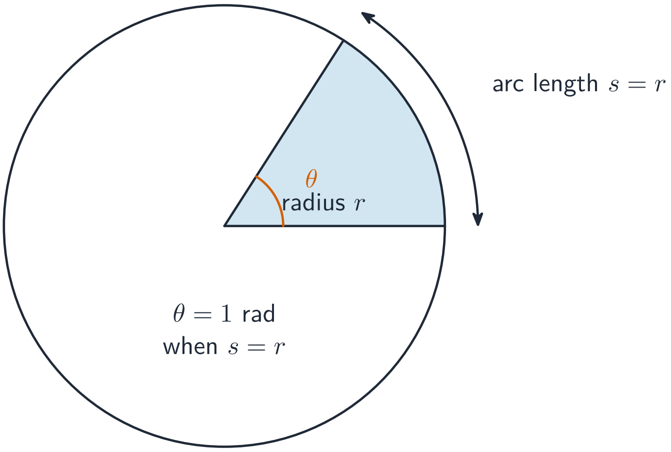
A-Level Physics

Angles in radians

The **radian** 弧度 is the angle made at the centre of a circle by an **arc** 弧 whose length equals the radius. For an arc of length s on a circle of radius r, the angle in radians is

$$\theta = \frac{s}{r}.$$

Radians have no unit (a ratio of lengths). A full circle has $s = 2\pi r$, so $\theta = 2\pi$ rad. A half-circle is π rad; a quarter is $\pi/2$ rad.



One radian is the angle whose arc length equals the radius

To convert: 1 rad $= 180°/\pi \approx 57.3°$. Set your calculator to **radians** for this topic; "degree" mode will give wrong answers.

Uniform circular motion: angular speed

A spinning fairground ride: every rider turns through the same angle each second.

An object moves in a circle of radius r at constant speed v. Define:

- **angular displacement** 角位移 θ — the angle (in radians) turned through by the radius from a chosen start line.
- **angular speed** 角速度 ω — the rate of change of angular displacement.

For uniform motion ω is constant and

$$\omega = \frac{\theta}{t}.$$

Unit: rad s^{-1}.

Period and frequency

If the object goes once round (2π rad, one **revolution** 圈) in time T (the **period** 周期), then

$$\omega = \frac{2\pi}{T} = 2\pi f,$$

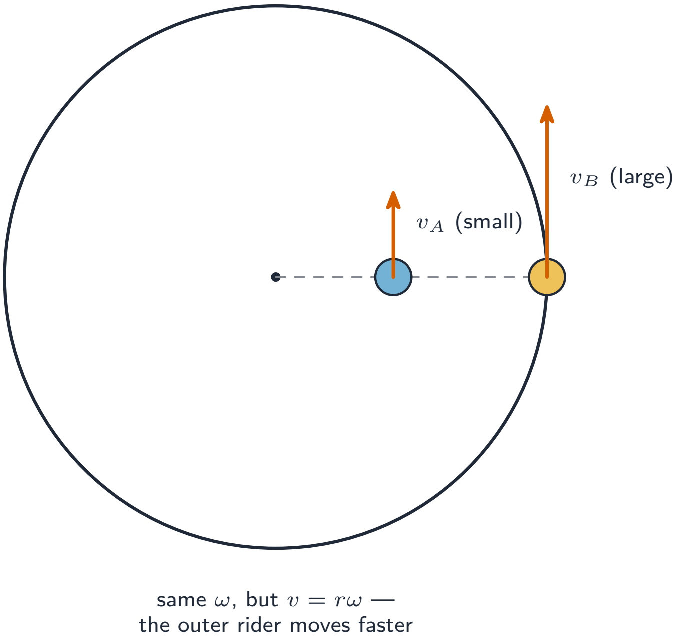
where $f = 1/T$ is the **frequency** 频率 of turning (Hz).

Linear and angular speed

In one period T the object travels a distance $2\pi r$ (the **circumference** 周长) at constant speed, so

$$v = \frac{2\pi r}{T} = r\omega.$$

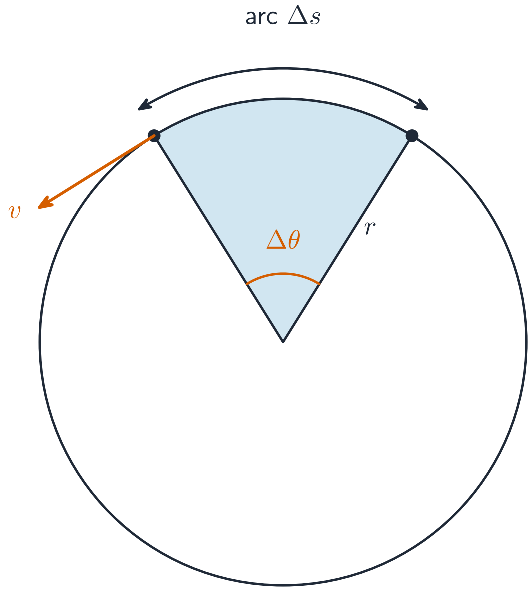
This links the linear (**tangential** 切向) speed v with the angular speed ω. At a larger radius (for the same angular speed) the linear speed is larger — a child on the edge of a merry-go-round moves faster than one near the centre, even though both go round once in the same time.



Same angular speed, but the rider at the larger radius has the larger linear speed ($v = r\omega$)

Worked example. A fairground ride of radius 4.0 m completes one turn every 8.0 s. Find its angular speed and the linear speed of a rider on the edge.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8.0} = 0.79 \text{ rad s}^{-1}, \qquad v = r\omega = 4.0 \times 0.79 = 3.1 \text{ m s}^{-1}.$$



As the radius turns through $\Delta\theta$ the object moves an arc Δs at speed v

Centripetal acceleration

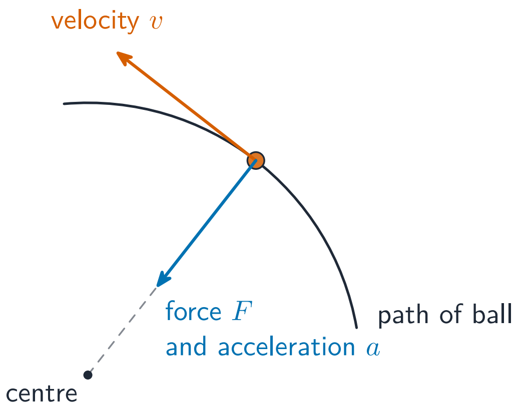
An object moving in a circle at constant speed still has a **changing velocity** 速度 — its direction keeps changing, even though its size stays the same. A changing velocity needs an **acceleration** 加速度. This acceleration points **towards the centre** and is the **centripetal acceleration** 向心加速度.

Size

$$a = \frac{v^2}{r} = r\omega^2.$$

The two forms are equal because $v = r\omega$. Pick the one with the quantities you have.

The centripetal acceleration is **perpendicular** 垂直 to the velocity at every instant — never along the direction of motion. (If part of it were along the motion, the speed would change.) Unit: m s^{-2}.



The velocity points along the tangent; the force and acceleration point to the centre

Centripetal force

A Ferris wheel: a centripetal force toward the centre keeps each car moving in a circle.

By Newton's second law, the resultant **force** 力 on a body in circular motion at constant speed is

$$F = ma = \frac{mv^2}{r} = mr\omega^2.$$

This is the **centripetal force** 向心力. It **always points towards the centre** — perpendicular to the velocity.

The centripetal force is not a new kind of force — it is the **net result** of the real forces acting (tension, gravity, friction, electric attraction, normal contact force, ...). In a problem, work out which real force(s) provide it.

Worked example. A 0.20 kg ball on a string is whirled in a horizontal circle of radius 0.50 m at 3.0 m s^{-1}. Find the centripetal force (the tension in the string).

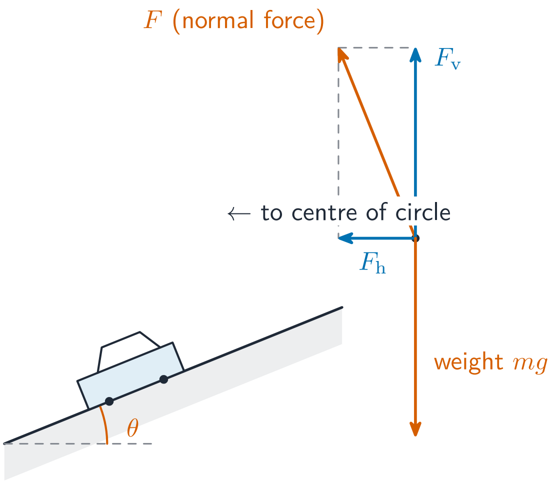
$$F = \frac{mv^2}{r} = \frac{0.20 \times 3.0^2}{0.50} = 3.6 \text{ N}.$$

Where the centripetal force comes from

- **Ball on a string** in a horizontal circle: the **tension** 张力 in the string.
- **Car turning a flat corner**: the **friction** 摩擦力 between tyres and road ($F = mv^2/r$). If the car goes too fast, friction is not enough and it skids outwards.
- **Banked corner** 倾斜 (no friction): the horizontal part of the **normal contact force** 支持力; $\tan\theta = v^2/(rg)$ for the angle that needs no friction.
- **Planet or satellite** 卫星 in **orbit** 轨道: the **gravitational attraction** 引力, $GMm/r^2 = mv^2/r$.
- **Electron** 电子 in a circular orbit (Bohr-style model): the **electrostatic** 静电 attraction between the electron and the positive **nucleus** 原子核:

$$\frac{kZe^2}{r^2} = \frac{m_e v^2}{r},$$

where $k = 1/(4\pi\varepsilon_0)$ and Z is the nuclear charge. Solve for v to get the orbital speed; then $T = 2\pi r/v$.

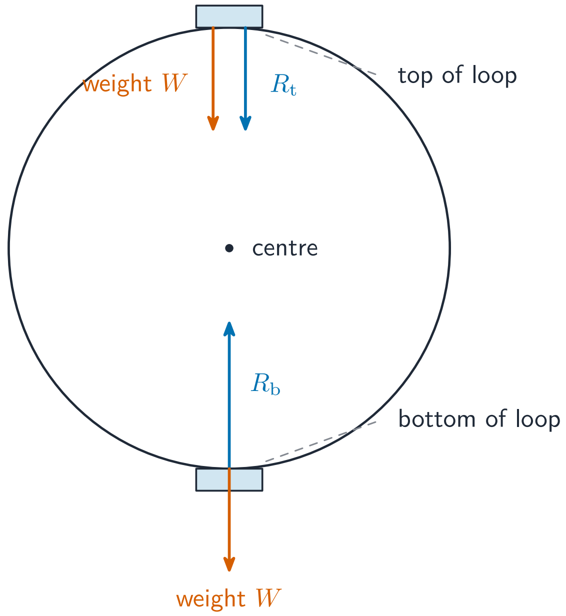


On a banked track the horizontal part of the road's force provides the centripetal force

Vertical circles

When the circle is upright, the speed is **not** constant (gravity does work) — but at each instant the **net force towards the centre** still equals mv^2/r:

- at the bottom of a loop: tension up, **weight** 重力 down, so $T - mg = mv^2/r$ — the tension is largest here.
- at the top of a loop: tension and weight both point down (towards the centre), so $T + mg = mv^2/r$ — the tension is smallest. For the slowest speed at the top with the string just tight, set $T = 0$: $mg = mv_{\min}^2/r$, giving $v_{\min} = \sqrt{gr}$.



Forces on a person at the top and bottom of a vertical circle

Worked example. A car goes round a vertical loop of radius 2.0 m. Find the minimum speed at the top for the car to keep contact with the track (take $g = 9.81 \text{ m s}^{-2}$).

At the slowest speed the track force is zero, so gravity alone provides the centripetal force: $mg = mv_{\min}^2/r$, giving $v_{\min} = \sqrt{gr}$:

$$v_{\min} = \sqrt{9.81 \times 2.0} \approx 4.4 \text{ m s}^{-1}.$$

The constant-speed result ($v = r\omega$, ω constant) holds for horizontal circles, or where the force only bends the path (orbits in gravity, charges in a **magnetic field** 磁场).

How to structure a circular-motion answer

1. **Find the radius** r and choose v or ω. Use $v = r\omega$ to switch between them.
2. **Find the centripetal acceleration** with $a = v^2/r$ or $r\omega^2$.
3. **List the real forces** and write Newton's second law in the **radial** 径向 direction (towards the centre is positive). Set the net inward force equal to mv^2/r.
4. For **period or frequency**: use $\omega = 2\pi/T$, or $T = 2\pi r/v$.
5. Check the **directions**: centripetal force and acceleration point to the centre; the velocity is along the tangent.

Exam tips

- Work in **radians**; angular speed $\omega = 2\pi/T = v/r$.
- **Centripetal acceleration** $a = v^2/r = \omega^2 r$; the net force acts **towards the centre** — it is provided by tension/gravity/friction, not an extra force.
- Always state **what provides the centripetal force** in the situation given.

Gravitational fields

A-Level Physics

Gravitational fields

Definition

A **gravitational field** 重力场 is a region where a **mass** 质量 feels a **force** 力 from other masses. The **gravitational field strength** 重力场强度 g at a point is the **gravitational force per unit mass** on a small **test mass** 检验质量 placed there:

$$g = \frac{F}{m}.$$

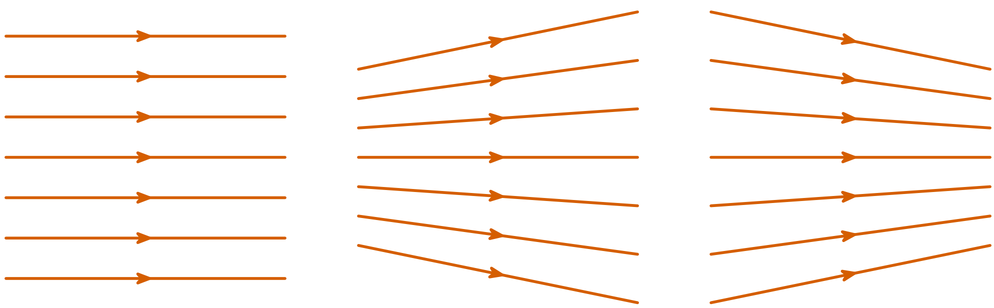
Unit: N kg^{-1} (the same as m s^{-2} — the acceleration of free fall in the field). g is a **vector** 矢量, pointing the way the force acts — towards the source mass.

Field lines

A gravitational field is drawn with **field lines** 场线 that point the way the force acts on a test mass:

- around a **point mass** 质点 or a uniform sphere (treated as a point mass from outside), the field lines are **radial** 径向, pointing **inwards**.
- near the Earth's surface over a small area, the field lines are nearly **parallel and equally spaced**, pointing straight down — a **uniform field** 匀强场.

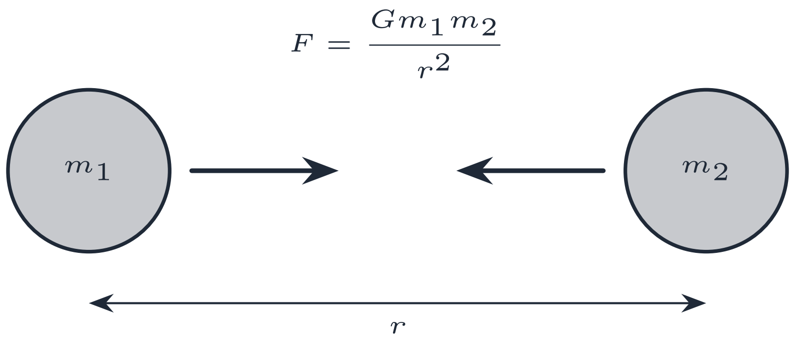
Closer lines mean a stronger field.



Field-line spacing shows the field strength — closer lines mean a stronger field

Newton's law of gravitation

For two point masses m_1, m_2 a distance r apart, the force on each is



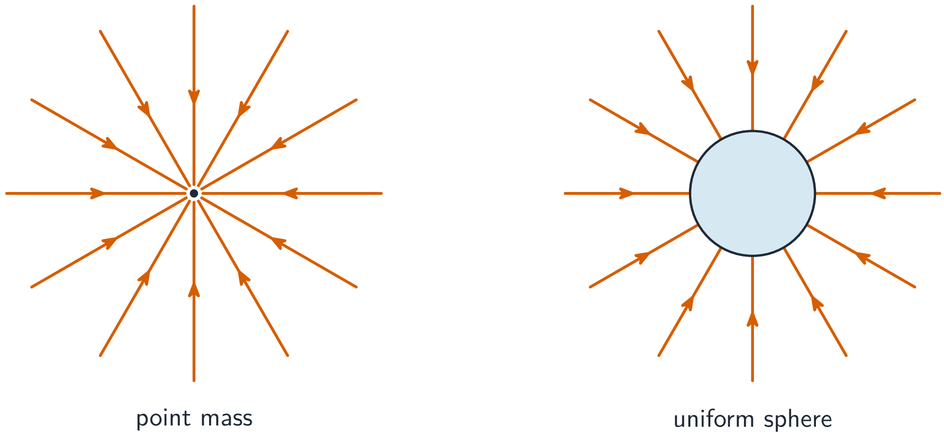
Two masses attract along the line joining them

$$F = \frac{Gm_1 m_2}{r^2},$$

pulling them together along the line joining them. This is **Newton's law of gravitation** 万有引力定律. The constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} is the **universal gravitational constant** 万有引力常量.

Spheres treated as point masses

For a uniform sphere (such as a planet or star), the field at any point **outside** is the same as that of a point mass equal to the total mass at the centre. So from above the surface, you can treat the Earth as a point mass at its centre. (Points inside a sphere are different, and are not in the syllabus.)



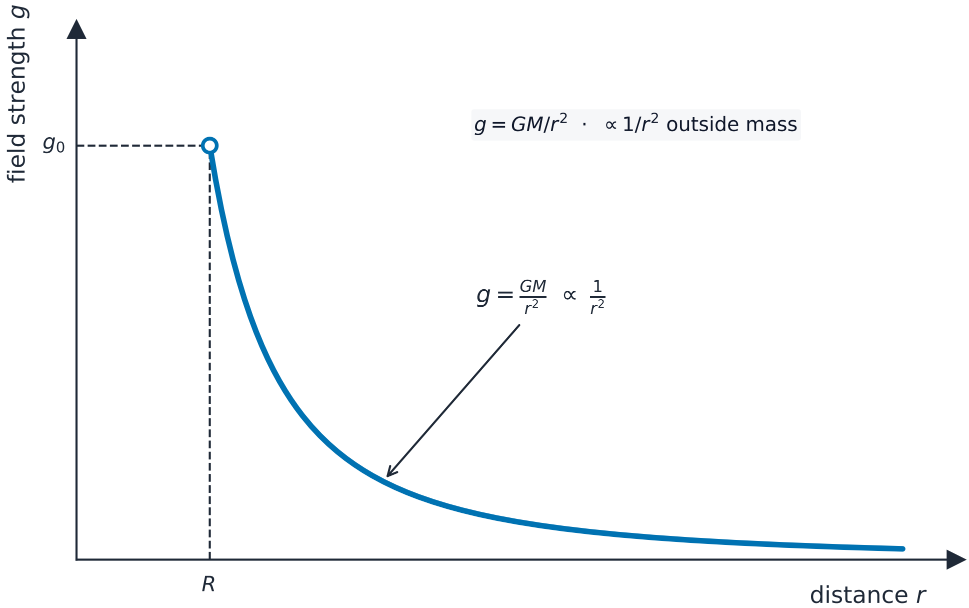
Outside a uniform sphere the field is radial, exactly like a point mass at the centre

Field strength from a point mass

Put the gravitational force on a test mass m at distance r from a point mass M into $g = F/m$:

$$F = \frac{GMm}{r^2}, \qquad g = \frac{GM}{r^2}.$$

So g falls off as $1/r^2$ as you move away from the source.



Field strength falls off as $1/r^2$ with distance from a point mass

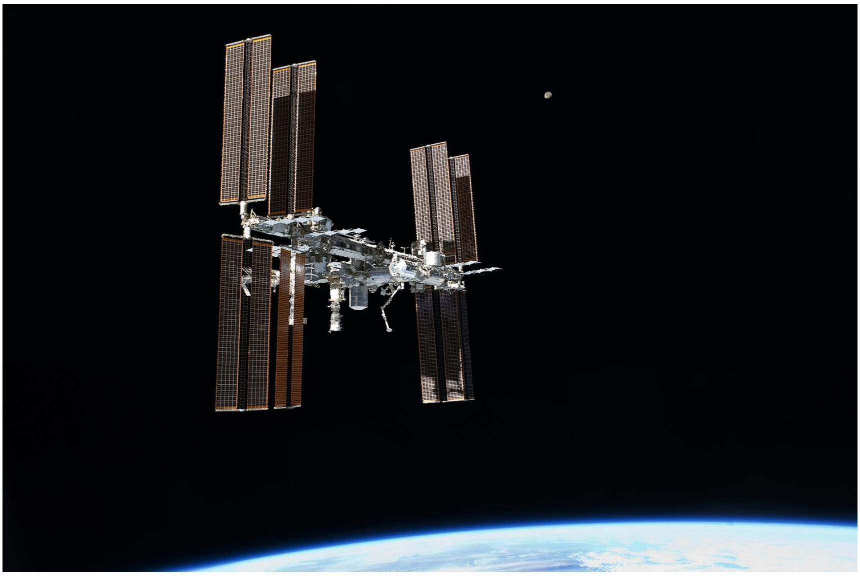
Worked example. Find the gravitational field strength at the Earth's surface. (Earth's mass $M = 6.0 \times 10^{24}$ kg, radius $R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.)

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{(6.4 \times 10^6)^2} \approx 9.8 \text{ N kg}^{-1}.$$

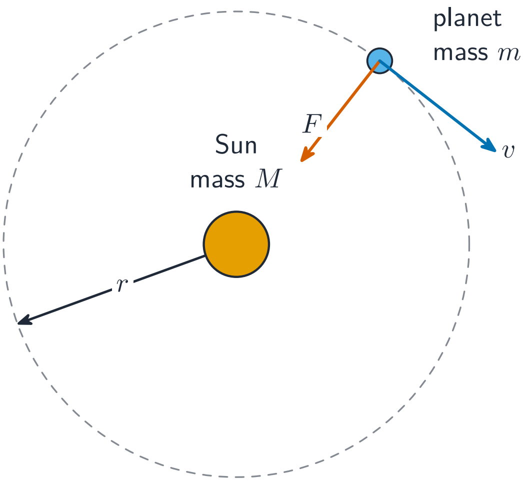
Why g is nearly constant near the Earth's surface

The Earth's radius is $R \approx 6.4 \times 10^6$ m. Rising to height h changes the distance from the centre from R to $R + h$. For $h \ll R$ (any building or mountain), $(R + h)/R \approx 1$, so g barely changes — going from 5 m to 10 m high changes r by about one part in a million. In the laboratory, g is effectively constant.

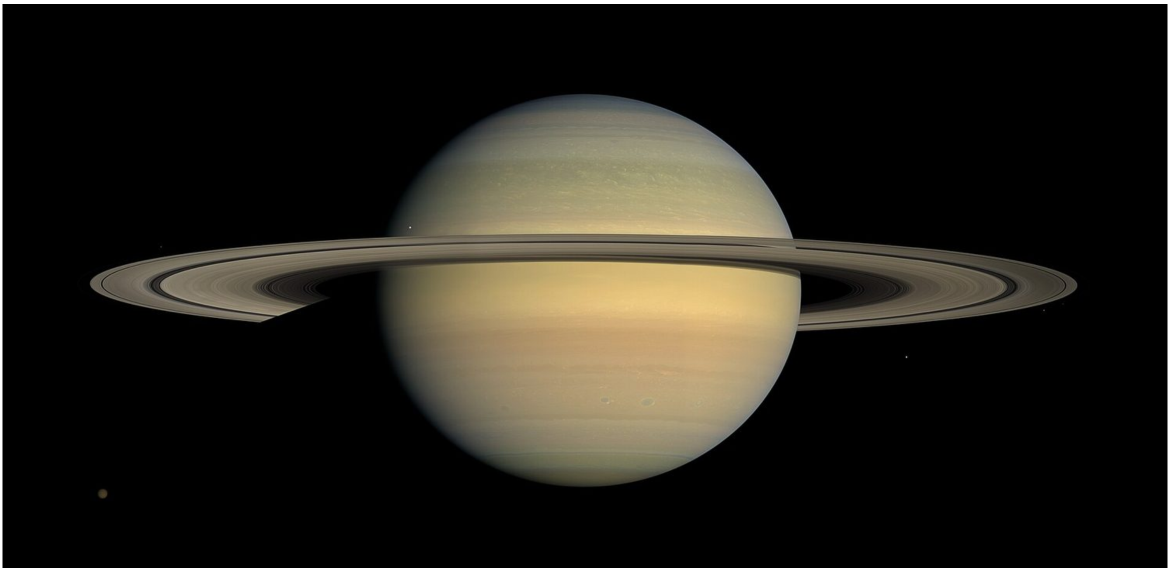
Orbital motion in a gravitational field



The International Space Station orbits Earth, held in its path by gravity.



Gravity provides the centripetal force that keeps a planet in a circular orbit



Saturn, its rings (countless small orbiting pieces) and its moons are all held in orbit by gravity

For a **satellite** 卫星 of mass m in a circular **orbit** 轨道 of radius r around a body of mass M, gravity provides the **centripetal force** 向心力:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}.$$

Cancel m (the orbital speed does not depend on the satellite's mass):

$$v = \sqrt{\frac{GM}{r}}.$$

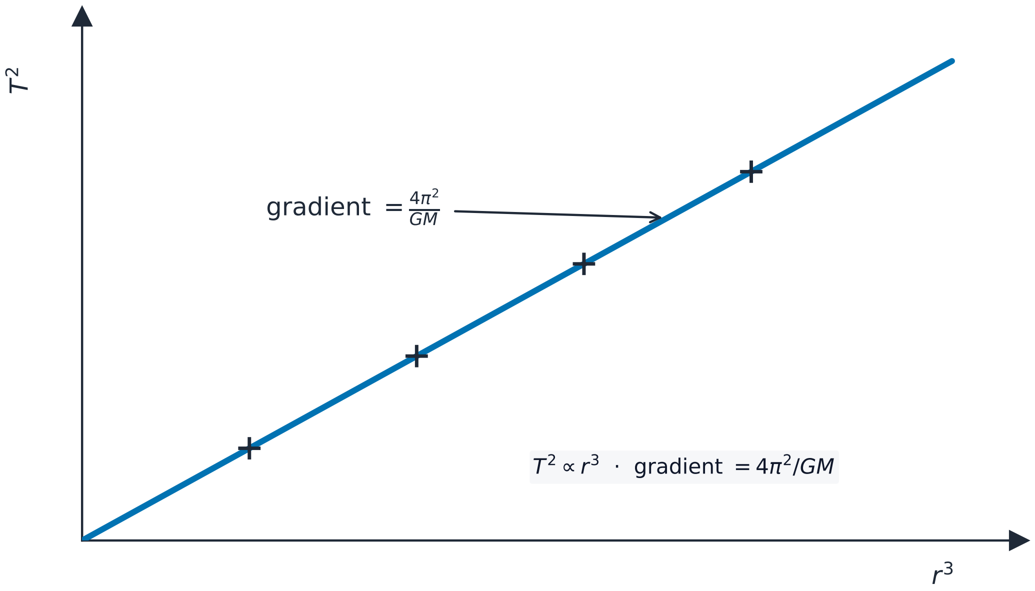
Worked example. A satellite orbits the Earth in a circular orbit of radius $r = 7.0 \times 10^6$ m. Find its orbital speed. (For the Earth, $GM = 4.0 \times 10^{14}$ m^3 s^{-2}.)

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{4.0 \times 10^{14}}{7.0 \times 10^6}} \approx 7.6 \times 10^3 \text{ m s}^{-1}.$$

The **period** 周期 follows from $T = 2\pi r/v$:

$$T = 2\pi\sqrt{\frac{r^3}{GM}}, \quad \text{so} \quad T^2 = \frac{4\pi^2}{GM} \cdot r^3.$$

This is **Kepler's third law** 开普勒第三定律 for circular orbits: $T^2 \propto r^3$. A plot of T^2 against r^3 is a straight line through the origin with gradient $4\pi^2/(GM)$, so orbital data gives the central mass.



Kepler's third law: $T^2 \propto r^3$, a straight line through the origin

Geostationary orbit

A **geostationary** 地球同步 satellite:

- stays directly above the same point on the Earth (so a fixed dish always points at it),
- has a period of **24 hours** (the same as the Earth's rotation),
- orbits **west to east** (the same way the Earth turns),
- must be directly above the **equator** 赤道.

It must have the same **angular speed** 角速度 as the Earth, in the same direction, in the equatorial plane (or it would drift north–south during the day). From $T = 24$ h and $T^2 = 4\pi^2 r^3/(GM)$, the radius is $r \approx 4.2 \times 10^7$ m (about 3.6×10^7 m above the surface).

Gravitational potential

Gravitational potential 引力势 ϕ at a point is the **work done per unit mass** in bringing a small test mass from **infinity** 无穷远 to that point:

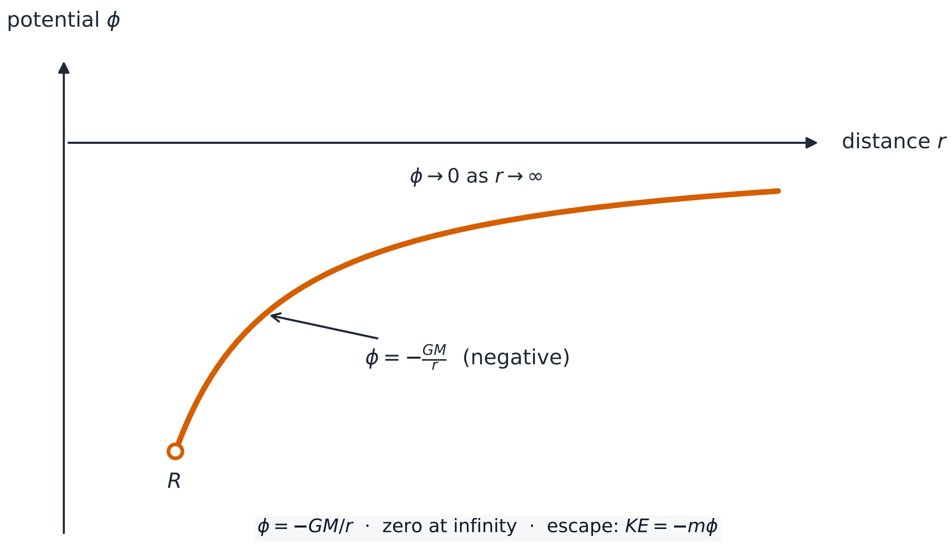
$$\phi = \frac{W}{m}.$$

Unit: J kg^{-1}.

The potential is taken as **zero at infinity**. As the test mass falls in towards the source, gravity does the work for you, so ϕ is **negative** everywhere except at infinity. For a point mass M at distance r:

$$\phi = -\frac{GM}{r}.$$

ϕ is a **scalar 标量**. For several masses, add the potentials.



The gravitational potential well: $\phi = -GM/r$ is negative, rising to zero at infinity

Gravitational potential energy of two point masses

If a test mass m sits where the potential is ϕ, the **gravitational potential energy 重力势能** of the pair is

$$E_{\mathrm{P}} = m\phi = -\frac{GMm}{r}.$$

Like the potential, E_{P} is **negative** and reaches zero only at infinite separation. Closer masses have more negative potential energy (more tightly bound).

Link with $\Delta E_\mathbf{P} = mg\Delta h$

For small height changes near the surface, r barely changes, so $\Delta E_\mathrm{P} \approx mg\Delta h$. For large changes (a satellite moving to a higher orbit) use $-GMm/r$ at each radius and take the difference:

$$\Delta E_\mathrm{P} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad (r_2 > r_1),$$

which is positive (energy must be supplied to raise the satellite).

Escape velocity (from conservation of energy)

To escape from radius r to infinity, an object's **kinetic energy 动能** must equal the size of its gravitational potential energy:

$$\tfrac{1}{2}mv_\mathrm{esc}^2 = \frac{GMm}{r}, \qquad v_\mathrm{esc} = \sqrt{\frac{2GM}{r}}.$$

At the Earth's surface, the **escape velocity 逃逸速度** is ≈ 11 km s^{-1}. It does not depend on the object's mass.

Worked example. Find the escape velocity from the Earth's surface. (For the Earth, $GM = 4.0 \times 10^{14}$ m^3 s^{-2}, $R = 6.4 \times 10^6$ m.)

$$v_\mathrm{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(4.0 \times 10^{14})}{6.4 \times 10^6}} \approx 1.1 \times 10^4 \text{ m s}^{-1} \ (= 11 \text{ km s}^{-1}).$$

Exam tips

- **Newton's law of gravitation** $F = GMm/r^2$ (inverse-square); field strength $g = GM/r^2$.
- Distinguish **gravitational potential** ($\phi = -GM/r$, always negative, zero at infinity) from field strength.
- For an orbit set gravity = centripetal force to get $T^2 \propto r^3$; a geostationary orbit has $T = 24$ h.

Temperature

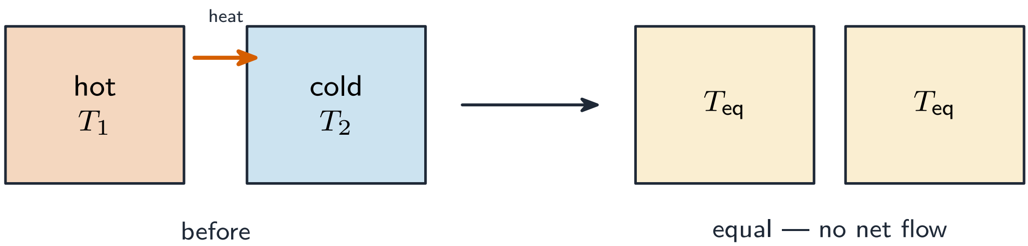
A-Level Physics

Thermal equilibrium

Heat 热量 (thermal energy) flows from a **higher temperature** to a **lower temperature**. When two bodies touch, energy moves until their temperatures are equal — they reach **thermal equilibrium** 热平衡. At equilibrium there is no net flow of energy.

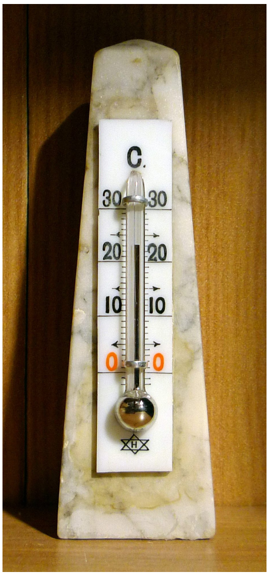
Two regions at the **same temperature** 温度 are in thermal equilibrium with each other — no net energy flows, even though particles still exchange energy.

Temperature decides the **direction** of heat flow. It is not a measure of how much **thermal energy** 热能 a body holds. A small cup of boiling water (100 °C) holds far less energy than a swimming pool at 25 °C, but a piece of metal put in the cup gains energy while one put in the pool loses it.



Heat flows from hot to cold until both reach the same temperature — thermal equilibrium

Measuring temperature

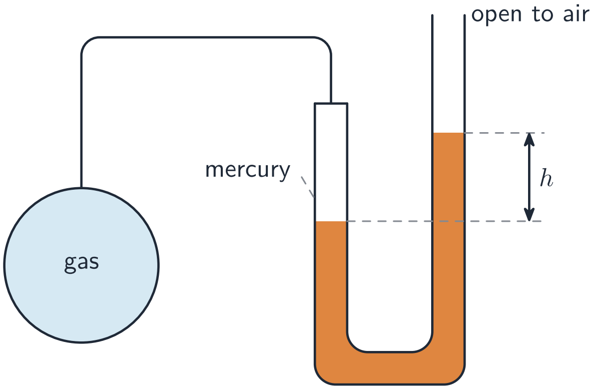


A thermometer measures temperature on a defined scale.

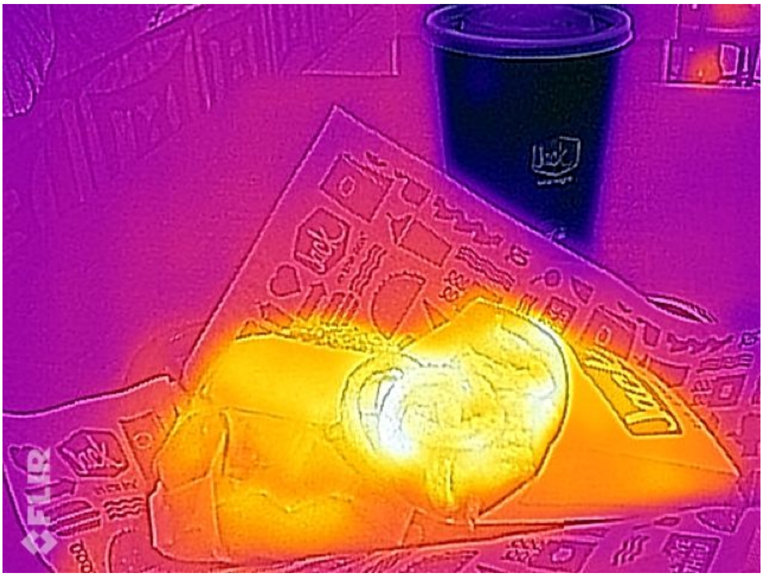
Any physical property that **changes in a repeatable way with temperature** can make a **thermometer** 温度计. Examples:

- **volume of a liquid** — a liquid-in-glass thermometer (mercury or alcohol). As the temperature rises, the liquid expands and rises up a narrow **capillary** 毛细管.
- **volume of a gas at constant pressure** — a gas thermometer. The gas volume rises in step with the absolute temperature.
- **resistance of a metal** — a resistance thermometer. A metal's **resistance** 电阻 rises nearly in step with temperature over a wide range.
- **e.m.f. of a thermocouple** — a **thermocouple** 热电偶 is two different metals joined at two points; the **electromotive force** 电动势 it makes depends on the temperature difference between the joins.

Different thermometers can read slightly differently if the property does not change in a straight line; they agree only at the **calibration** 校准 points.



A constant-volume gas thermometer — the gas pressure is found from the height difference h



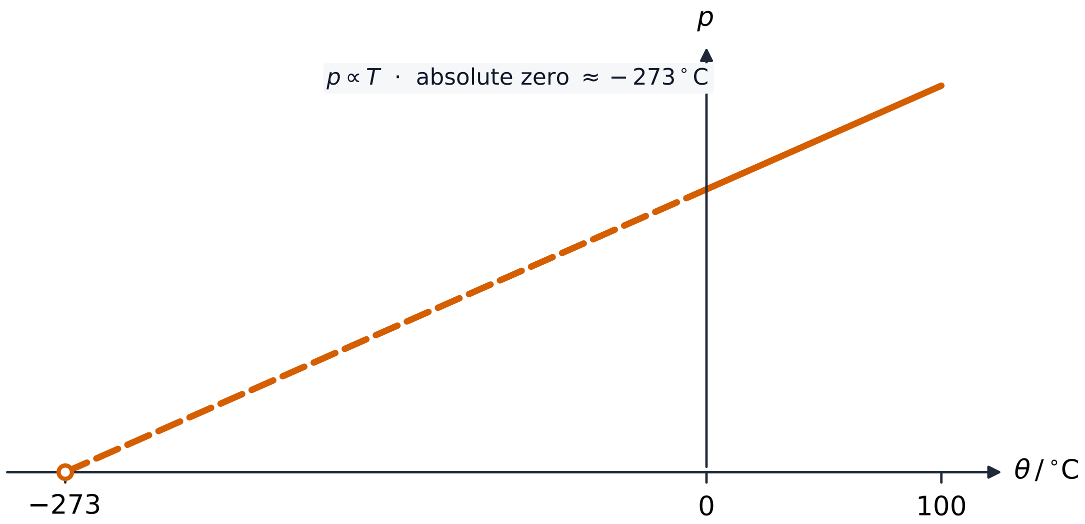
A thermal (infrared) camera maps temperature to colour: the hot fries glow bright orange, the cold drink stays dark

Thermodynamic temperature scale

The **thermodynamic temperature** 热力学温度 (or **absolute temperature** 绝对温度) scale does not depend on any one substance — only on the laws of thermodynamics. Its unit is the **kelvin** 开尔文 (K).

Absolute zero

The lowest possible temperature is **zero kelvin** (0 K), called **absolute zero** 绝对零度. There a system has its least possible **internal energy** 内能 — particles have no random motion to speak of. Nothing can be cooled below this.



Extrapolating the pressure–temperature line back to zero pressure gives absolute zero, about −273 °C

Celsius scale

The **Celsius** 摄氏度 scale θ is shifted from the thermodynamic scale by a fixed amount:

$$T/\mathrm{K} = \theta/^\circ\mathrm{C} + 273.15.$$

So 0 °C = 273.15 K and 100 °C = 373.15 K. A kelvin and a degree Celsius are the **same size**, so a temperature **difference** of 1 K equals 1 °C — but the absolute values differ by 273.15.

In gas-law calculations you must always use **absolute** temperatures in kelvin. Using °C gives wrong answers.

Specific heat capacity

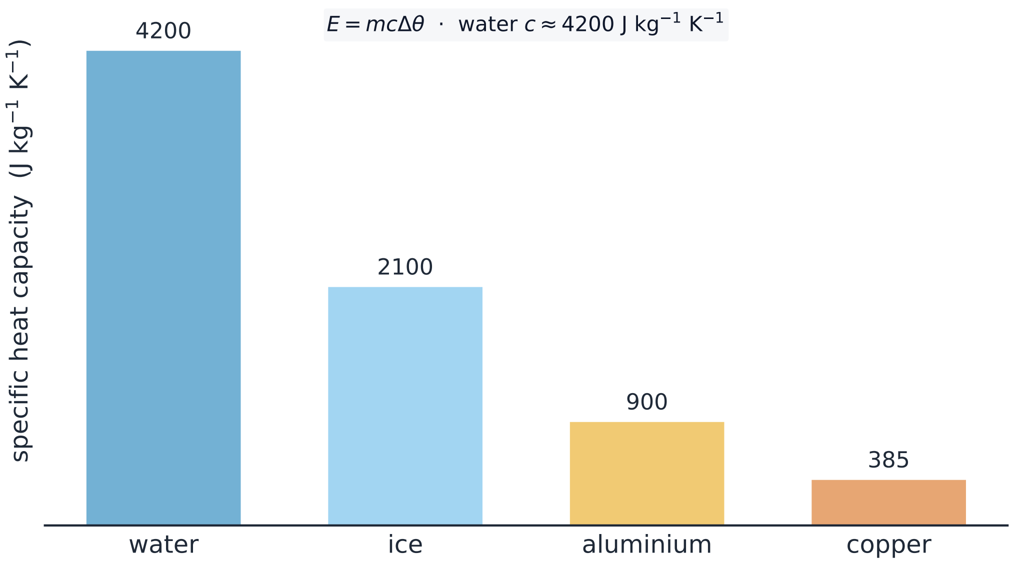
The **specific heat capacity** 比热容 c of a substance is the **energy** 能量 needed to raise the temperature of **unit mass** by **one kelvin**:

$$c = \frac{Q}{m\Delta T} \qquad \Longleftrightarrow \qquad Q = mc\Delta T.$$

Unit: J kg^{-1} K^{-1}.

Examples:

- water: $c \approx 4200$ J kg^{-1} K^{-1} (high — why water is a good coolant and why oceans steady the climate).
- aluminium: $c \approx 900$ J kg^{-1} K^{-1}.
- copper: $c \approx 385$ J kg^{-1} K^{-1}.



Water's specific heat capacity is far higher than common solids — why it is such a good coolant

To find an unknown c by experiment: supply known energy Q electrically ($Q = VIt$, from the **power** 功率), then measure the temperature rise ΔT of a known **mass** 质量 m. Then $c = Q/(m\Delta T)$. Reduce heat loss with **insulation** 隔热 and use a rise of about 10 K (big enough to measure well, small enough to limit losses).

Worked example. How much energy is needed to heat 0.50 kg of water from 20 °C to 100 °C? (Specific heat capacity of water $c = 4200$ J kg^{-1} K^{-1}.)

A temperature difference is the same in K and °C, so $\Delta T = 80$:

$$Q = mc\Delta T = 0.50 \times 4200 \times 80 = 1.68 \times 10^5\ \text{J}\ (= 168\ \text{kJ}).$$

When two bodies reach thermal equilibrium with no heat lost to the surroundings, the energy gained by the colder one equals the energy lost by the hotter one:

$$m_1 c_1 (T_{\text{eq}} - T_1) = m_2 c_2 (T_2 - T_{\text{eq}}).$$

Worked example. 0.20 kg of water at 80 °C is mixed with 0.30 kg of water at 20 °C, with no heat lost. Find the final temperature.

The heat lost by the hot water equals the heat gained by the cold water (the c of water cancels):

$$0.20\,(80 - T_{\text{eq}}) = 0.30\,(T_{\text{eq}} - 20) \quad \Rightarrow \quad T_{\text{eq}} = 44\ ^\circ\text{C}.$$

Specific latent heat

When a substance changes state (solid ↔ liquid, or liquid ↔ gas) at constant temperature, energy must be supplied (or removed) with **no** temperature change. This energy is the **latent heat** 潜热.

The **specific latent heat** 比潜热 L is the energy to change the state of **unit mass** at constant temperature:

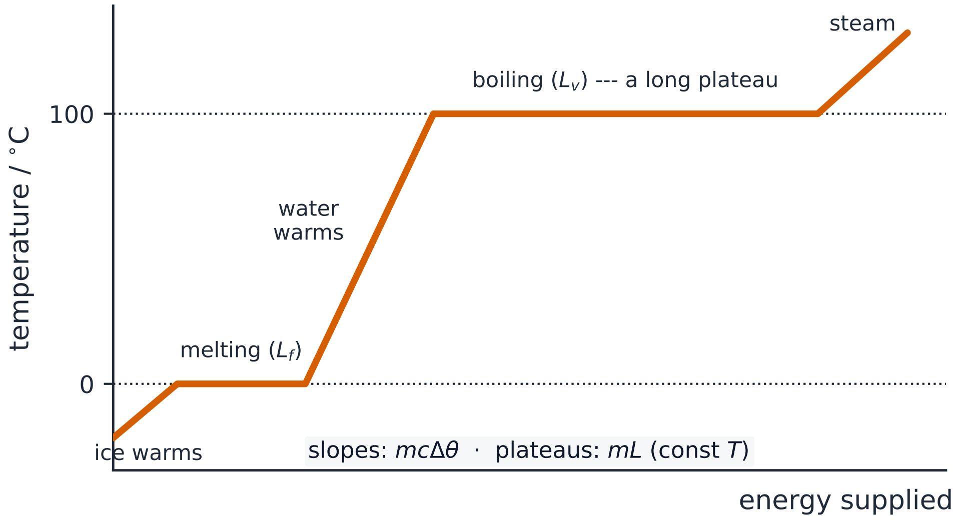
$$L = \frac{Q}{m} \qquad \Longleftrightarrow \qquad Q = mL.$$

Unit: J kg^{-1}.

Two kinds:

- **specific latent heat of fusion** 熔化 L_f — for melting or freezing (solid ↔ liquid).
- **specific latent heat of vaporisation** 汽化 L_v — for boiling or condensing (liquid ↔ gas).

For water at atmospheric pressure: $L_\text{f} \approx 3.34 \times 10^5$ J kg^{-1} (at 0 °C); $L_\text{v} \approx 2.26 \times 10^6$ J kg^{-1} (at 100 °C). So L_v is about 7 times L_f.



Heating curve: the sloped parts warm the substance ($mc\Delta T$); the flat plateaus are the phase changes (mL)

Worked example. A 2.0 kW heater boils water already at 100 °C. How long does it take to turn 0.10 kg of this water into steam? ($L_\mathrm{v} = 2.26 \times 10^6$ J kg^{-1}, no heat lost.)

The energy needed is $Q = mL_\mathrm{v} = 0.10 \times 2.26 \times 10^6 = 2.26 \times 10^5$ J. From $Q = Pt$,

$$t = \frac{Q}{P} = \frac{2.26 \times 10^5}{2000} \approx 110 \text{ s.}$$

Why $L_\mathrm{v} > L_\mathrm{f}$

Two reasons, both from the particle picture of matter:

1. **Bonds**: in melting, only some of the **intermolecular** 分子间 bonds break; the particles stay close as a liquid. In boiling, **all** the bonds must break so the particles can separate. Breaking all of them needs more energy.
2. **Work against the atmosphere**: when a liquid turns to gas it expands hugely (vapour has about 10^3 times the liquid's **volume** 体积), so it does work pushing back the surrounding **atmospheric pressure** 大气压强. That work comes from the energy supplied.

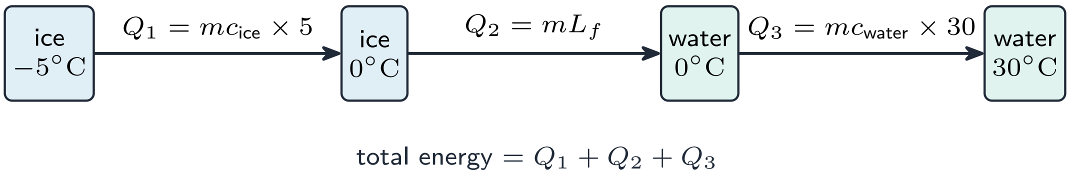
Multi-step problems

If a problem mixes temperature change and a **phase change** 相变 (e.g. ice at −5 °C warming to water at 30 °C):

1. heat the solid from −5 °C to 0 °C: $Q_1 = mc_\mathrm{ice} \times 5$.

2. melt at 0 °C: $Q_2 = mL_\text{f}$.
3. heat the water from 0 °C to 30 °C: $Q_3 = mc_\text{water} \times 30$.

Total: $Q_1 + Q_2 + Q_3$. A phase change is at constant temperature, so use mL there, not $mc\Delta T$.



Split a mixed problem into stages — warm, change state, warm — then add the energies

When a question gives heater power P and asks for the time, use $Q = Pt$ (assuming no heat loss). Insulating (lagging) the container and using a small mass are common ways to improve the experiment.

Exam tips

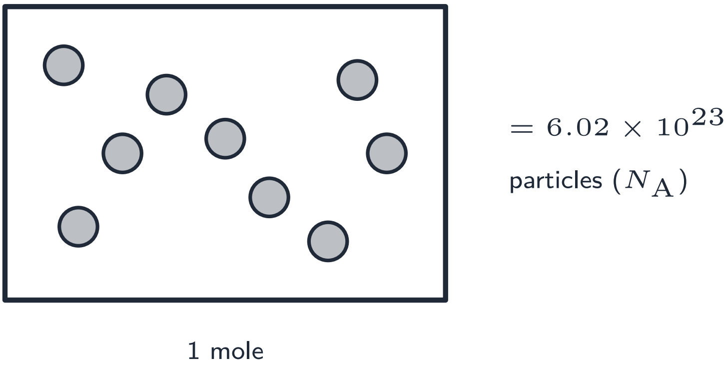
- **Thermal equilibrium** means no net heat flow (equal temperature); the thermodynamic scale is fixed by absolute zero and the triple point.
- Convert temperatures with $T/\text{K} = \theta/°\text{C} + 273.15$ and use **kelvin** in energy and gas equations.
- Use $Q = mc\Delta T$ for heating and $Q = mL$ for a change of state (no temperature change) — never mix the two.

Ideal gases

A-Level Physics

The mole

Amount of substance 物质的量 is an SI base quantity. Its unit is the **mole** 摩尔 (mol) — one of the seven SI base units, with the kilogram, metre, second, ampere and **kelvin** 开尔文 from Topic 1.



One mole contains the Avogadro number of particles

One mole of any substance has a number of particles equal to the **Avogadro constant** 阿伏伽德罗常量:

$$N_\mathrm{A} = 6.02 \times 10^{23}\ \mathrm{mol}^{-1}.$$

A "particle" means whatever you are counting — **atoms** 原子 for a **monatomic** 单原子 element like helium, **molecules** 分子 for O_2 or H_2O. Always say what you are counting.

For n moles, the number of particles is $N = nN_\mathrm{A}$.

The **molar mass** 摩尔质量 M_m is the mass of one mole (kg mol^{-1} or g mol^{-1}). Mass of n moles is $M = nM_\mathrm{m}$. Mass of one particle is $m_0 = M_\mathrm{m}/N_\mathrm{A}$.

Equation of state of an ideal gas

Compressed gas cylinders store a fixed mass of gas at high pressure.

An **ideal gas** 理想气体 obeys $pV \propto T$ exactly, where T is the **thermodynamic temperature** 热力学温度.

The **equation of state** 状态方程 can be written two equal ways:

$$pV = nRT \qquad \text{or} \qquad pV = NkT.$$

Here:

- p — **pressure** 压强 (Pa).
- V — **volume** 体积 (m^3).
- T — thermodynamic temperature in kelvin (never °C).
- n — number of moles; N — number of molecules.
- R — **molar gas constant** 摩尔气体常量, $R = 8.31\ \mathrm{J\ mol^{-1}\ K^{-1}}$.
- k — **Boltzmann constant** 玻尔兹曼常量, $k = 1.38 \times 10^{-23}\ \mathrm{J\ K^{-1}}$.

Since $N = nN_\mathrm{A}$, we get $k = R/N_\mathrm{A}$: k is the gas constant **per molecule**, as R is per mole.

Using the equation of state

List the variables you have, find the unknown, and choose the form that matches your "amount" (moles $\rightarrow nRT$; molecules $\rightarrow NkT$). Always use SI units: Pa, m^3, K.

Worked example. A cylinder of volume 0.020 m^3 holds gas at 27 °C and a pressure of 2.0×10^5 Pa. How many moles of gas are there? ($R = 8.31$ J mol^{-1} K^{-1}.)

Convert to kelvin: $T = 27 + 273 = 300$ K. Then from $pV = nRT$,

$$n = \frac{pV}{RT} = \frac{(2.0 \times 10^5)(0.020)}{8.31 \times 300} \approx 1.6 \text{ mol.}$$

If a fixed amount of gas changes from state 1 to state 2:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}.$$

Worked example. A fixed mass of gas at 300 K occupies 0.50 m^3. It is heated to 450 K at constant pressure. Find the new volume.

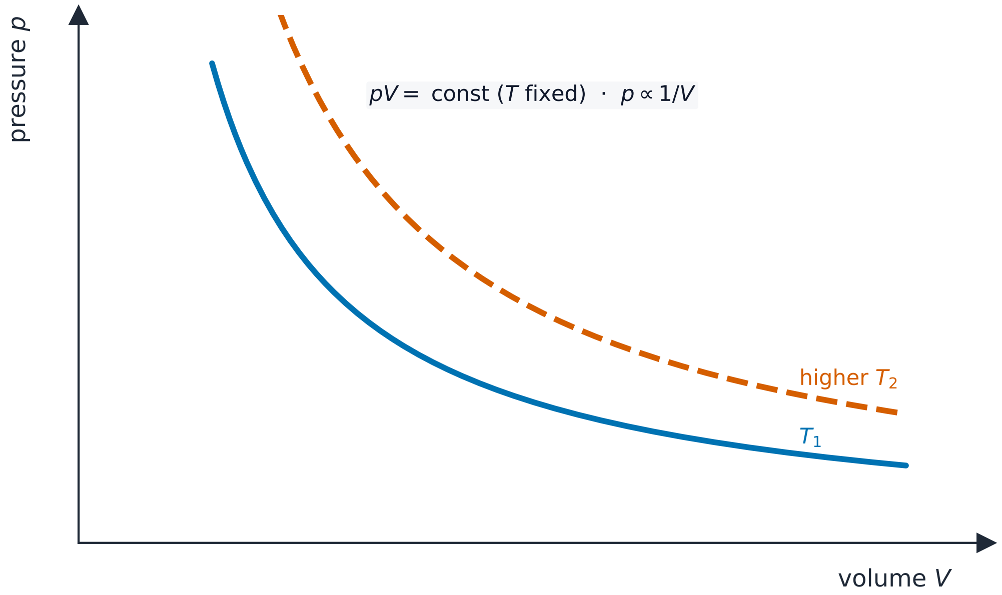
At constant pressure V/T is constant, so

$$V_2 = V_1 \frac{T_2}{T_1} = 0.50 \times \frac{450}{300} = 0.75 \text{ m}^3.$$

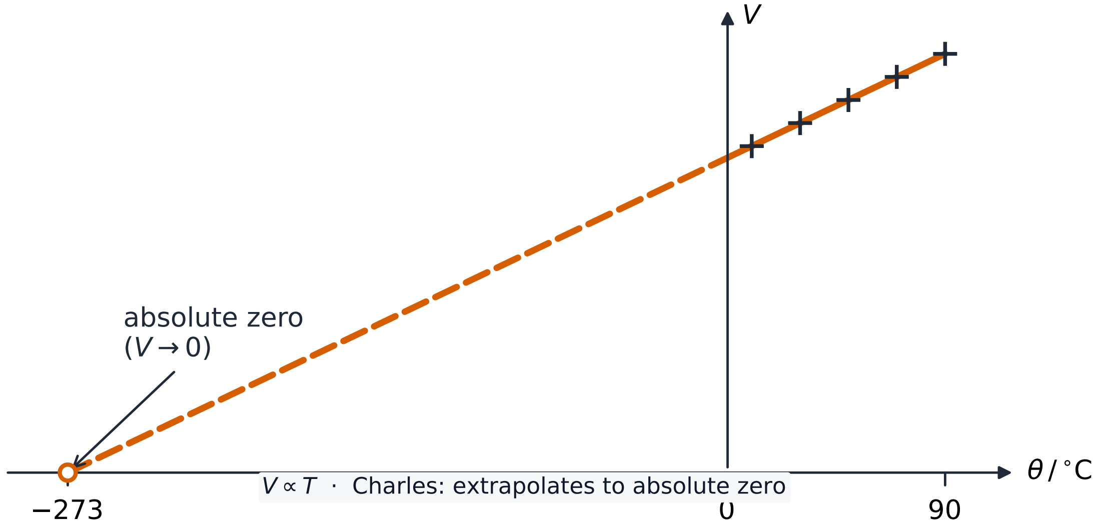
Special cases:

- **constant temperature** (**Boyle's law 玻意耳定律**): $p_1 V_1 = p_2 V_2$.
- **constant pressure** (**Charles's law 查理定律**): $V/T =$ constant.
- **constant volume** (**pressure law 气体压强定律**): $p/T =$ constant.

A common mistake is using °C instead of K — $pV \propto T$ only holds with T in kelvin.



Boyle's law: at constant temperature pV is constant, so a p–V graph is a hyperbola



At constant pressure the volume of a gas rises linearly with temperature, reaching zero at absolute zero (Charles's law)

Kinetic theory of gases

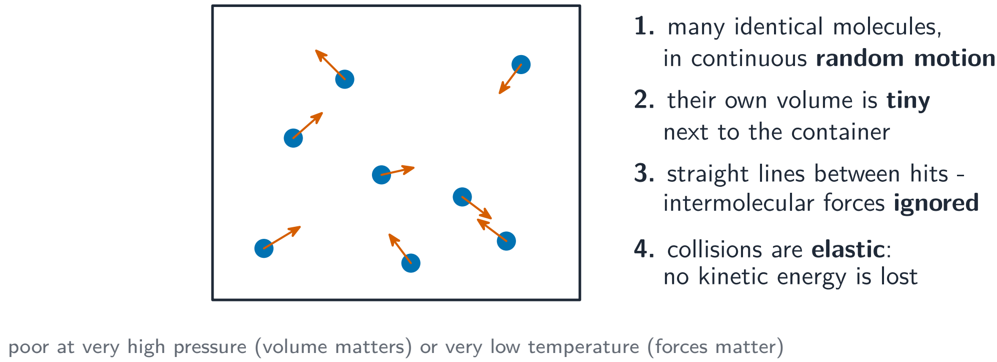
A scuba diver breathes compressed gas; its pressure, volume and temperature are all linked.

The **kinetic theory** 分子动理论 explains a gas's large-scale behaviour from the **random motion** 无规则运动 of its molecules.

Assumptions

For an ideal gas:

1. a **large number** of identical molecules in continuous random motion.
2. the molecules' own volume is too small to matter compared with the container.
3. the time of each collision is too short to matter compared with the time between collisions.
4. **intermolecular 分子间** forces are ignored except during collisions (molecules go in straight lines between them).
5. collisions (with the walls and with each other) are **elastic** — an **elastic collision 弹性碰撞** loses no kinetic energy, so the gas does not cool down by itself.
6. Newton's laws apply.

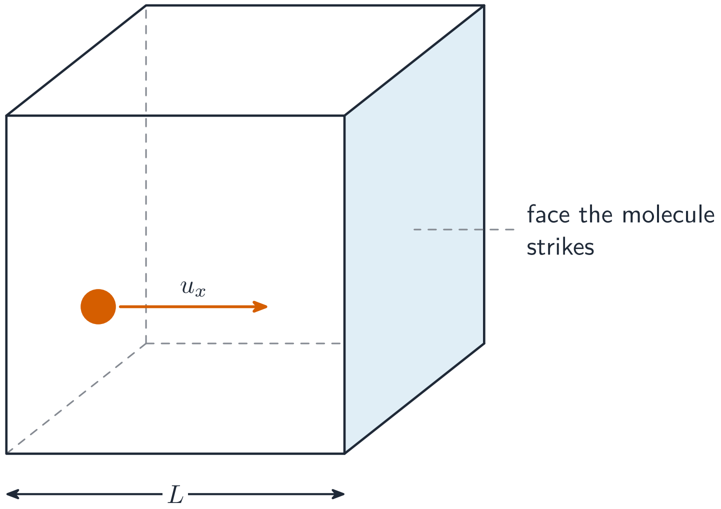


The key assumptions of the kinetic theory of an ideal gas

These assumptions become poor at very high pressure (molecular volume matters) or very low temperature (intermolecular forces matter).

Pressure of a gas — outline of the derivation

Take a cubic box of side L with N molecules, each of mass m. Look at one molecule moving along the x-axis with velocity u_1.



The pressure derivation considers one molecule's velocity component u_x normal to a face of a cube of gas

- **one collision with the right wall**: velocity reverses to $-u_1$, change in **momentum 动量** $\Delta p_x = -2mu_1$. By Newton's third law the wall gets an **impulse 冲量** of $+2mu_1$.
- **time between hits on that wall**: travel $2L$ there and back, so $\Delta t = 2L/u_1$.
- **average force from this molecule**: $F_1 = \Delta p/\Delta t = mu_1^2/L$.
- **add over all molecules**: $F = (Nm/L)\langle u_x^2\rangle$, where $\langle u_x^2\rangle$ is the **mean square 均方** of the x-velocity.
- **pressure**: $p = F/L^2 = Nm\langle u_x^2\rangle/V$.

In 3-D, by symmetry $\langle u_x^2\rangle = \frac{1}{3}\langle c^2\rangle$, where $\langle c^2\rangle$ is the **mean-square speed 均方速率**. So

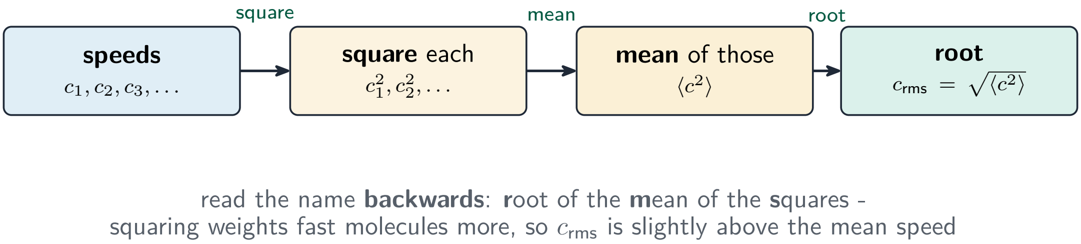
$$pV = \frac{1}{3}Nm\langle c^2\rangle.$$

Root-mean-square speed

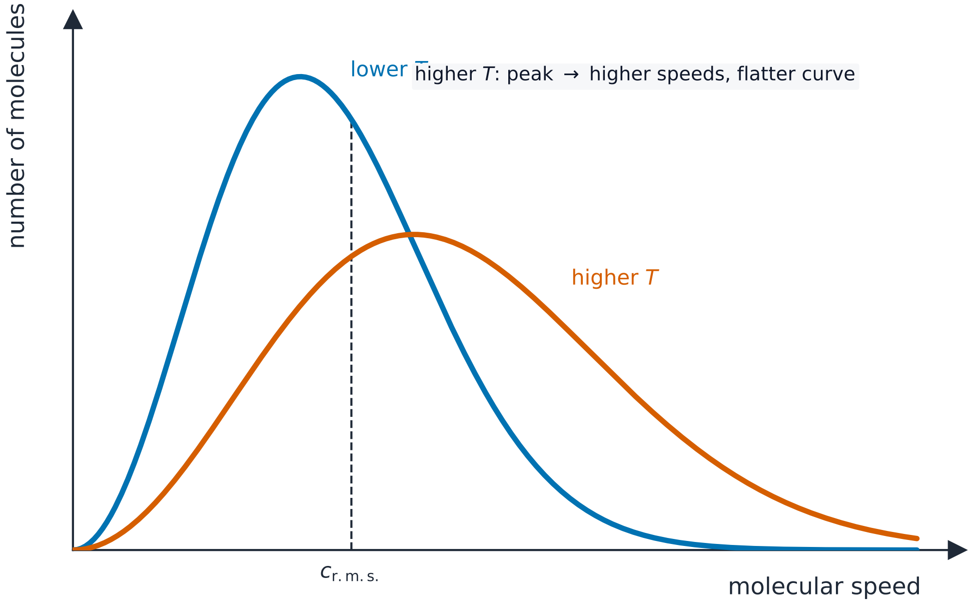
The square root of $\langle c^2 \rangle$ is the **root-mean-square** 均方根 (r.m.s.) speed:

$$c_{\text{r.m.s.}} = \sqrt{\langle c^2 \rangle}.$$

It is a useful single measure of how fast the molecules move, slightly larger than the mean speed (squaring weights fast molecules more).



Root-mean-square speed: square each speed, take the mean, then the square root



The spread of molecular speeds: a higher temperature broadens the curve and shifts it to faster speeds

Average translational kinetic energy

Compare the two expressions for pV:

$$pV = NkT \quad \text{and} \quad pV = \tfrac{1}{3}Nm\langle c^2\rangle.$$

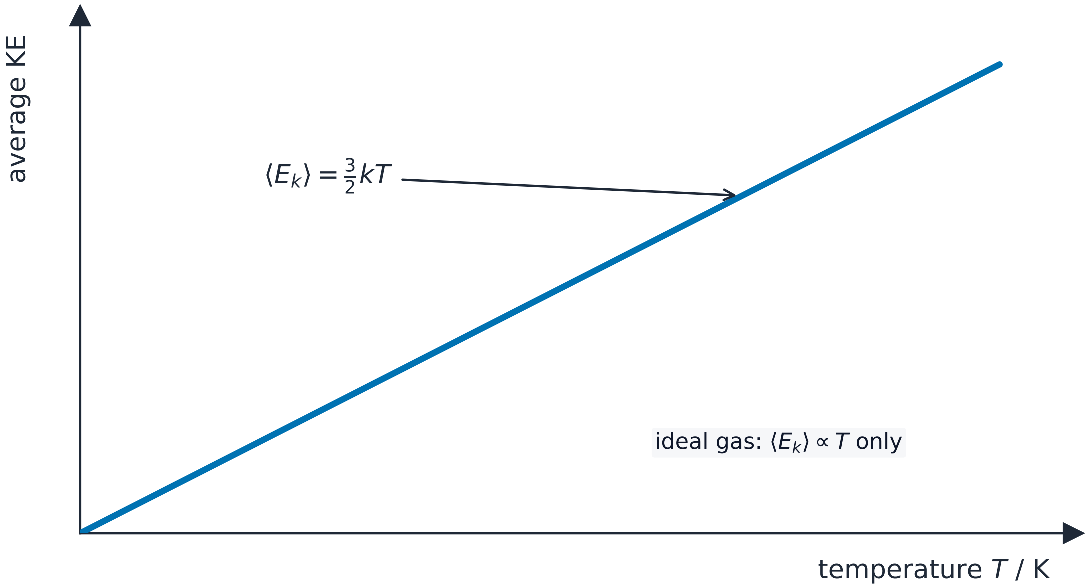
Set them equal, cancel N, and multiply by $\frac{3}{2}$:

$$\tfrac{3}{2}kT = \tfrac{1}{2}m\langle c^2\rangle.$$

The right side is the **average translational kinetic energy** 平动动能 $\langle E_\mathrm{k}\rangle$ of one molecule. So

$$\langle E_\mathrm{k}\rangle = \tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT.$$

This is a key result: **the average translational kinetic energy of an ideal-gas molecule depends only on the thermodynamic temperature**, not on the type of gas or its pressure.



Average molecular KE is proportional to thermodynamic temperature: $\langle E_k\rangle = \frac{3}{2}kT$

Worked example. Find the root-mean-square speed of oxygen molecules at 300 K. (Mass of one O_2 molecule $= 5.3 \times 10^{-26}$ kg, $k = 1.38 \times 10^{-23}$ J K^{-1}.)

From $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$, the mean-square speed is $\langle c^2\rangle = 3kT/m$:

$$c_{\text{r.m.s.}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23})(300)}{5.3 \times 10^{-26}}} \approx 480 \text{ m s}^{-1}.$$

Consequences

- **doubling the absolute temperature doubles the average KE** of each molecule, so $\langle c^2 \rangle$ doubles and $c_{\text{r.m.s.}}$ grows by $\sqrt{2}$.
- for **two gases at the same temperature**, the lighter gas has a larger $\langle c^2 \rangle$. Hydrogen molecules move faster on average than oxygen molecules in the same room.
- **total translational KE** of N molecules: $\frac{3}{2}NkT = \frac{3}{2}nRT$.

Internal energy of an ideal gas

For an ideal gas the molecules are point particles with no intermolecular potential energy and (in this simple model) no rotation or vibration. So the **internal energy** 内能 is just the total **kinetic energy** 动能:

$$U = \tfrac{3}{2}NkT = \tfrac{3}{2}nRT.$$

So the internal energy of an ideal gas is proportional to the thermodynamic temperature — doubling T doubles U.

Changing pressure at fixed temperature

Doubling p at fixed T (by squeezing the gas to half its volume) does **not** change $\langle E_\text{k} \rangle$ — that depends only on T. There are more wall collisions per second, but each molecule has the same average kinetic energy.

Exam tips

- Use $pV = nRT$ (n in mol) or $pV = NkT$ (N molecules), with temperature in **kelvin**.
- Learn the **kinetic-theory assumptions** (random motion, negligible molecular volume, elastic collisions, no intermolecular forces).
- Mean translational KE $= \frac{3}{2}kT$ — it depends **only on temperature**.

Thermodynamics

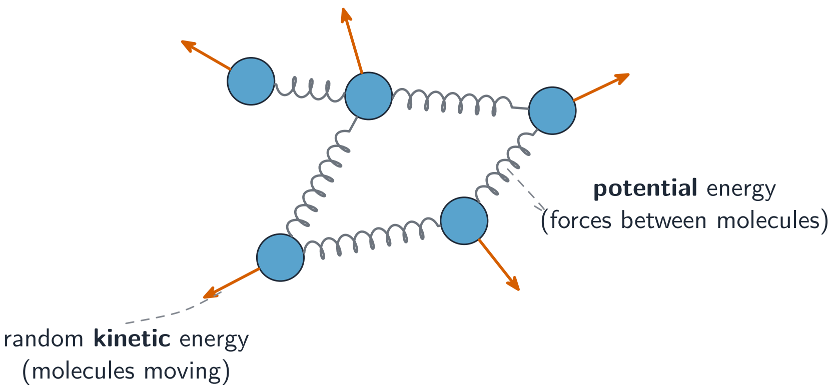
A-Level Physics

Internal energy

The **internal energy** 内能 U of a system is the sum of:

- the **random kinetic energies** 动能 of its **molecules** 分子 — they fly through space (**translational** 平动 motion), and unless they are single atoms they also spin (**rotational** 转动) and shake (**vibrational** 振动), and
- the **potential energies** from the forces between the molecules.

For a real solid, liquid or gas, both parts matter. In the **ideal-gas** 理想气体 model the **intermolecular** 分子间 forces are ignored, so the molecular potential energy is zero and the internal energy is purely kinetic.



Internal energy is the molecules' random kinetic energy (the arrows) plus the potential energy of the forces between them (the springs)

Two key points:

1. U **depends only on the state** of the system (its **temperature** 温度, **pressure** 压强, **volume** 体积, **amount of substance** 物质的量) — not on the path taken to get there.
2. U **is a sum over the molecules**, not the kinetic energy of the whole object moving. A moving train of gas has bulk kinetic energy, but that is separate from U — U is the energy of the *random* molecular motion.

Temperature and internal energy

Raising an object's temperature raises the random kinetic energy of its molecules, and so raises its internal energy.

For an ideal gas every molecule has average translational kinetic energy $\frac{3}{2}kT$ (Topic 15). With zero intermolecular potential energy, the total internal energy is

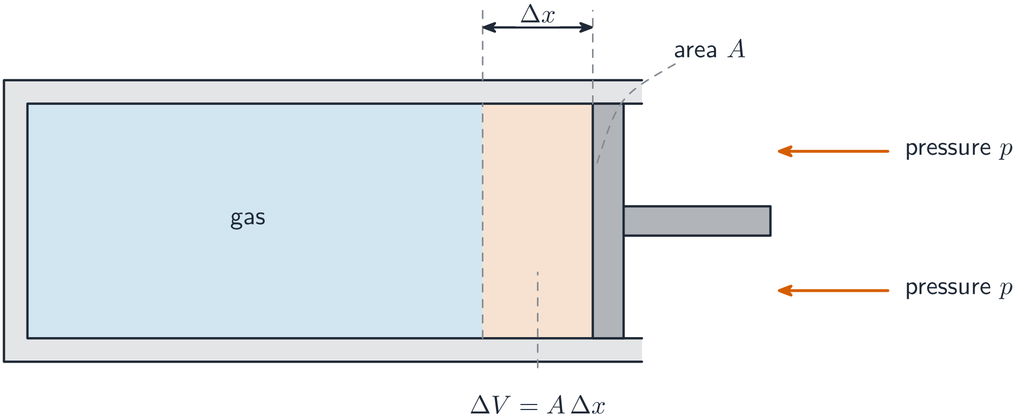
$$U = \frac{3}{2}NkT = \frac{3}{2}nRT.$$

So the internal energy of an ideal gas is directly proportional to the **thermodynamic temperature** 热力学温度. Doubling T doubles U. This is **only** exact for an ideal gas.

During a **phase change** 相变 (melting or boiling) of a real substance, U rises because the molecular potential energy rises (bonds breaking), even though the temperature stays constant.

Work done on or by a gas

A steam turbine does work as expanding steam pushes its blades around.



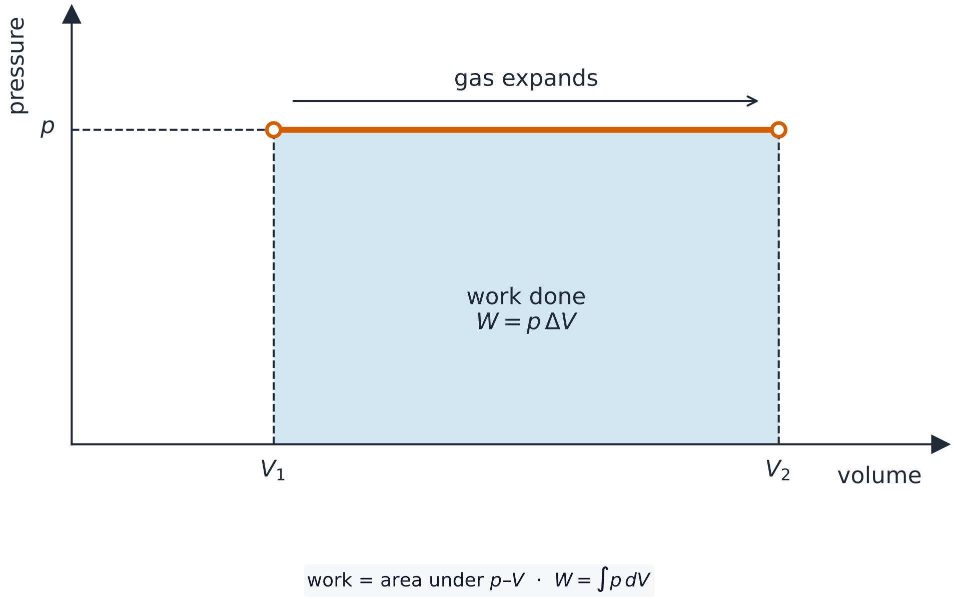
A gas pushing a piston of area A out by Δx does work $W = p\,\Delta V$ on the surroundings (swept volume $\Delta V = A\,\Delta x$)

When a gas changes volume against an outside pressure, mechanical work is done. At constant pressure p with a small volume change ΔV, the size of the work is

$$W = p\Delta V.$$

Worked example. A gas at a constant pressure of 1.0×10^5 Pa expands from 2.0×10^{-3} m^3 to 5.0×10^{-3} m^3. Find the work done by the gas.

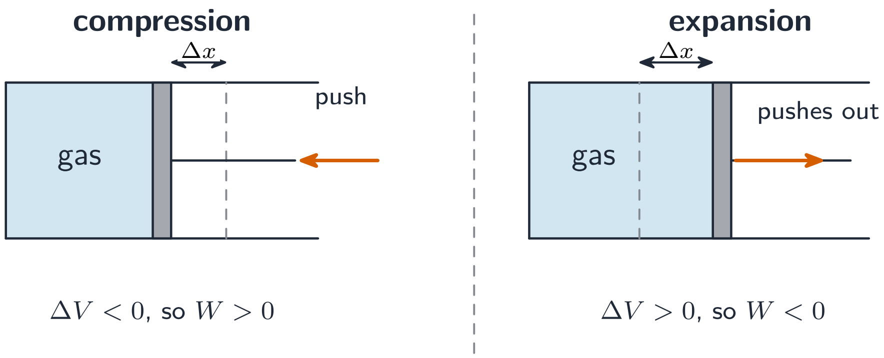
$$W = p\,\Delta V = (1.0\times10^5)(5.0\times10^{-3} - 2.0\times10^{-3}) = 300\ \text{J}.$$



On a pressure–volume graph, the work done at constant pressure is the area under the line: $W = p\,\Delta V$

Sign convention in this syllabus

This syllabus writes the first law as $\Delta U = q + W$, where W is the work done **on** the gas and q is the energy put in **by heating**.



Work done on the gas: positive when it is compressed, negative when it expands

- when the gas is **compressed**, ΔV is negative and the work done **on** the gas is positive — the gas gains energy.

- when the gas **expands**, ΔV is positive and the work done **on** the gas is negative — the gas loses energy (it does work on the surroundings).

Watch which form a question wants:

- "work done **on** the gas" — positive when compressing.
- "work done **by** the gas" — the opposite sign, positive when expanding.

At constant volume ($\Delta V = 0$), no work is done.

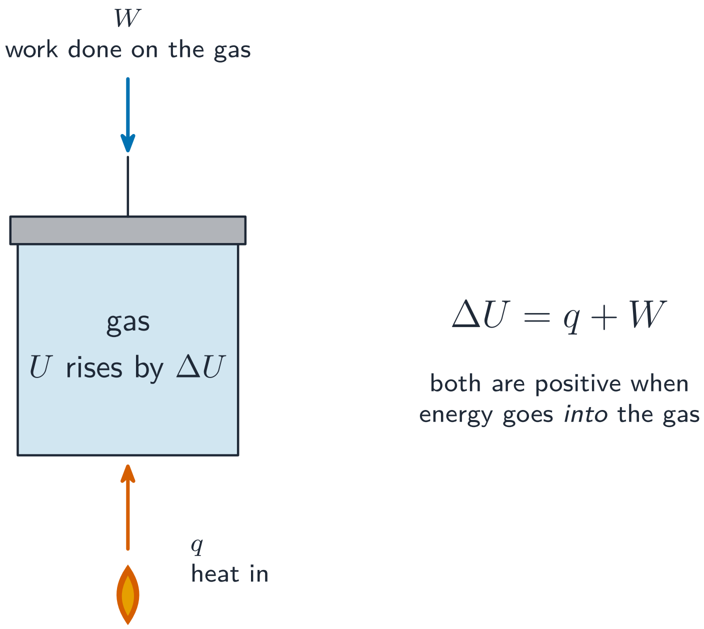
First law of thermodynamics

A power station is a heat engine: it converts heat into useful work.

The **first law of thermodynamics** 热力学第一定律 says that **energy is conserved when heat and work pass between a system and its surroundings**:

$$\Delta U = q + W,$$

where ΔU is the rise in internal energy, q is the energy added **by heating** (positive in, negative out), and W is the work done **on** the gas (positive when compressed). This is **conservation of energy** 能量守恒 for a gas.



Both heating (q) and work done on the gas (W) put energy in, raising the internal energy by ΔU

Worked example. A gas absorbs 500 J of heat while it expands and does 200 J of work on its surroundings. Find the change in its internal energy.

The gas does work, so the work done *on* it is $W = -200$ J:

$$\Delta U = q + W = 500 + (-200) = 300 \text{ J}.$$

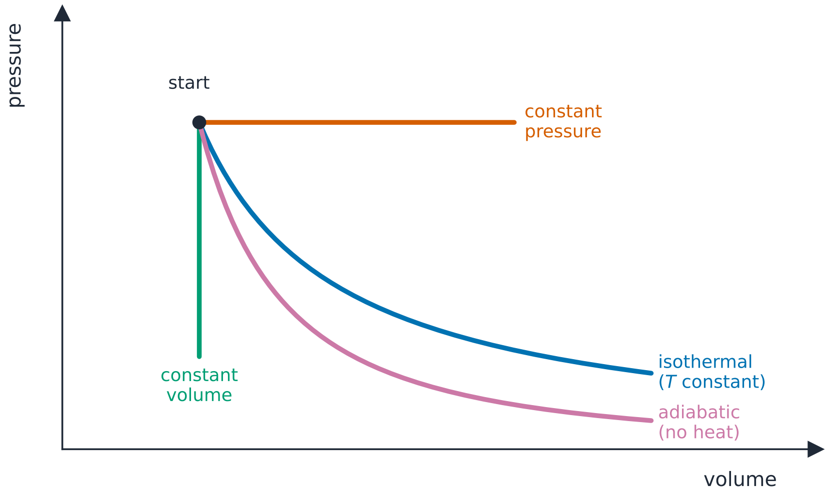
Reading the equation

ΔU is fixed by the change of state (for an ideal gas, by the change in temperature). The same ΔU can come from different mixes of q and W:

- all heat, no work: $\Delta U = q$ (constant-volume heating).
- all work, no heat: $\Delta U = W$ (insulated compression or expansion).

Standard processes

For an ideal gas, $\Delta U = \frac{3}{2}nR\Delta T$ — it depends only on ΔT.



isothermal, isobaric, isometric on p–V plane

The four standard processes, all starting from the same state

Process	**What stays constant**	ΔU	W **(on gas)**	q
Isothermal	T	0	W	$-W$
Constant volume	V	$\frac{3}{2}nR\Delta T$	0	ΔU
Constant pressure	p	$\frac{3}{2}nR\Delta T$	$-p\Delta V$	$\Delta U - W$
Adiabatic	(no heat)	varies	W	0

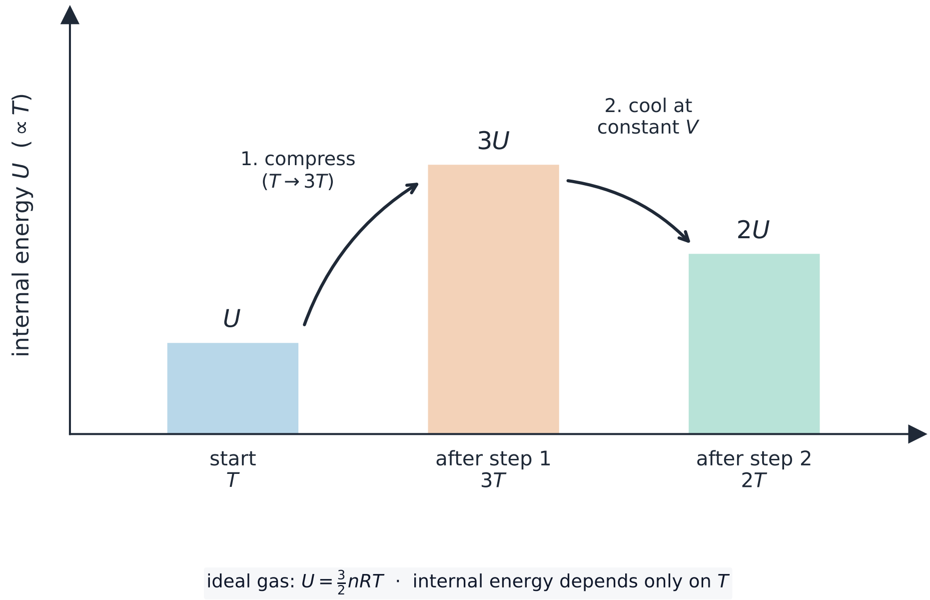
Read each row with the first law $\Delta U = q + W$:

- **Isothermal** 等温 (constant T): $\Delta T = 0$, so $\Delta U = 0$. Then $q = -W$ — any heat that goes in comes straight back out as work.
- **Adiabatic** 绝热 (no heat flow): $q = 0$, so $\Delta U = W$. The gas warms up only because work is done on it.
- **Constant volume** (sealed rigid container): no work is done ($\Delta V = 0$), so all the heat goes into internal energy: $q = \Delta U$.
- **Constant pressure** (gas pushing a **piston** 活塞): the gas does work as it expands, so the heat you supply does two jobs — it raises the internal energy *and* does the expansion work.

Worked example: two-step process

A sample of ideal gas at temperature T with internal energy U goes through:

1. **compression** to temperature $3T$; work W is done on the gas.
2. **cooling at constant volume** to temperature $2T$.



Internal energy tracks temperature: $U \to 3U$ on compressing to $3T$, then $3U \to 2U$ on cooling to $2T$

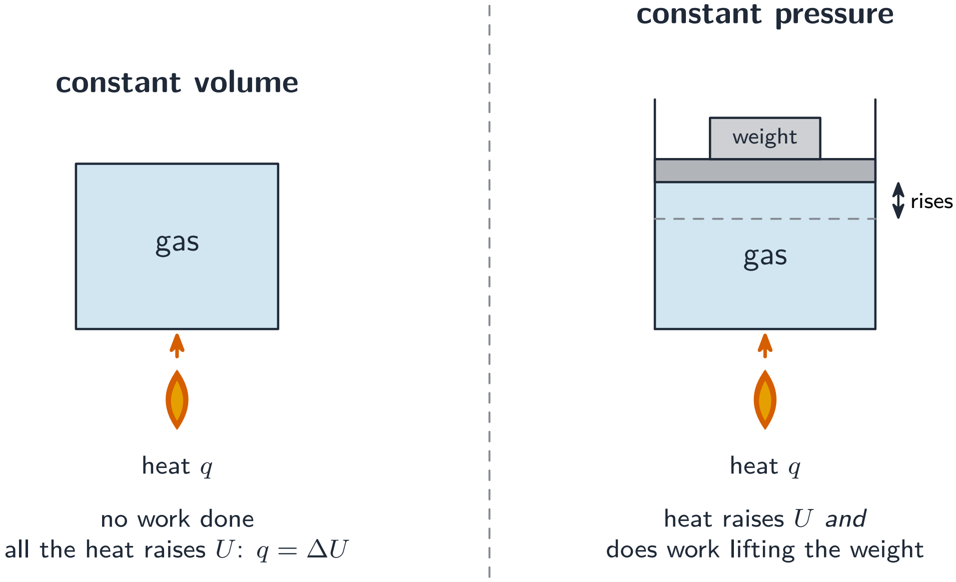
Step 1 ($T \to 3T$): $U = \frac{3}{2}nRT$, so $U \to 3U$, giving $\Delta U_1 = 2U$. $W_1 = +W$. So $q_1 = \Delta U_1 - W_1 = 2U - W$.

Step 2 ($3T \to 2T$, constant volume): $\Delta U_2 = -U$. $W_2 = 0$. So $q_2 = -U$ (heat flows out).

Check: total $\Delta U = 2U - U = U$, taking the gas from T to $2T$ ($U \to 2U$) — consistent.

Heat capacity at constant volume

For constant-volume heating of an ideal gas, $q = \Delta U = \frac{3}{2}nR\Delta T$. So the molar **heat capacity** 热容 at constant volume is $\frac{3}{2}R$ for a **monatomic** 单原子 ideal gas. (You are not required to use the symbol C_V, but the result $q = \frac{3}{2}nR\Delta T$ for constant-volume heating is.)



At constant volume all the heat raises U ($q = \Delta U$); at constant pressure the gas also does work

Heating without a temperature change

If heat is supplied during a phase change at constant pressure (e.g. boiling water), the temperature stays constant but the internal energy still rises (the **latent heat** 潜热 separates the molecules), and the gas does expansion work. The first law still holds: $\Delta U = q + W$.

Exam tips

- **First law:** $\Delta U = q + W$ — be careful with the sign convention (W is the work done **on** the gas).
- For an ideal gas, **internal energy depends only on temperature** ($\Delta U \propto \Delta T$).
- Work done by a gas at constant pressure $= p\Delta V$; read the sign from expansion (by) or compression (on).

Oscillations

A-Level Physics

Simple harmonic motion: definition

A pendulum clock keeps time using simple harmonic motion.

A particle moves with **simple harmonic motion** 简谐运动 (SHM) when its **acceleration** 加速度 is:

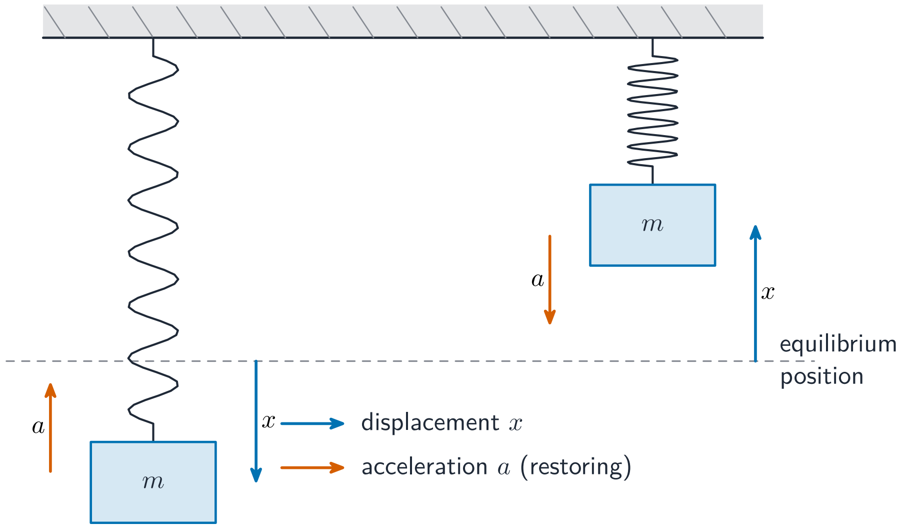
- **proportional to its displacement** from a fixed **equilibrium** 平衡 point, and
- **directed back towards that point** — opposite in sign to the displacement.

The defining equation is

$$a = -\omega^2 x,$$

where x is the **displacement** 位移 from equilibrium and ω is a positive constant, the **angular frequency** 角频率. The minus sign means "directed back towards equilibrium".

Many systems do this near a stable equilibrium: a mass on a **spring** 弹簧, a **pendulum** 单摆 (small swing), a floating block pushed down, the charge on a **capacitor** 电容器 in an LC circuit, atoms in a solid.



Acceleration always points back towards equilibrium, opposite to the displacement

Key terms

- **displacement** x — distance from equilibrium at a moment (a vector along the line of motion).
- **amplitude** 振幅 x_0 — the largest displacement from equilibrium. Always positive.
- **period** 周期 T — the time for one full oscillation.
- **frequency** 频率 f — the number of oscillations per second; $f = 1/T$. Unit: Hz.
- **angular frequency** ω — $\omega = 2\pi/T = 2\pi f$. Unit: rad s^{-1}.
- **phase difference** 相位差 — the fraction of a cycle (in radians) by which one oscillation leads or lags another. A quarter-cycle apart is a phase difference of $\pi/2$.

So $T = 2\pi/\omega$ and $f = \omega/(2\pi)$ — given any one of ω, f, T you can find the others.

Worked example. A mass on a spring oscillates with SHM of amplitude 0.050 m and frequency 2.5 Hz. Find its maximum acceleration.

The angular frequency is $\omega = 2\pi f = 2\pi \times 2.5 = 15.7$ rad s^{-1}. The acceleration is largest at the extremes, where $|a| = \omega^2 x_0$:

$$a_{\max} = \omega^2 x_0 = 15.7^2 \times 0.050 \approx 12 \text{ m s}^{-2}.$$

Displacement, velocity, acceleration in SHM

If the particle starts at $x = 0$ moving in the positive direction at $t = 0$, then

$$x = x_0 \sin(\omega t).$$

Differentiating once gives the **velocity 速度**:

$$v = x_0\omega \cos(\omega t) = v_0 \cos(\omega t),$$

where $v_0 = x_0\omega$ is the **maximum speed** (as the particle passes through equilibrium).

Differentiating again gives the acceleration:

$$a = -x_0\omega^2 \sin(\omega t) = -\omega^2 x,$$

which is the SHM defining equation again. (If the particle instead starts at the extreme position $x = x_0$ at $t = 0$, use $x = x_0 \cos(\omega t)$. Choose the one that fits the start conditions.)

Velocity in terms of displacement

A useful relation that does not use time:

$$v = \pm\omega\sqrt{x_0^2 - x^2}.$$

- at equilibrium ($x = 0$): $v = \pm\omega x_0$ (maximum speed). Both signs, because the particle passes through equilibrium twice each cycle.
- at the extremes ($x = \pm x_0$): $v = 0$ (at rest for an instant).

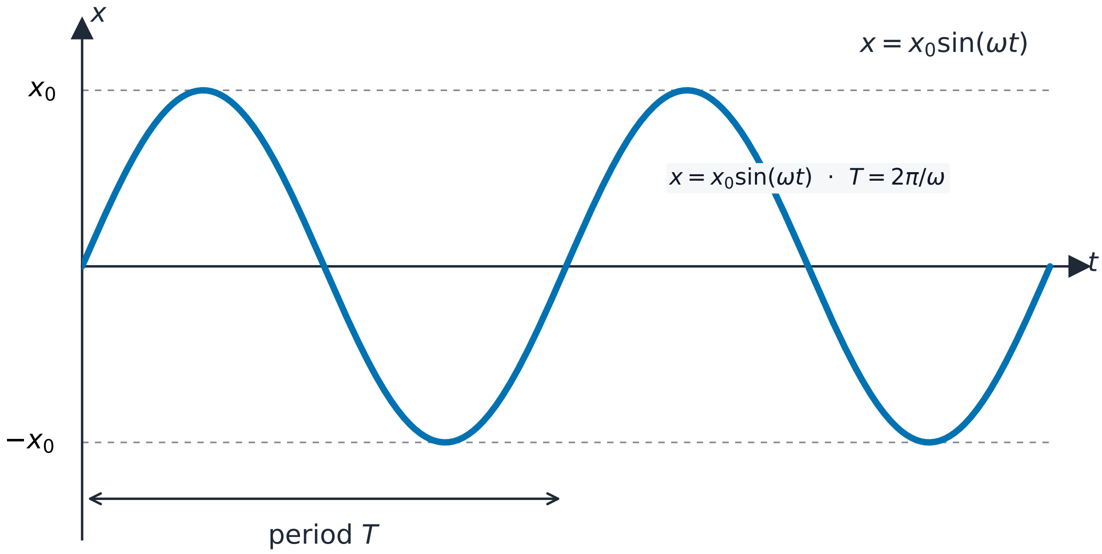
Worked example. The same oscillation has amplitude $x_0 = 0.050$ m and angular frequency $\omega = 15.7$ rad s^{-1}. Find the speed when the displacement is $x = 0.030$ m.

$$v = \omega\sqrt{x_0^2 - x^2} = 15.7\sqrt{0.050^2 - 0.030^2} = 15.7 \times 0.040 \approx 0.63 \text{ m s}^{-1}.$$

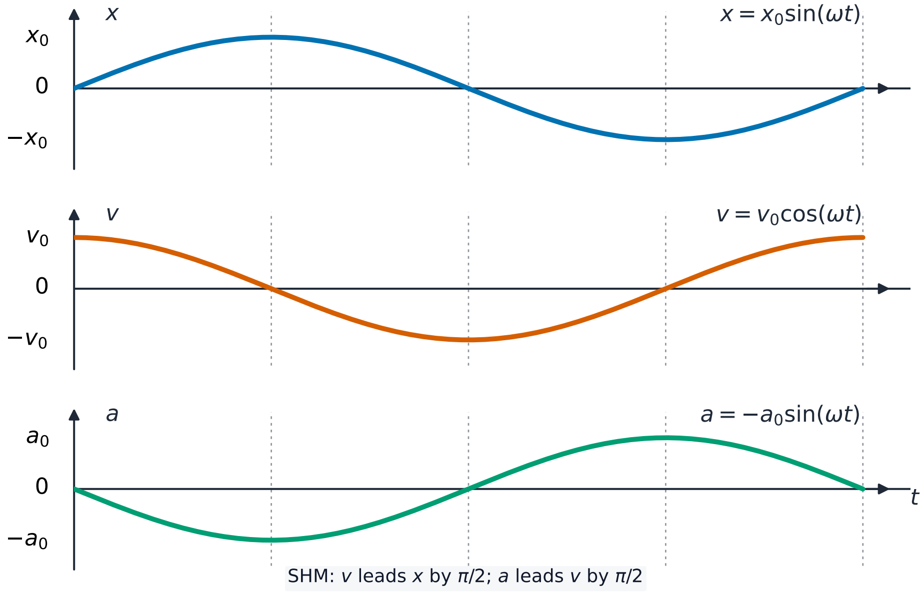
Graphs against time

For $x = x_0 \sin \omega t$:

- x vs t — a sine curve, amplitude x_0, period $T = 2\pi/\omega$.
- v vs t — a cosine curve, **leading** x **by** $\pi/2$, amplitude ωx_0.
- a vs t — a negative sine curve, **out of phase with** x **by** π (180°), amplitude $\omega^2 x_0$.



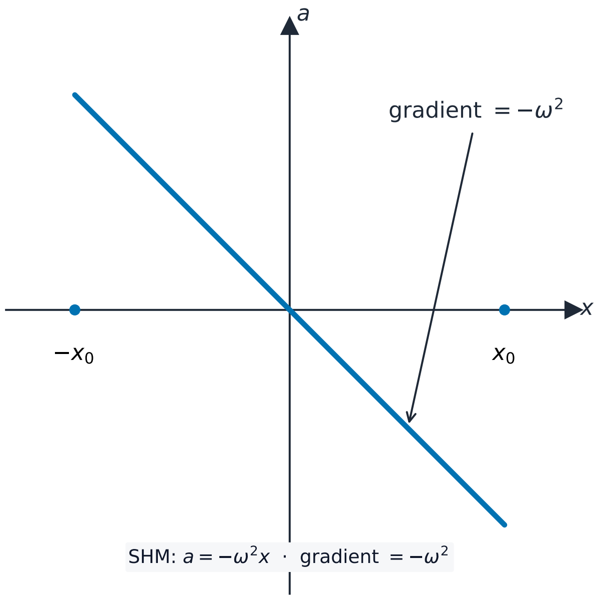
Displacement varies sinusoidally with time in simple harmonic motion



Velocity leads displacement by a quarter cycle; acceleration is exactly out of phase with displacement

Graph of a against x

A straight line through the origin with **negative gradient** $-\omega^2$. So you can read ω from the graph: gradient $= -\omega^2$, so $\omega = \sqrt{|\text{gradient}|}$, then $T = 2\pi/\omega$. This is a common exam pattern.



The acceleration–displacement graph is a straight line through the origin with gradient $-\omega^2$

Energy in simple harmonic motion

A simple harmonic oscillator keeps swapping energy between two forms:

- **kinetic energy** 动能 $E_K = \frac{1}{2}mv^2$.
- **potential energy** E_P (elastic for a spring, gravitational for a pendulum).

With no **damping** 阻尼, the **total energy** is constant (this is **conservation of energy** 能量守恒).

Maximum and minimum

- at **equilibrium** ($x = 0$): v is largest, so E_K is largest and E_P is smallest (zero, by choice).
- at the **extremes** ($x = \pm x_0$): $v = 0$, so $E_K = 0$ and E_P is largest.

Total energy

Using $v_{\max} = \omega x_0$:

$$E_{\text{total}} = \tfrac{1}{2}mv_{\max}^2 = \tfrac{1}{2}m\omega^2 x_0^2.$$

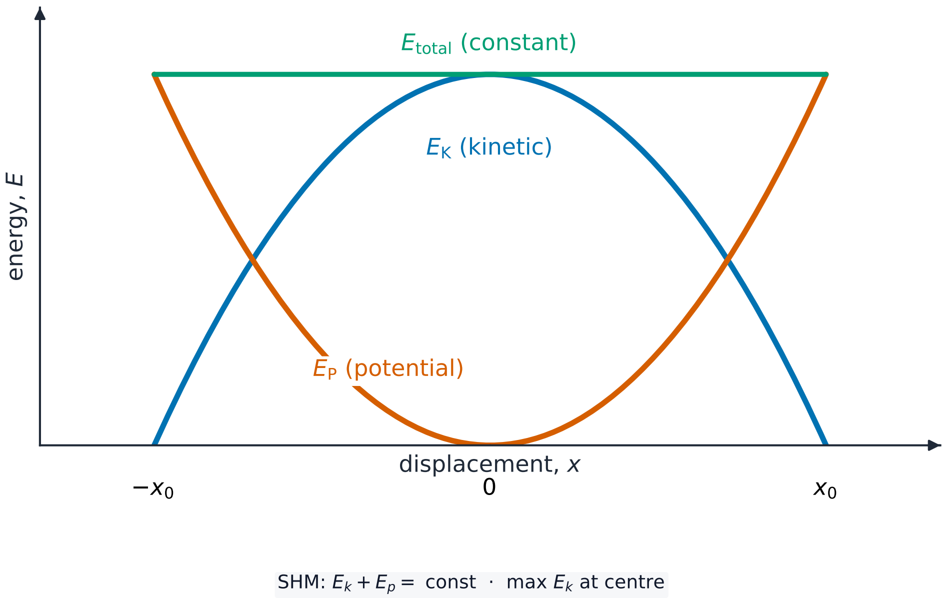
Two key facts: the total energy is proportional to the **square of the amplitude** (doubling x_0 gives four times the energy), and to ω^2.

Energy against displacement

Using $v^2 = \omega^2(x_0^2 - x^2)$:

$$E_{\text{K}} = \tfrac{1}{2}m\omega^2(x_0^2 - x^2), \qquad E_{\text{P}} = \tfrac{1}{2}m\omega^2 x^2.$$

So E_{K} is a downward parabola (peak at $x = 0$, zero at $x = \pm x_0$) and E_{P} is an upward parabola (zero at $x = 0$, largest at $x = \pm x_0$). Their sum is constant.



Kinetic and potential energy swap over a cycle while the total energy stays constant

Worked example. A 0.20 kg mass oscillates with amplitude $x_0 = 0.050$ m and angular frequency $\omega = 15.7$ rad s^{-1}. Find the total energy of the oscillation.

The total energy equals the maximum kinetic energy, as the mass passes through $x = 0$ at $v_{\max} = \omega x_0$:

$$E = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2}(0.20)(15.7)^2(0.050)^2 \approx 0.062 \text{ J}.$$

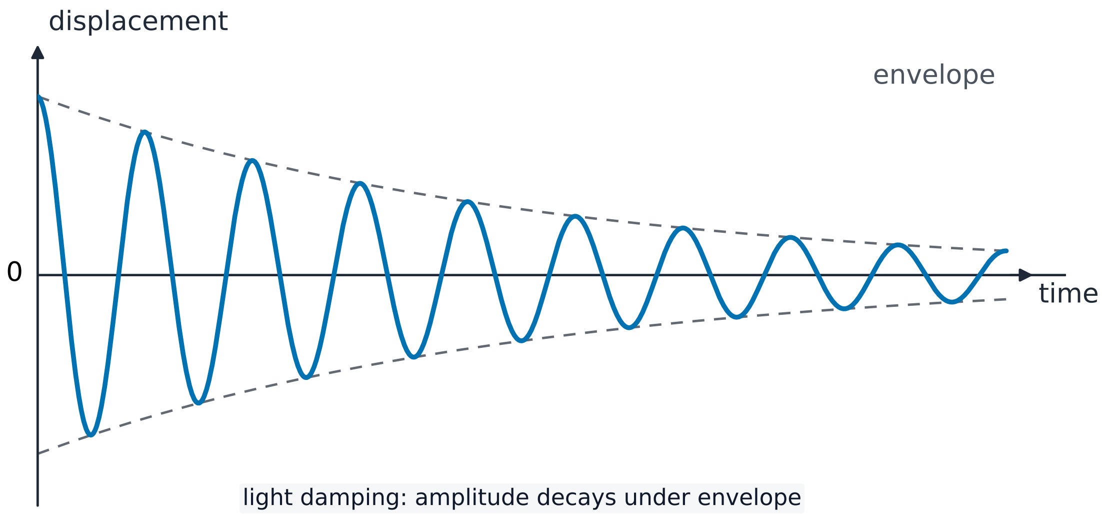
This stays constant, swapping between kinetic and potential form twice each cycle.

Damped oscillations

A resistive force (**friction** 摩擦力, **drag** 阻力, **air resistance** 空气阻力) causes **damping** — the amplitude shrinks over time as energy is lost as heat. Three named cases:

Light damping

The amplitude **shrinks slowly** over many cycles (a **light damping** 轻阻尼 case). The system still oscillates near its natural frequency, but each cycle is smaller than the last. A car's suspension is light-to-medium damped, so bumps die away but the ride stays smooth.



In light damping the amplitude dies away slowly over many cycles

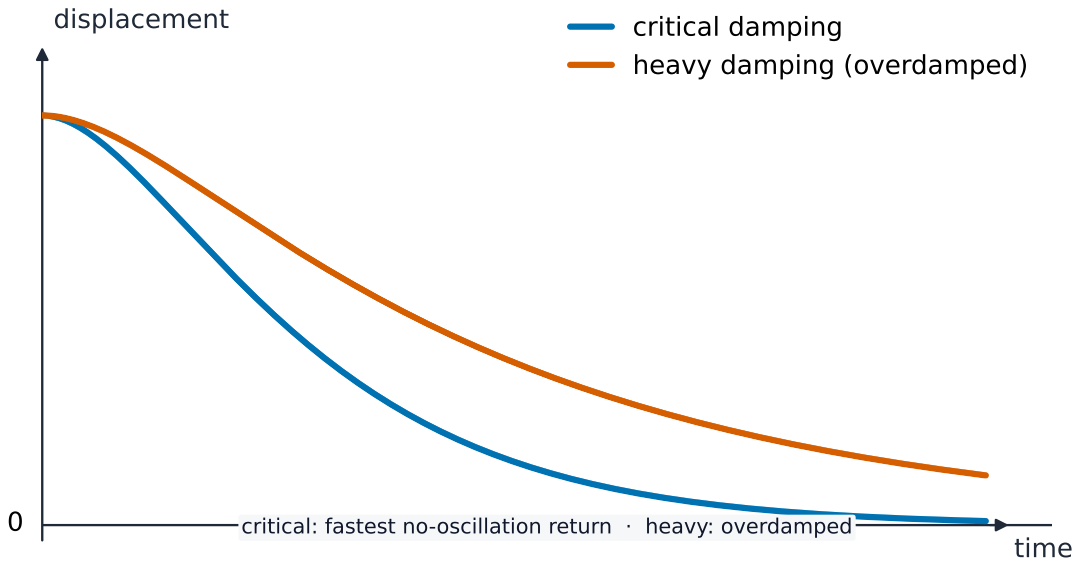
Critical damping

The least damping that brings the system back to equilibrium **without overshooting and without oscillating** — a **critical damping** 临界阻尼 case. It returns in the shortest time. A **galvanometer** 检流计 or analogue **voltmeter** 电压表 is critically damped so the needle settles quickly.

Heavy damping

So much resistance that the system returns slowly, with no oscillation, but more slowly than the critical case — a **heavy damping** 过阻尼 case. A door with a strong closer is heavily damped.

On a displacement–time graph: light damping is a wave whose size dies away smoothly; critical damping returns quickly with no overshoot; heavy damping returns slowly.



Critical damping returns to equilibrium fastest without overshoot; overdamping returns more slowly

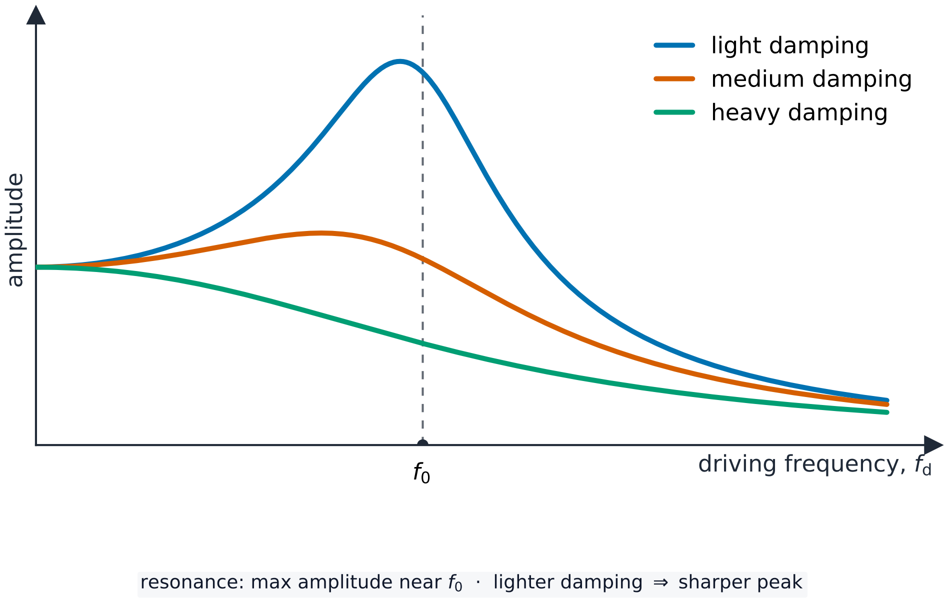
Forced oscillations and resonance

A plucked guitar string vibrates at its resonant frequencies.

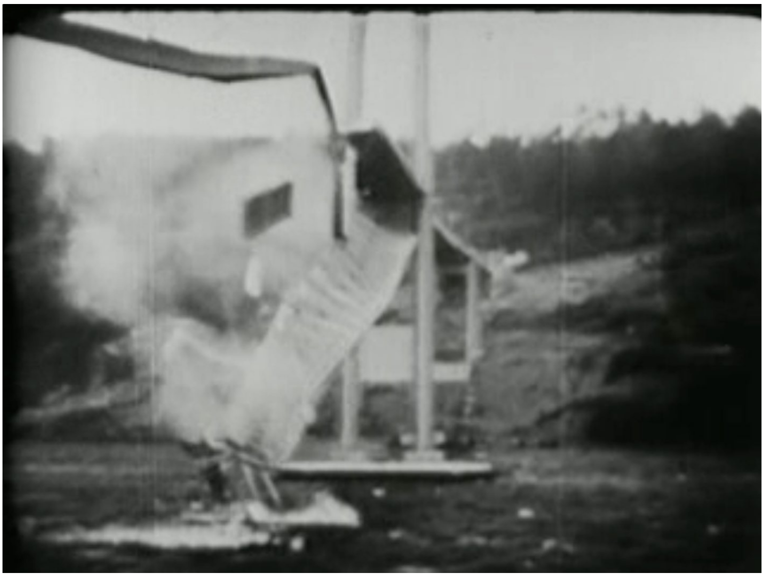
A **forced oscillation** 受迫振动 is driven by an outside periodic force at a frequency f_d chosen by the experimenter. The system then oscillates at this **driving frequency** 驱动频率 f_d, not at its own natural frequency. A plot of amplitude against f_d is a **resonance curve** 共振曲线 with a peak.

Resonance

Resonance 共振 happens when the driving frequency equals the system's **natural frequency** 固有频率 f_0. At resonance the amplitude is **largest** and the energy transfer from the driver is most efficient.



The amplitude of a forced oscillation peaks at resonance, when the driving frequency equals the natural frequency



Resonance can destroy. In 1940 the wind pushed the Tacoma Narrows Bridge close to its natural frequency; with little damping the twisting grew and grew until the deck ripped apart. Engineers now design bridges and buildings so their natural frequencies avoid such driving forces

Examples:

- a swing pushed at the right rate builds up a large amplitude.
- a wine glass broken by a sound at its natural ringing frequency.
- a building shaken by an earthquake whose frequency matches a natural frequency — engineers design buildings so their natural frequencies avoid the main earthquake range.

The peak's shape depends on damping: **lighter damping → a sharper, higher peak**; **heavier damping → a broader, lower peak**, shifted slightly to lower frequency.

Exam tips

- The **SHM condition is** $a = -\omega^2 x$ (acceleration proportional to displacement, directed back to equilibrium).
- Learn $x = x_0 \sin \omega t$ (or cos), $v_{max} = \omega x_0$, $a_{max} = \omega^2 x_0$; KE and PE interchange with the **total energy constant**.
- Velocity is **zero at the extremes** and maximum at the centre.
- **Resonance** occurs when the driving frequency equals the natural frequency; damping lowers and broadens the peak.

Electric fields

A-Level Physics

Electric fields

Lightning is a giant spark driven by a huge electric field.

An **electric field** 电场 is a region where a charge feels a **force** 力 from other charges. The **electric field strength** 电场强度 E at a point is the **force per unit positive charge** on a small positive **test charge** 检验电荷 placed there:

$$E = \frac{F}{q}.$$

Unit: N C^{-1} (the same as V m^{-1}, as we will see). E is a **vector** 矢量, pointing the way the force acts on a **positive** charge. The force on a charge q is

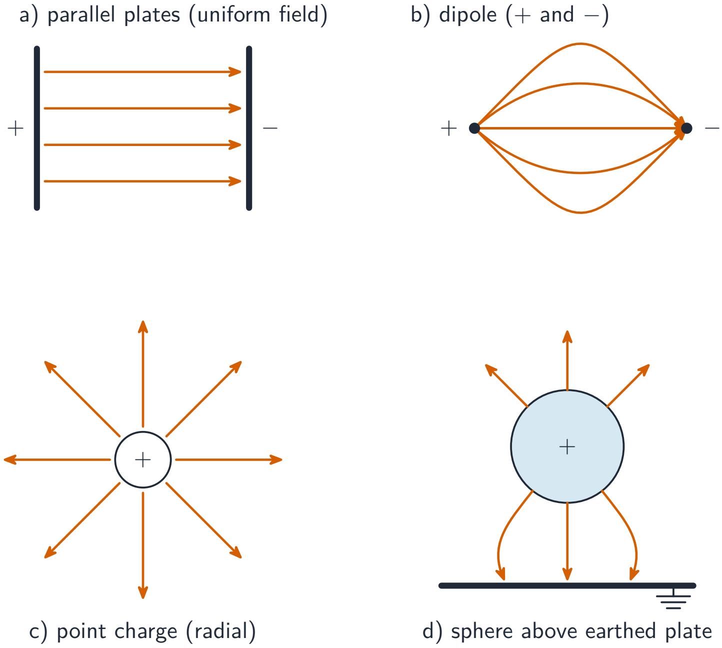
$$F = qE,$$

opposite to the field if q is negative.

Field lines

- **field lines** 场线 point the way the force acts on a positive test charge.
- lines start on **positive** charges and end on **negative** charges (or go to infinity).
- lines never cross; closer lines mean a stronger field.

Examples: a positive **point charge** 点电荷 has radial lines pointing out; a negative one has lines pointing in; two opposite charges (a **dipole** 偶极子) have lines curving from + to −; two parallel charged plates give a **uniform field** 匀强场 of equally spaced parallel lines.

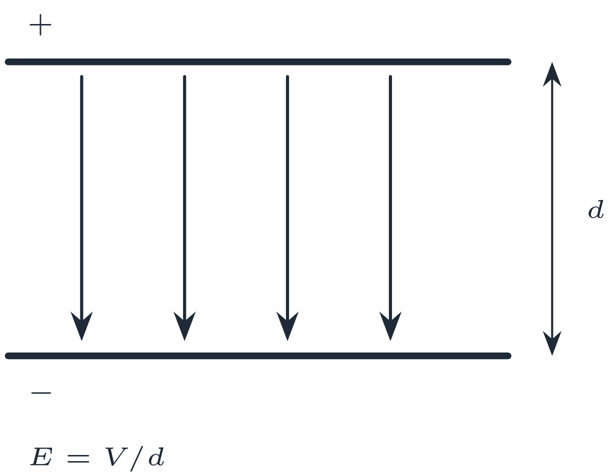


Field-line patterns for parallel plates, a dipole, a point charge, and a charged sphere above an earthed plate

A Van de Graaff generator stores a large static charge on its metal dome, making a strong electric field around it

Uniform electric fields

Between two parallel plates a distance d apart with **potential difference 电势差** V between them, the field is uniform (apart from edge effects) with size



Between parallel plates the field is uniform, $E = V/d$

$$E = \frac{V}{d}.$$

It points from the higher-potential plate to the lower one. The unit V m^{-1} comes straight from this and equals N C^{-1}.

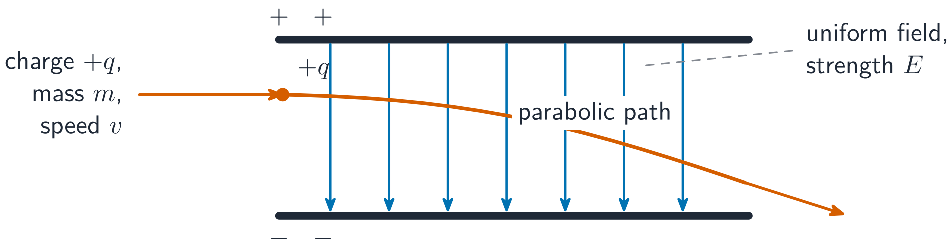
Worked example. Two parallel plates 5.0 mm apart have a p.d. of 200 V between them. Find the field strength, and the force on an electron in the gap. ($e = 1.6 \times 10^{-19}$ C.)

$$E = \frac{V}{d} = \frac{200}{5.0 \times 10^{-3}} = 4.0 \times 10^4 \text{ V m}^{-1}, \qquad F = qE = (1.6 \times 10^{-19})(4.0 \times 10^4) = 6.4 \times 10^{-15} \text{ N}.$$

A charged particle in a uniform field

A charge q in a uniform field feels a **constant force** $F = qE$, so a **constant acceleration** $a = qE/m$ — just like a mass in a uniform gravitational field.

- released **at rest**, it speeds up along the field (positive charge) or against it (negative charge), gaining **kinetic energy** 动能.
- entering **at right angles** to the field, it follows a **parabolic** 抛物线 path — like a **projectile** 抛体 in gravity. This is how a **cathode-ray tube** 阴极射线管 used to steer its beam.



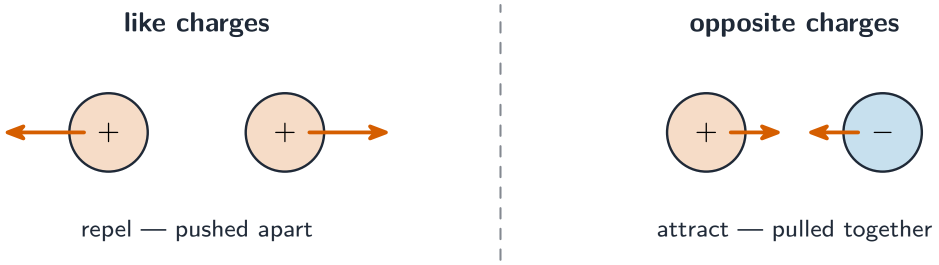
A charge entering a uniform field at right angles follows a parabolic path, like a projectile

Coulomb's law

For two point charges Q_1 and Q_2 a distance r apart in free space, each feels a force of size

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}.$$

This is **Coulomb's law** 库仑定律. Here $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1} is the **permittivity of free space** 真空电容率, and $1/(4\pi\varepsilon_0) \approx 9.0 \times 10^9$ N m^2 C^{-2}. The force is along the line joining the charges: **repulsive** for like charges, **attractive** for opposite charges.



Like charges repel; opposite charges attract — the force is along the line joining them

Worked example. Find the electrostatic force between point charges of +2.0 nC and +3.0 nC placed 4.0 cm apart. ($1/(4\pi\varepsilon_0) = 9.0 \times 10^9$ N m^2 C^{-2}.)

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \frac{(9.0 \times 10^9)(2.0 \times 10^{-9})(3.0 \times 10^{-9})}{(0.040)^2} \approx 3.4 \times 10^{-5}\ \text{N (repulsive)}.$$

Spheres treated as point charges

A spherical **conductor** 导体 with total charge Q gives, at any point **outside**, the same field as a point charge Q at its centre (measure r from the centre). **Inside** a hollow charged conductor the field is zero, so the conductor is an **equipotential** 等势面.

Electric field due to a point charge

Coulomb's law gives the force between **two** charges. Divide it by the test charge ($E = F/q$) and you are left with the field of the source charge Q **alone**. The field at distance r from a point charge Q is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}.$$

It points out from a positive Q, in towards a negative Q, and falls as $1/r^2$ — just like **gravitational field** 重力场 strength, except gravity is always attractive. For several charges, add the fields as a **vector sum** 矢量和.

Electric potential

Electric potential 电势 V at a point is the **work done per unit positive charge** in bringing a small positive test charge from **infinity** 无穷远 to that point:

$$V = \frac{W}{q}.$$

Unit: V. The potential is **zero at infinity**. For a positive source charge $V > 0$ everywhere outside; for a negative source charge $V < 0$.

Potential due to a point charge

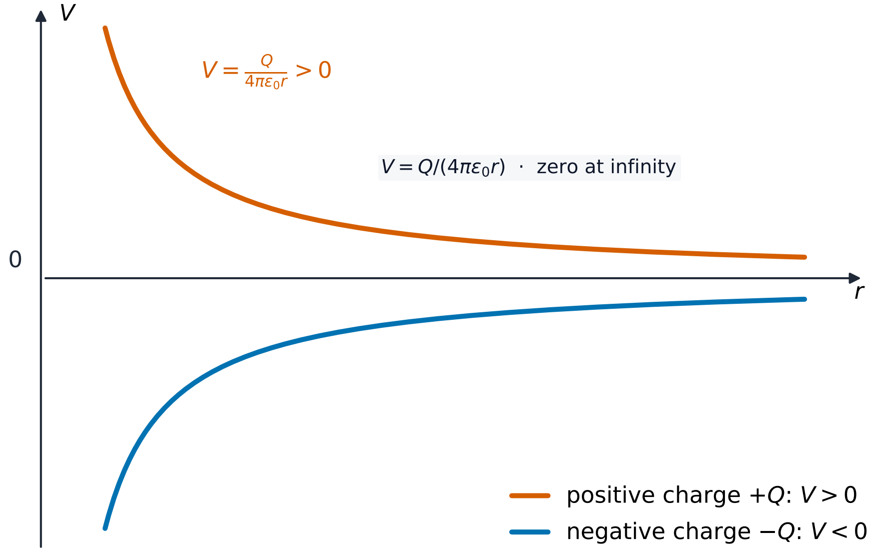
$$V = \frac{Q}{4\pi\varepsilon_0 r}.$$

Note the $1/r$ here (compared with $1/r^2$ for the field). V is a **scalar** 标量; for several charges, add the potentials (with sign).

Worked example. Find the electric potential 4.0 cm from a point charge of +3.0 nC. ($1/(4\pi\varepsilon_0) = 9.0 \times 10^9$ N m^2 C^{-2}.)

$$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{(9.0 \times 10^9)(3.0 \times 10^{-9})}{0.040} \approx 6.8 \times 10^2 \text{ V}.$$

Because potential is a scalar, the potential from several charges is simply their sum (each with its own sign) — no directions to resolve.



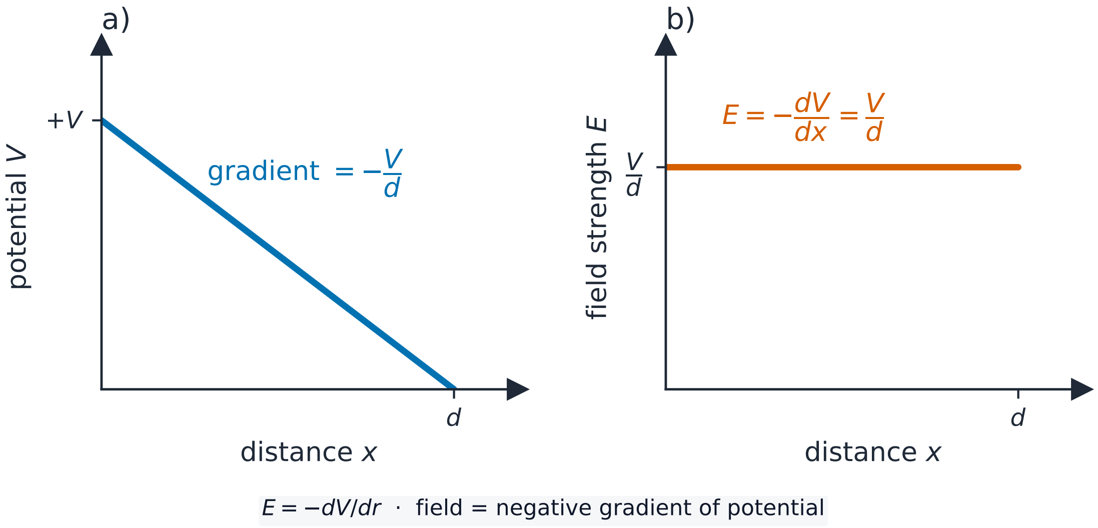
The potential near a point charge varies as 1/r — positive for a positive charge, negative for a negative one

Link between field and potential

The field equals the **negative potential gradient** 电势梯度:

$$E = -\frac{dV}{dx}.$$

Between parallel plates V changes evenly with position, giving $E = V/d$ as before. The minus sign means the field points towards **lower** potential. For a point charge, $-\dfrac{dV}{dr} = \dfrac{Q}{4\pi\varepsilon_0 r^2} = E$.



In a uniform field the potential falls steadily with distance, so the field strength V/d is constant

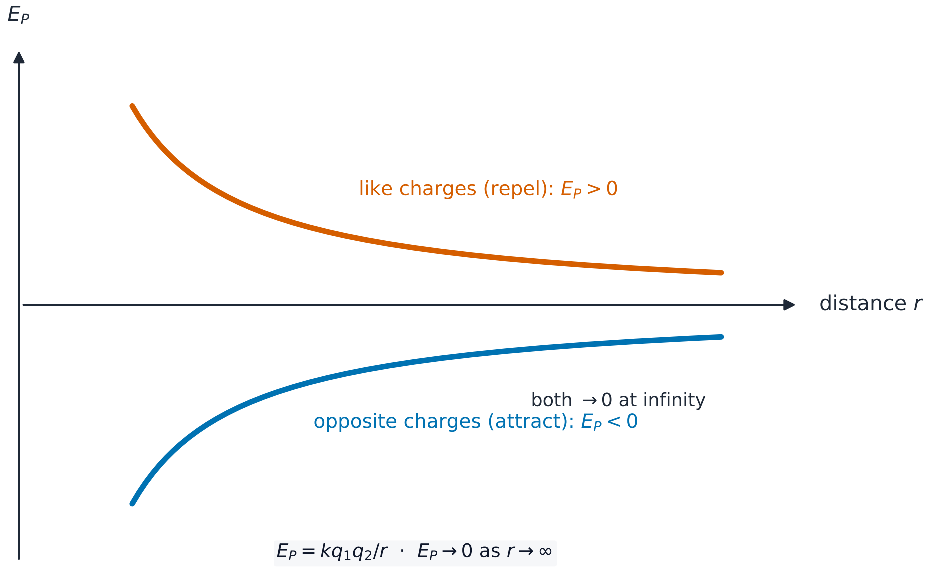
Electric potential energy

A charge q at a point of potential V has **electric potential energy** 电势能 $E_P = qV$. For two point charges Q and q a distance r apart:

$$E_P = \frac{Qq}{4\pi\varepsilon_0 r}.$$

- **like** charges: $E_P > 0$ — stored energy that would be released if they flew apart.
- **opposite** charges: $E_P < 0$ — a **bound** 束缚 system; energy must be supplied to separate them.

In both cases $E_P \to 0$ as $r \to \infty$.



Electric PE of two charges: positive for like charges, a negative well for opposite charges

Worked-example pattern

An **electron** 电子 orbits a **nucleus** 原子核 of charge $+Ze$ at distance r. The Coulomb attraction provides the **centripetal force** 向心力:

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r}, \qquad v = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 m_e r}}.$$

The total energy is kinetic plus potential:

$$E_{\text{total}} = \tfrac{1}{2}m_e v^2 - \frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{Ze^2}{8\pi\varepsilon_0 r},$$

which is negative (a bound state).

Gravitational versus electric

The two field theories look alike:

Quantity	Gravitational	Electric
Source	mass M (always positive)	charge Q (can be $\pm$)
Field strength	$g = GM/r^2$	$E = Q/(4\pi\varepsilon_0 r^2)$
Force on test object	$F = mg$	$F = qE$
Potential	$\phi = -GM/r$	$V = Q/(4\pi\varepsilon_0 r)$
PE of two	$-GMm/r$	$Qq/(4\pi\varepsilon_0 r)$
Nature	always attractive	attractive or repulsive

The minus sign in the **gravitational potential** 引力势 reflects that gravity is always attractive; the electric potential takes the sign of the source charge.

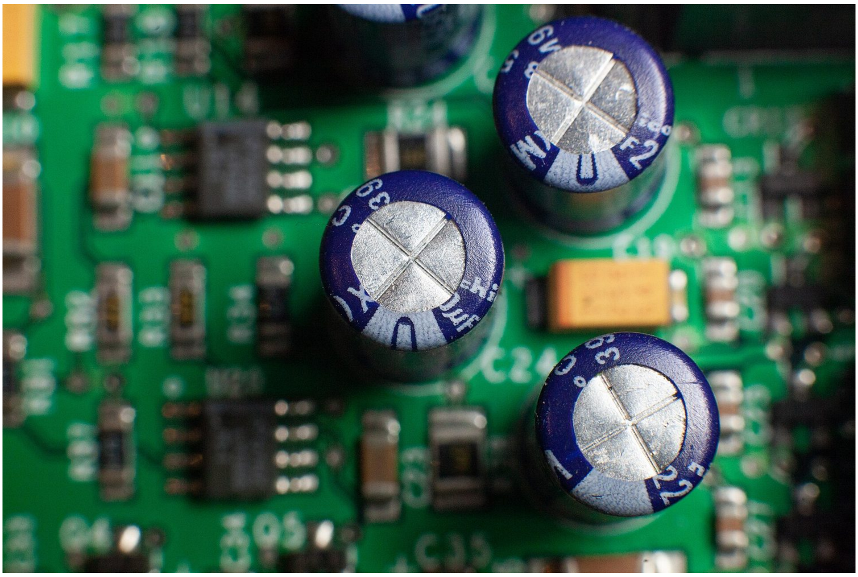
Exam tips

- **Coulomb's law** $F = Q_1Q_2/4\pi\varepsilon_0 r^2$ (inverse-square, but can attract or repel).
- Distinguish a **uniform field** ($E = V/d$, between plates) from a **radial field** ($E = Q/4\pi\varepsilon_0 r^2$).
- Electric potential $V = Q/4\pi\varepsilon_0 r$ (sign follows the charge); the field points from high to low potential.

Capacitance

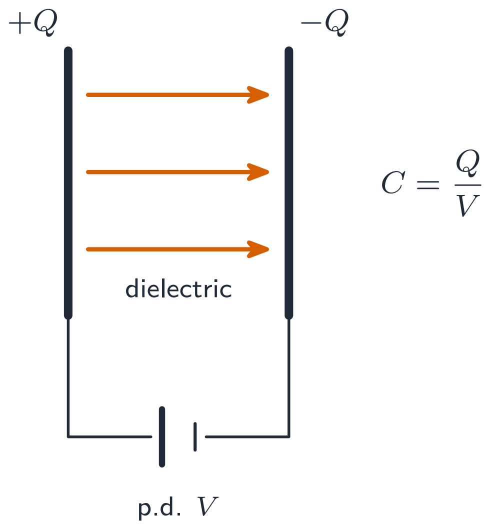
A-Level Physics

Capacitance



Capacitors store electric charge in a circuit.

A **capacitor** 电容器 stores charge. The simplest one is two parallel **conductor** 导体 plates with an **insulator** 绝缘体 (a **dielectric** 电介质, or just **vacuum** 真空 / air) between them. Connected to a battery, charge $+Q$ builds up on one plate and $-Q$ on the other, with a **potential difference** 电势差 V across the gap.



A parallel-plate capacitor: equal and opposite charges on two plates with a p.d. V across the gap

The **capacitance** 电容 C of any capacitor (or any isolated conductor) is

$$C = \frac{Q}{V}.$$

This applies to:

- an **isolated sphere** holding charge Q at potential V (zero at infinity). For radius r, $V = Q/(4\pi\varepsilon_0 r)$, so $C = 4\pi\varepsilon_0 r$.
- a **parallel-plate capacitor**: charges $\pm Q$ on the plates, p.d. V between them.

Unit: **farad** 法拉 (F) = C V^{-1}. A farad is huge, so real capacitors run from pF to mF.

Capacitance is **constant** for a given capacitor (set by its size and dielectric). Doubling the charge doubles the voltage, so $C = Q/V$ stays the same.

Worked example. A 100 μF capacitor is charged to 12 V. Find the charge stored.

$$Q = CV = (100 \times 10^{-6})(12) = 1.2 \times 10^{-3}\ \text{C}\ (= 1.2\ \text{mC}).$$

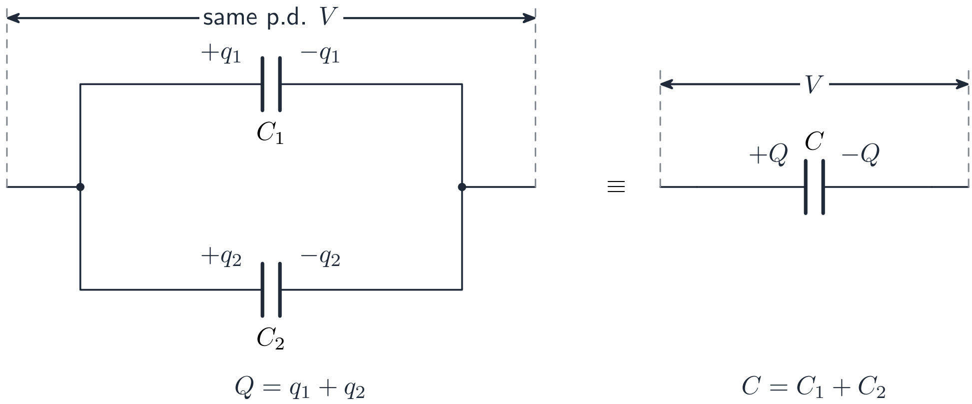
Real capacitors range from large electrolytic cans (high capacitance) down to tiny film and ceramic types – from pF up to mF

Combining capacitors

Capacitors in parallel share the same p.d. V. The total charge is the sum:

$$Q_{\text{total}} = C_1V + C_2V + \ldots, \qquad \text{so} \qquad C_{\text{parallel}} = C_1 + C_2 + \ldots$$

A parallel combination has **larger** capacitance than any one capacitor.

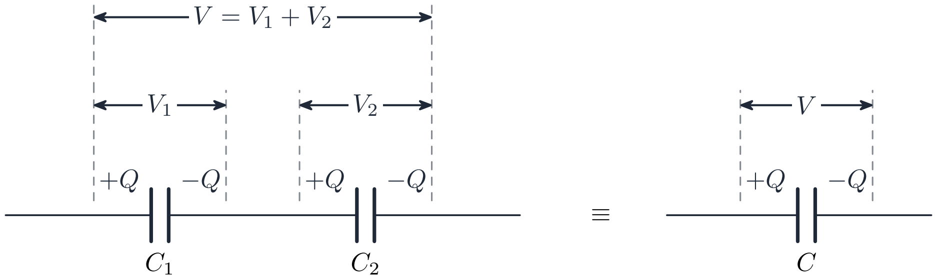


Capacitors in parallel share the same p.d.; the charges add

Capacitors in series carry the **same charge** Q. The total p.d. is the sum:

$$V_{\text{total}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \dots, \qquad \text{so} \qquad \frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

A series combination has **smaller** capacitance than any one capacitor.



Capacitors in series carry the same charge; the p.d.s add

Note: these rules are the **opposite** of those for resistors (resistors sum in series; capacitors sum in parallel), because $C = Q/V$ has V on the bottom while $R = V/I$ has I on the bottom.

Energy stored in a capacitor

Charging a capacitor from 0 to Q needs work, because each extra bit of charge is pushed against the p.d. already there. When the charge is q, the p.d. is $V(q) = q/C$, so adding a small charge dq needs work $V\,dq$. The total work is

$$W = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C}.$$

Using $V = Q/C$, this is the **energy** 能量 stored:

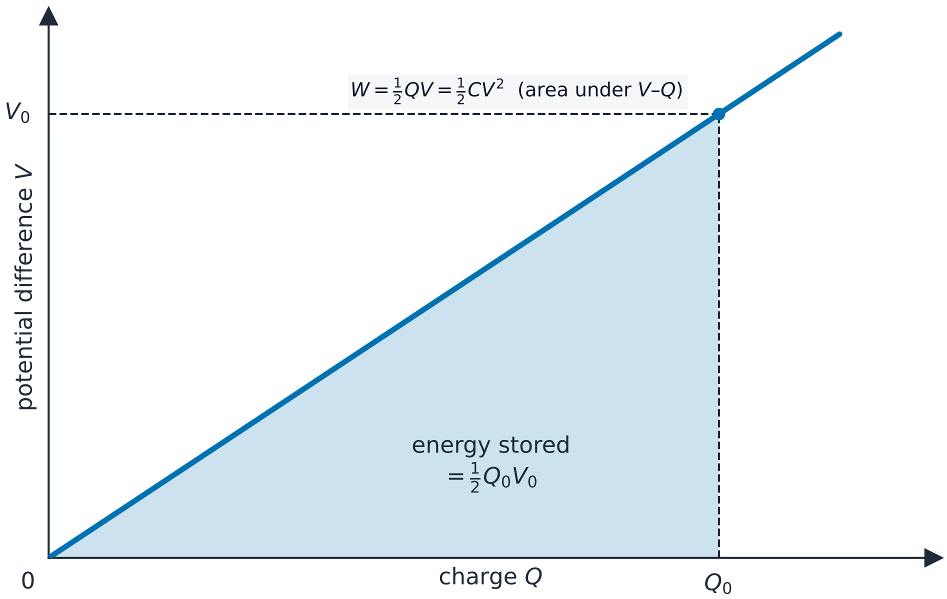
$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}.$$

Worked example. Find the energy stored in a 100 μF capacitor charged to 12 V.

$$W = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(100 \times 10^{-6})(12)^2 = 7.2 \times 10^{-3}\ \text{J}\ (= 7.2\ \text{mJ}).$$

Reading the Q–V graph

A plot of V against Q is a straight line through the origin with gradient $1/C$. The energy stored is the **area** under the line up to a given charge Q, which is the triangle $\frac{1}{2}QV$. The factor $\frac{1}{2}$ is there because the average p.d. during charging is $V/2$ (it grows from zero to V), not V.



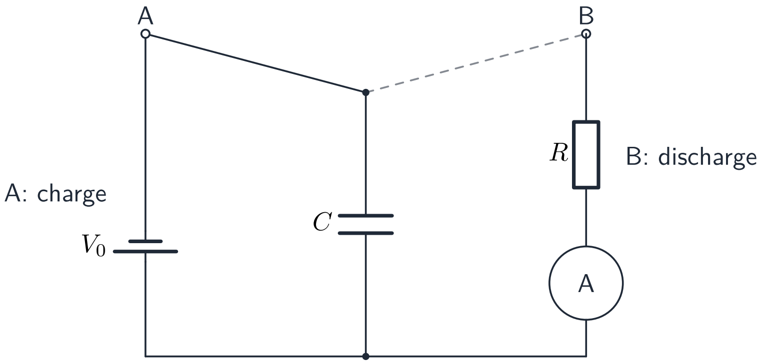
The energy stored is the area under the potential–charge line (the triangle $\frac{1}{2}QV$)

Why charging is "half efficient"

Connect a capacitor C to an ideal battery of e.m.f. V through a wire. The capacitor stores $\frac{1}{2}CV^2$, but the battery supplies charge $Q = CV$ at e.m.f. V, giving out $QV = CV^2$. The other half is lost as heat in the wire — whatever the wire's resistance.

Capacitor discharging through a resistor

A capacitor C charged to V_0 is connected through a switch to a **resistor** 电阻器 of **resistance** 电阻 R. When the switch closes at $t = 0$, the capacitor discharges.



A capacitor charges through switch A, then discharges through the resistor via switch B

Setting up the equation

By **Kirchhoff's second law** 基尔霍夫第二定律 around the loop, $V_C = V_R$. Using $V_C = Q/C$, $V_R = IR$ and $I = -dQ/dt$:

$$\frac{Q}{C} = -R\frac{dQ}{dt}.$$

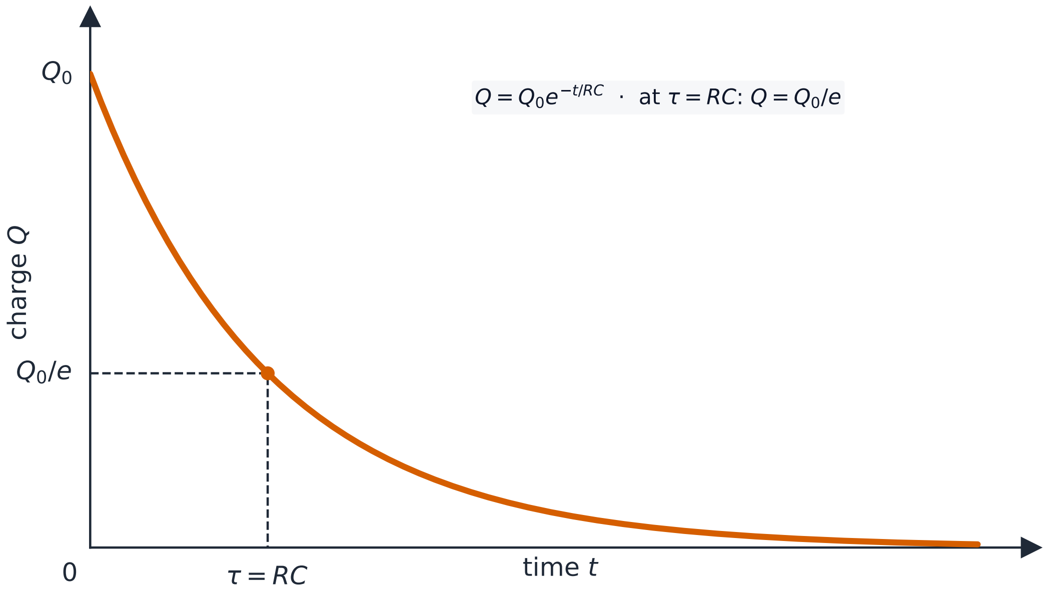
This is solved by an **exponential decay** 指数衰减 with time constant RC.

Discharge equations

Charge Q, p.d. V and current I all decay exponentially with the same time constant:

$$Q = Q_0 e^{-t/(RC)}, \qquad V = V_0 e^{-t/(RC)}, \qquad I = I_0 e^{-t/(RC)},$$

with $I_0 = V_0/R$.



Charge decays exponentially during discharge, falling to Q_0/e after one time constant RC

Time constant

$$\tau = RC$$

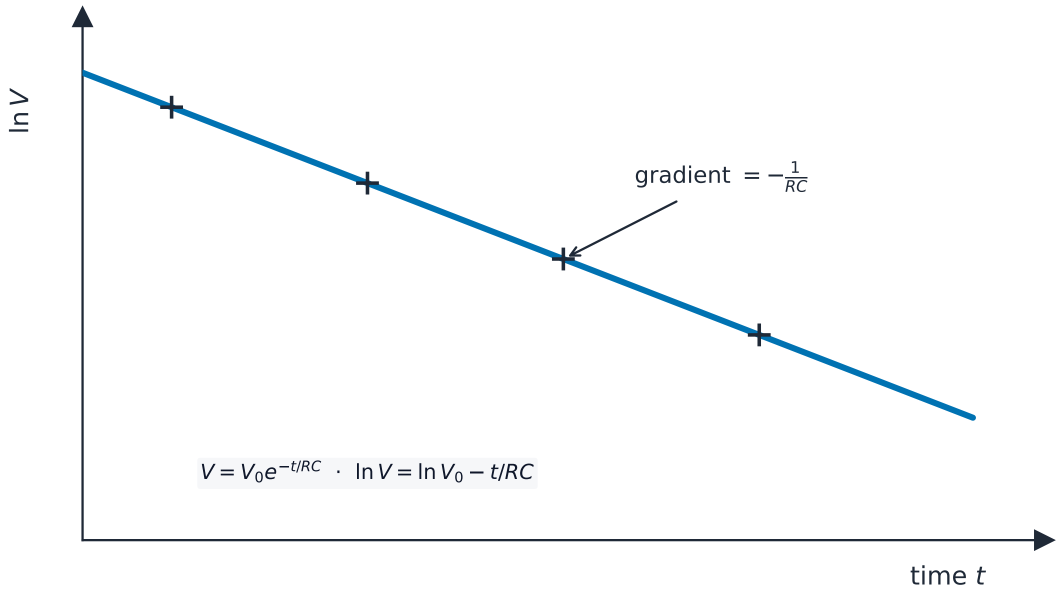
is the **time constant** 时间常数 (in seconds: $\Omega \cdot \mathrm{F} = \mathrm{s}$). It is the time for a decaying quantity to fall to $1/e \approx 0.37$ (about 37%) of its starting value. After 2τ it is at about 13.5%; after 5τ, below 1%.

Worked example. A 100 μF capacitor charged to 12 V is discharged through a 47 kΩ resistor. Find the time constant and the voltage after one time constant.

$$\tau = RC = (47 \times 10^3)(100 \times 10^{-6}) = 4.7 \text{ s}.$$

After one time constant the voltage falls to $1/e$ of its start: $V = 12 \times 0.37 \approx 4.4$ V.

To find τ from a curve: read the time to fall to $1/e$ of the start. Or take logs: $\ln(V/V_0) = -t/(RC)$, so a plot of $\ln V$ against t is a straight line with gradient $-1/(RC)$.



A graph of $\ln V$ *against time is a straight line of gradient* $-1/(RC)$

Reading graphs during discharge

- Q against V_C: since $Q = CV$ always, this is a straight line through the origin with gradient C. Discharge moves the point from (V_0, Q_0) down to $(0, 0)$.
- I against V_C: since $I = V_C/R$, this is a straight line through the origin with gradient $1/R$, so you can find R.

Common exam questions

Given a discharge curve $V(t)$ or $Q(t)$:

- read the start value V_0 or Q_0 at $t = 0$.
- read the time to fall to $V_0/e \to$ time constant $\tau = RC$.
- given R, find $C = \tau/R$ (or the other way round).
- predict a later value with the exponential formula.

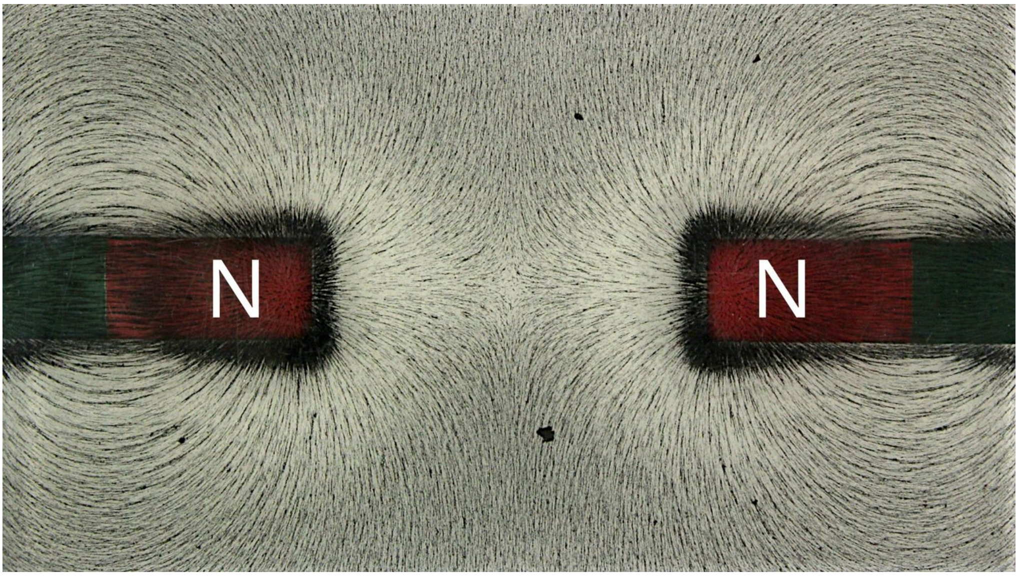
Exam tips

- $C = Q/V$; energy stored $W = \frac{1}{2}QV = \frac{1}{2}CV^2$ (the $\frac{1}{2}$ is the area under the Q-V graph).
- Combine capacitors the **opposite way to resistors** (parallel add, series reciprocal).
- Discharge is exponential: $Q = Q_0 e^{-t/RC}$; the time constant $\tau = RC$ is when the charge falls to 37%.

Magnetic fields

A-Level Physics

The magnetic field



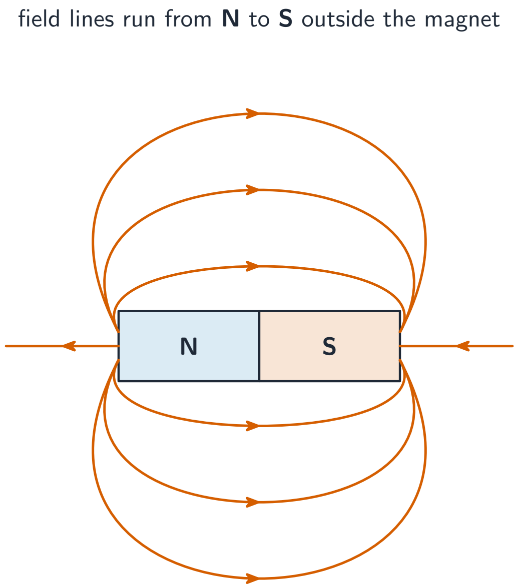
Iron filings trace the field lines around bar magnets.

A **magnetic field** 磁场 is a region where a moving charge (or a **current** 电流) feels a **force** 力. It is made by:

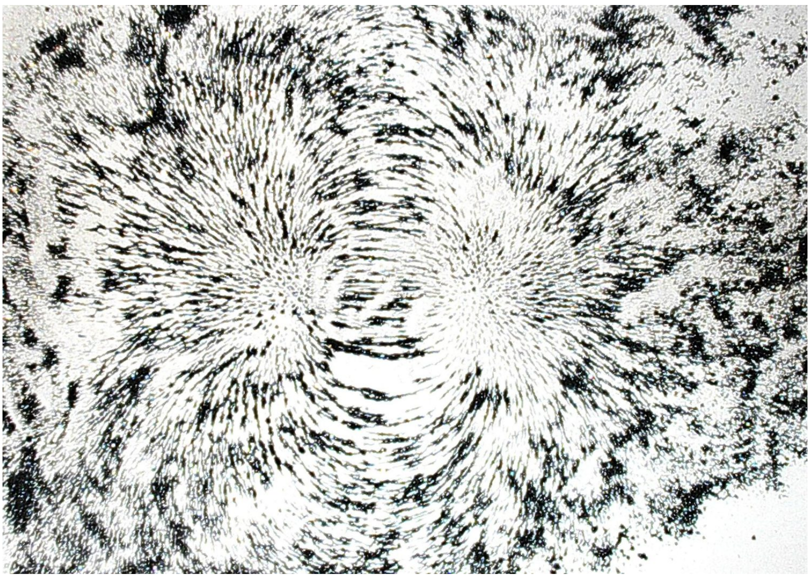
- **moving charges** (usually a current in a wire), or
- **permanent magnets** 永磁体 (where it comes from tiny atomic currents).

Field lines

- **field lines** 场线 point from **N** to **S** outside a magnet, and S to N inside (so they form closed loops).
- lines never cross; closer lines mean a stronger field.



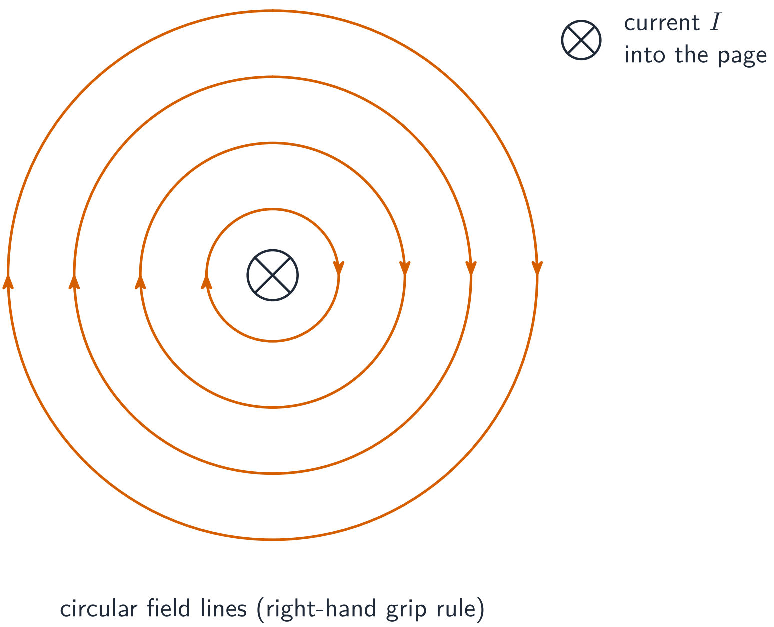
The field of a bar magnet: lines run from N to S, strongest near the poles



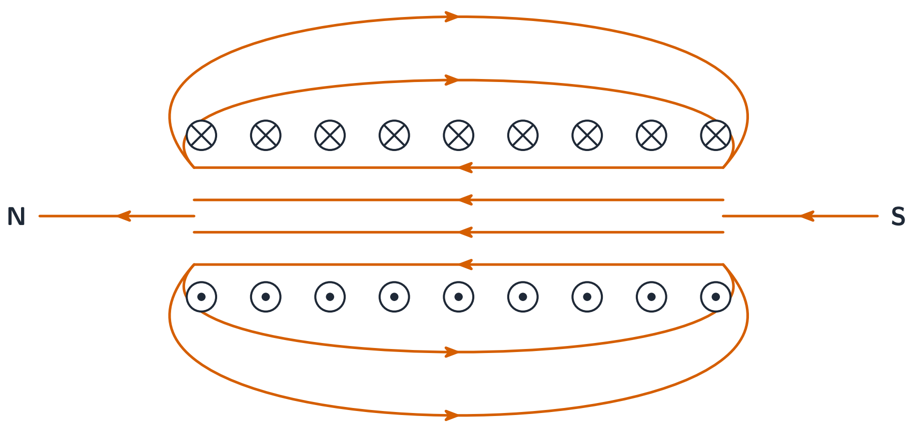
Iron filings around a bar magnet line up along the field, showing its real shape

Patterns to know:

- **bar magnet** 条形磁铁 — curved lines from N to S outside, strongest near the poles.
- **long straight wire** — circles around the wire; the direction comes from the **right-hand grip rule** 右手定则 (thumb along the current, fingers curl the way the field points).
- **flat circular coil** — the field through the centre is at right angles to the coil; the coil acts like a small bar magnet.
- **long solenoid** 螺线管 — the field inside is nearly **uniform** along the axis, like a stretched bar magnet; outside it falls off fast.



Field around a long straight wire



Field of a solenoid

An **iron core** 铁芯 inside a solenoid greatly **increases** the field, because the iron's atomic magnets line up and add to it. This is why **electromagnets** 电磁铁 and **transformers** 变压器 have iron cores.

Force on a current-carrying conductor

A current I in a wire of length L in a magnetic field of flux density B feels a force

$$F = BIL\sin\theta,$$

where θ is the angle between the wire and the field. The force is largest when the wire is at right angles to the field ($F = BIL$) and zero when the wire is along the field.

Worked example. A wire of length 0.20 m carries a current of 3.0 A at right angles to a magnetic field of flux density 0.50 T. Find the force on it.

$$F = BIL = 0.50 \times 3.0 \times 0.20 = 0.30 \text{ N}.$$

Magnetic flux density

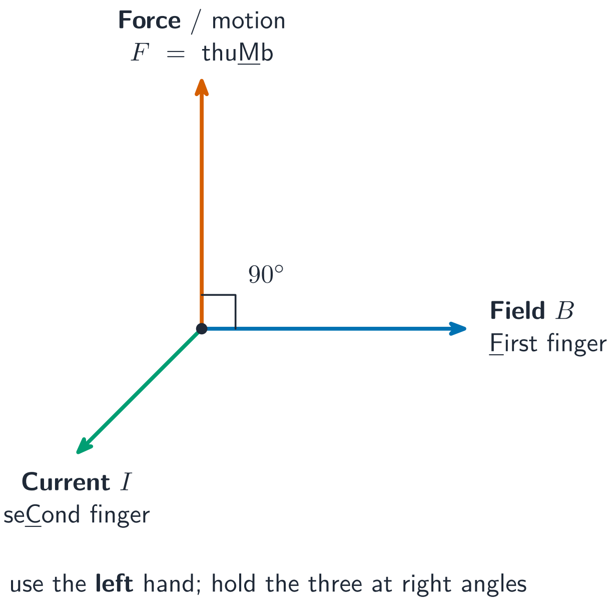
This equation also **defines** the **magnetic flux density** 磁通密度 B:

$$B = \frac{F}{IL} \quad \text{(wire at right angles to the field).}$$

So B is the force per unit current per unit length on a wire at right angles to the field. Unit: **tesla** 特斯拉, $\mathrm{T} = \mathrm{N\ A^{-1}\ m^{-1}}$.

Direction — Fleming's left-hand rule

Use the **left hand** (**Fleming's left-hand rule** 弗莱明左手定则): first finger = **F**ield, second finger = **C**urrent, thumb = force (thrust). Hold the three at right angles.



Fleming's left-hand rule: thumb = force, first finger = field, second finger = current

Force on a moving charge

A charge Q moving at **velocity** 速度 v through a field feels

$$F = BQv\sin\theta,$$

with θ the angle between v and B. Same left-hand rule (the second finger is the motion of a **positive** charge — reverse it for a negative charge). The force is largest when v is at right angles to B, and zero when v is along B.

Circular motion in a uniform field

A charge moving at right angles to a **uniform field** 匀强场 feels a force at right angles to both v and B. This force does **no work** (always at right angles to the motion), so the **kinetic energy** 动能 and speed stay constant — the particle moves in a **circle**. Set the magnetic force equal to the **centripetal force** 向心力:

$$BQv = \frac{mv^2}{r}, \qquad r = \frac{mv}{BQ}.$$

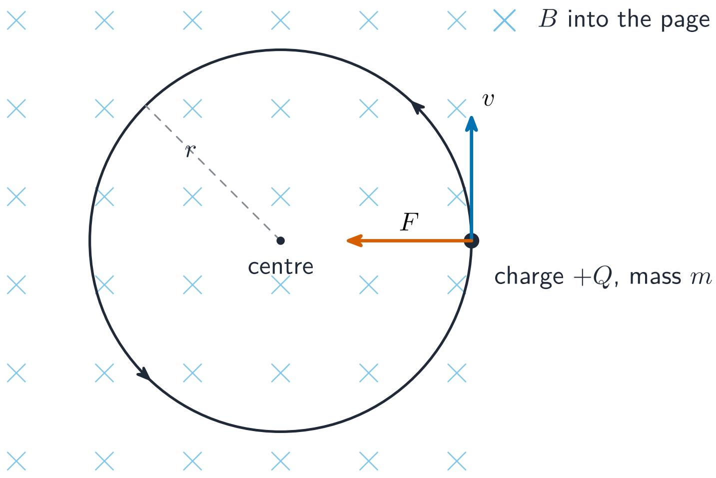
Worked example. A proton (mass 1.7×10^{-27} kg, charge 1.6×10^{-19} C) moves at 2.0×10^{6} m s^{-1} at right angles to a 0.50 T field. Find the radius of its circular path.

$$r = \frac{mv}{BQ} = \frac{(1.7 \times 10^{-27})(2.0 \times 10^{6})}{(0.50)(1.6 \times 10^{-19})} \approx 0.043 \text{ m}.$$

So the radius depends on the **momentum** 动量 mv. The period is

$$T = \frac{2\pi m}{BQ},$$

which does **not** depend on the speed — a faster particle goes in a bigger circle but takes the same time per turn. If v also has a part along B, that part is unchanged, and the path is a **helix** 螺旋.



$$BQv = \frac{mv^2}{r} \;\Rightarrow\; r = \frac{mv}{BQ}$$

Circular path of a charged particle in a magnetic field

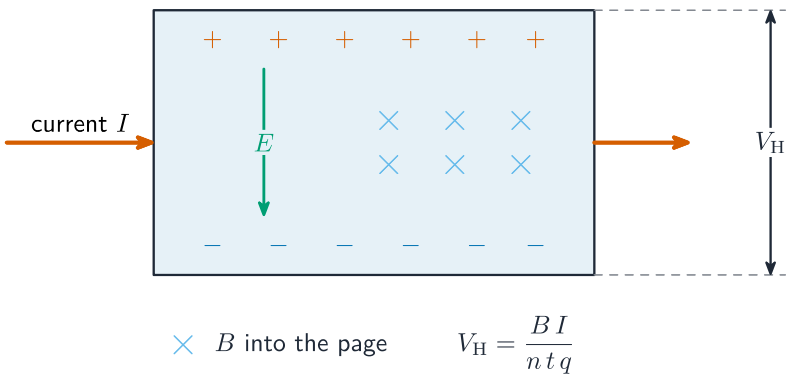
Hall effect

A slab of conductor carrying current I, in a field B at right angles to the current, develops a voltage across its faces — the **Hall voltage 霍尔电压** V_{H} (the **Hall effect** 霍尔效应).

The moving charges feel a magnetic force BQv_{d} (v_{d} is the **drift velocity** 漂移速度), so they build up on one face, making an **electric field** 电场 E that opposes more build-up. At steady state $eE = Bev_{\mathrm{d}}$, so $E = Bv_{\mathrm{d}}$. With $V_{\mathrm{H}} = Ew$ and $I = nev_{\mathrm{d}}wt$:

$$V_{\mathrm{H}} = \frac{BI}{ntq},$$

where q is the carrier charge. A **Hall probe** 霍尔探头 uses this to measure B: pass a known current through a thin **semiconductor** 半导体 slab and read V_{H} (largest when the slab is at right angles to B).



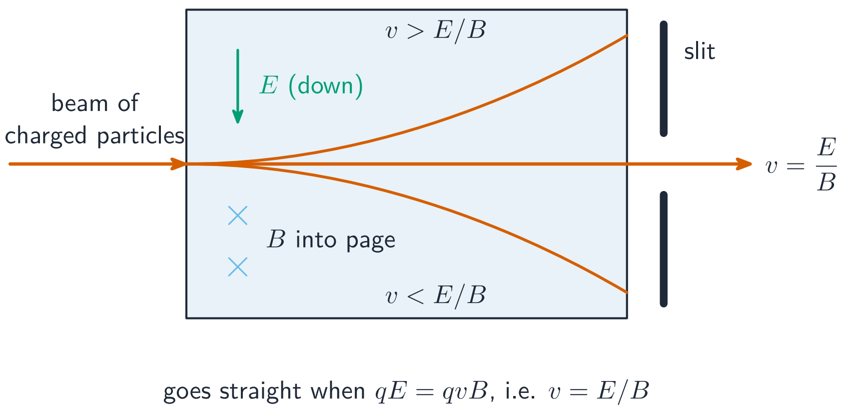
The Hall effect

Velocity selector

A **velocity selector** 速度选择器 uses crossed electric and magnetic fields to let through only one speed. With the electric force qE and magnetic force qvB set to oppose each other, the net force is zero only when

$$qE = qvB \quad \Rightarrow \quad v = \frac{E}{B}.$$

Particles at speed E/B go straight through; faster or slower ones are deflected.



Velocity selector

Force between parallel currents

Two long parallel wires each sit in the other's magnetic field. Using Fleming's left-hand rule: **parallel currents (same direction) attract; antiparallel currents (opposite directions) repel**. This is the basis of the SI definition of the ampere.

Electromagnetic induction

Magnetic flux

The **magnetic flux** 磁通量 Φ through a flat area A at right angles to B is

$$\Phi = BA.$$

If the area's normal is at angle θ to B, use $\Phi = BA\cos\theta$. Unit: **weber** 韦伯, $\mathrm{Wb} = \mathrm{T\ m^2}$. For a **coil** 线圈 of N turns, the **flux linkage** 磁链 is $N\Phi = NBA$.

Faraday's and Lenz's laws

When the flux linkage through a circuit **changes**, an **electromotive force** 电动势 (e.m.f.) is induced — this is **electromagnetic induction** 电磁感应.

Faraday's law 法拉第定律: the induced e.m.f. equals the rate of change of flux linkage:

$$|\varepsilon| = N\frac{d\Phi}{dt}.$$

Worked example. A coil of 200 turns and area 0.010 m^2 sits with its plane at right angles to a 0.50 T field. The field falls steadily to zero in 0.20 s. Find the average induced e.m.f.

The flux linkage changes from $N\Phi = NBA = 200 \times 0.50 \times 0.010 = 1.0$ Wb to zero, so

$$|\varepsilon| = \frac{\Delta(N\Phi)}{\Delta t} = \frac{1.0}{0.20} = 5.0 \text{ V}.$$

Lenz's law 楞次定律: the induced e.m.f. acts to **oppose the change** that makes it. This is **conservation of energy 能量守恒** — if it reinforced the change, energy would come from nothing. Combined:

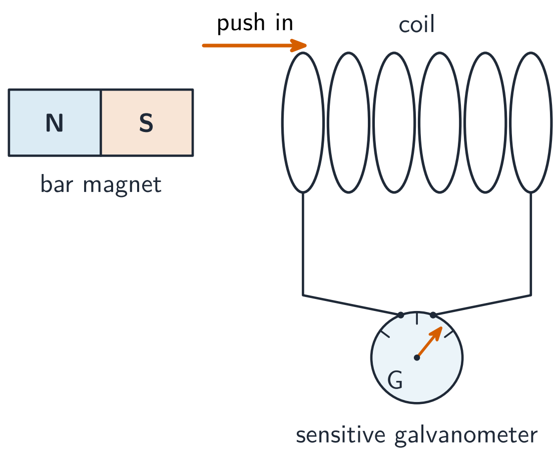
$$\varepsilon = -\frac{d(N\Phi)}{dt}.$$

What changes the flux?

- **changing** B (moving a magnet near a coil),
- **changing area** A (a rod sliding along rails),
- **changing orientation** (a coil turning in a field — the a.c. generator, next topic).

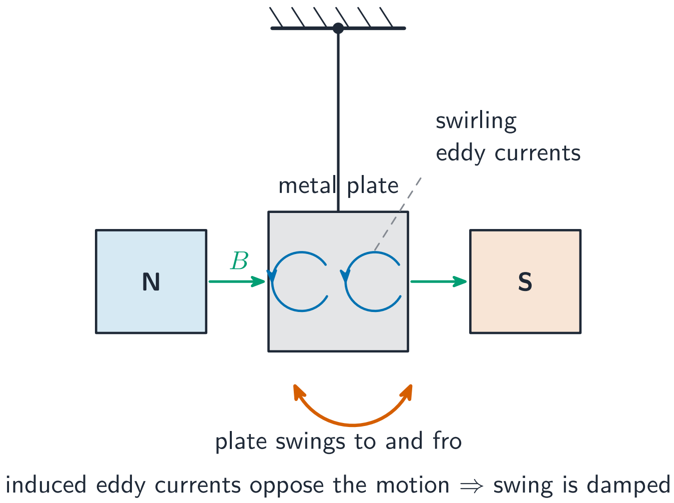
Demonstrations

- moving a bar magnet into a coil deflects a **galvanometer 检流计**; the deflection reverses when the magnet is pulled out (Lenz's law), and is larger for faster motion (Faraday's law).



Demonstrating electromagnetic induction

- a copper disc swinging into a field is quickly slowed — **eddy currents** 涡流 are induced that oppose the motion.



Eddy-current damping

What makes the induced e.m.f. larger

From $\varepsilon = N\,d\Phi/dt$ with $\Phi = BA$: more turns N, a stronger B, a larger area A, or a faster change — each gives a larger induced e.m.f.

Exam tips

- Force on a current $F = BIL\sin\theta$; on a moving charge $F = BQv$; find the direction with **Fleming's left-hand rule**.
- A charge moving perpendicular to B moves in a **circle** ($BQv = mv^2/r$).
- **Electromagnetic induction**: e.m.f. = rate of change of flux linkage (Faraday); its direction opposes the change (Lenz's law).

Alternating currents

A-Level Physics

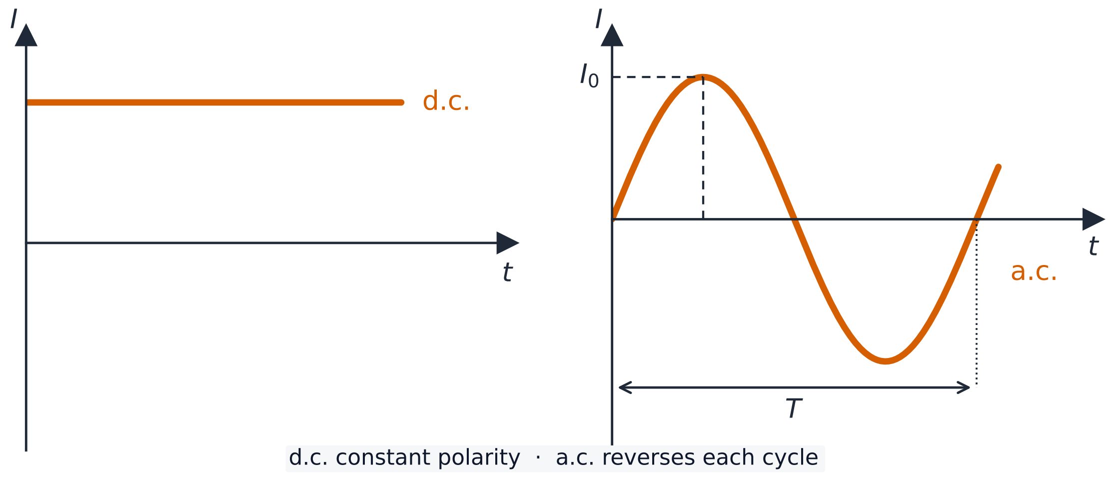
Alternating current basics

A substation's transformers step alternating voltage up or down.

An **alternating current** 交流电 (a.c.) keeps reversing direction. Mains supply is sinusoidal a.c.: I or V follows a sine wave in time:

$$I = I_0 \sin(\omega t), \qquad V = V_0 \sin(\omega t).$$

(For a purely resistive load the **voltage** 电压 and **current** 电流 are in phase, which is the case in this syllabus.)



A steady direct current compared with a sinusoidal alternating current of peak I_0 and period T

Key terms

- **period** 周期 T — the time for one full cycle. Unit: s.
- **frequency** 频率 f — cycles per second; $f = 1/T$. Mains is often 50 Hz or 60 Hz.
- **angular frequency** 角频率 $\omega = 2\pi f = 2\pi/T$.
- **peak value** 峰值 I_0 or V_0 — the largest value in a cycle (also called the amplitude).
- **peak-to-peak value** 峰峰值 $2I_0$ — from $+I_0$ to $-I_0$. Useful when reading an oscilloscope.

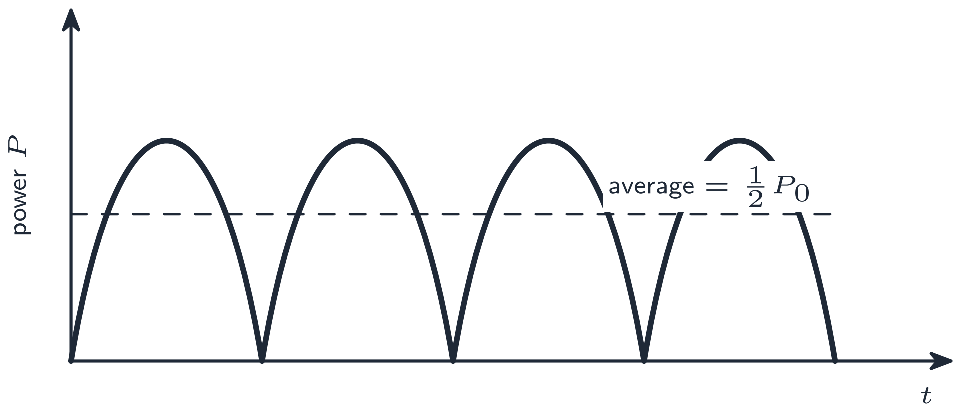
Reading a CRO trace

Same as for any wave (Topic 7), using a **cathode-ray oscilloscope** 示波器:

- horizontal divisions $\times$ **time-base** 时基 $\rightarrow$ period T, so $f = 1/T$.
- vertical divisions $\times$ y-gain $\rightarrow$ peak voltage V_0 (measure centre to peak, or peak-to-peak then halve).

Power delivered to a resistor

For a resistive load R, the instant **power** 功率 is $P(t) = I(t)^2R$. With $I = I_0 \sin(\omega t)$:



The power in a resistor pulses; its average is half the peak

$$P(t) = I_0^2 R \sin^2(\omega t).$$

This is always positive, with peak $I_0^2 R$ and minimum zero, oscillating at twice the frequency of I. The mean of $\sin^2(\omega t)$ over a cycle is $\frac{1}{2}$, so the **average power** is

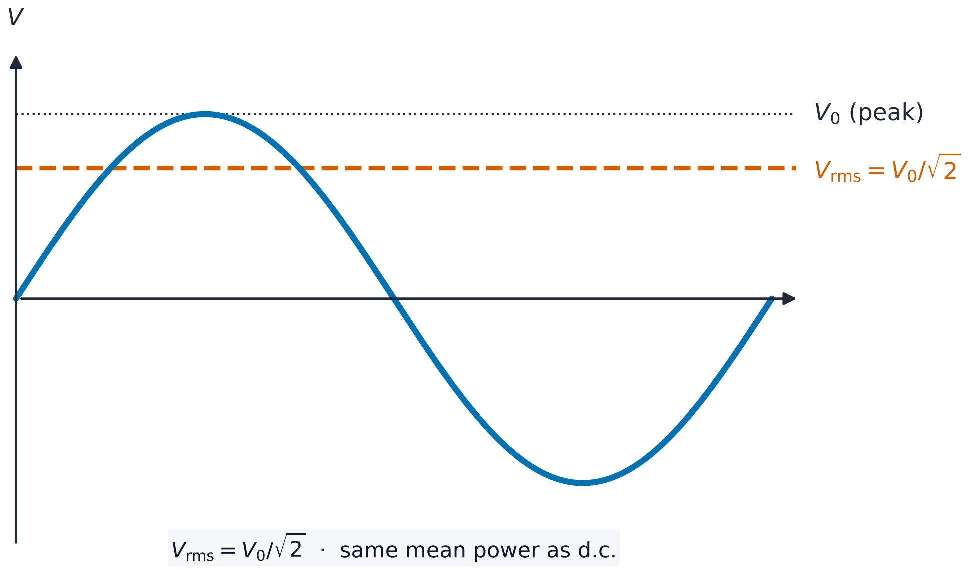
$$\langle P \rangle = \tfrac{1}{2} I_0^2 R = \tfrac{1}{2} P_{\text{peak}}.$$

Average a.c. power in a resistor is **half the peak power**.

Root-mean-square (r.m.s.) values

The r.m.s. current $I_{\text{r.m.s.}}$ is the **steady direct current** that would give the same average power in the same **resistance** 电阻 R. From $\langle P\rangle = I_{\text{r.m.s.}}^2 R = \frac{1}{2}I_0^2 R$:

$$I_{\text{r.m.s.}} = \frac{I_0}{\sqrt{2}}, \qquad V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}}.$$



The r.m.s. value is the steady d.c. level ($V_0/\sqrt{2}$) that delivers the same average power as the a.c.

Worked example. An a.c. supply has a peak voltage of 12 V. Find its r.m.s. voltage.

$$V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}} = \frac{12}{\sqrt{2}} \approx 8.5 \text{ V}.$$

The $\sqrt{2}$ comes from the name **root-mean-square** 均方根: $I_{\text{r.m.s.}} = \sqrt{\langle I^2\rangle}$ and $\langle \sin^2\rangle = \frac{1}{2}$. (Only the sinusoidal case is needed.)

Why r.m.s. matters

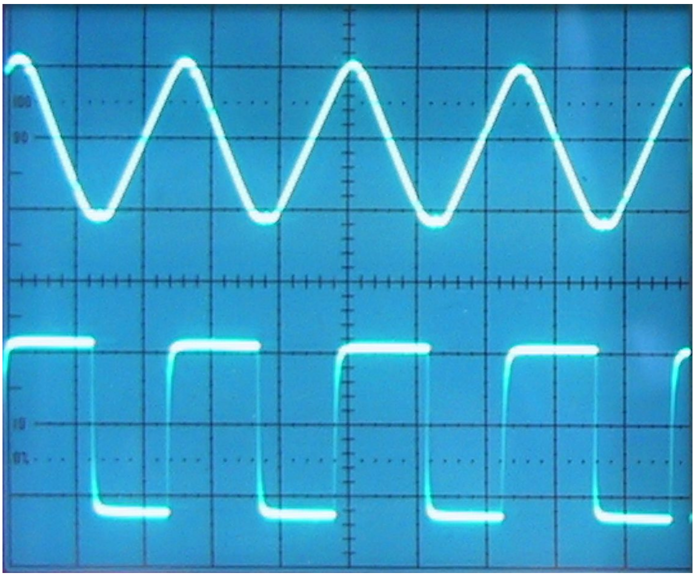
Quoted a.c. values are r.m.s. values. "230 V mains" means $V_{\text{r.m.s.}} = 230$ V, with peak $V_0 = 230\sqrt{2} \approx 325$ V. Components must be rated for the peak, not the r.m.s. Average power then takes the d.c. form:

$$\langle P \rangle = I_{\text{r.m.s.}}^2 R = V_{\text{r.m.s.}}^2 / R = V_{\text{r.m.s.}} I_{\text{r.m.s.}}.$$

Worked example. A heater of resistance 50 Ω is connected to the 230 V r.m.s. mains. Find the r.m.s. current and the average power dissipated.

$$I_{\text{r.m.s.}} = \frac{V_{\text{r.m.s.}}}{R} = \frac{230}{50} = 4.6 \text{ A}, \qquad \langle P \rangle = V_{\text{r.m.s.}} I_{\text{r.m.s.}} = 230 \times 4.6 \approx 1.1 \times 10^3 \text{ W}.$$

Rectification



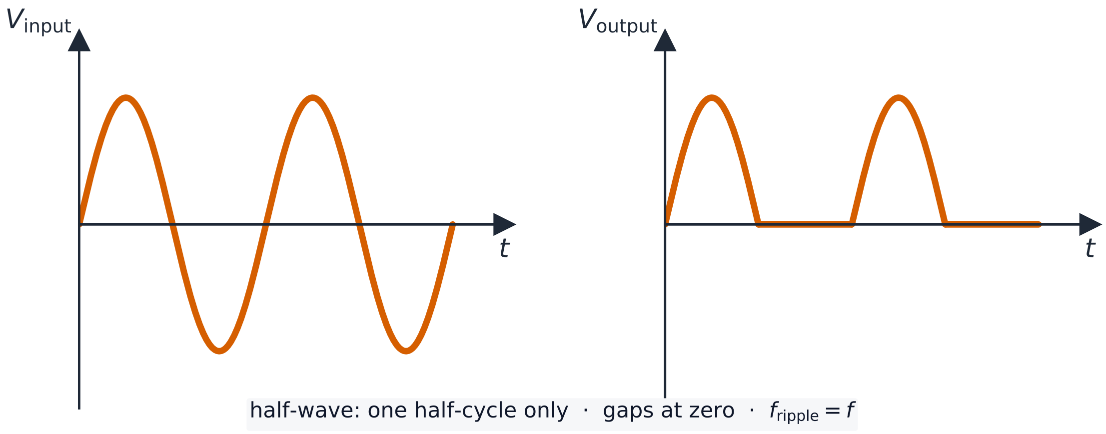
An oscilloscope shows how a voltage varies with time.

Rectification 整流 turns an alternating voltage into a one-direction (d.c.-like) voltage, using **diodes** 二极管 (which conduct in only one direction).

Half-wave rectification

A **single diode** in series with the load passes only the positive half of each cycle; in the negative half the diode is **reverse-biased** 反向偏置 and no current flows. This is **half-wave rectification** 半波整流.

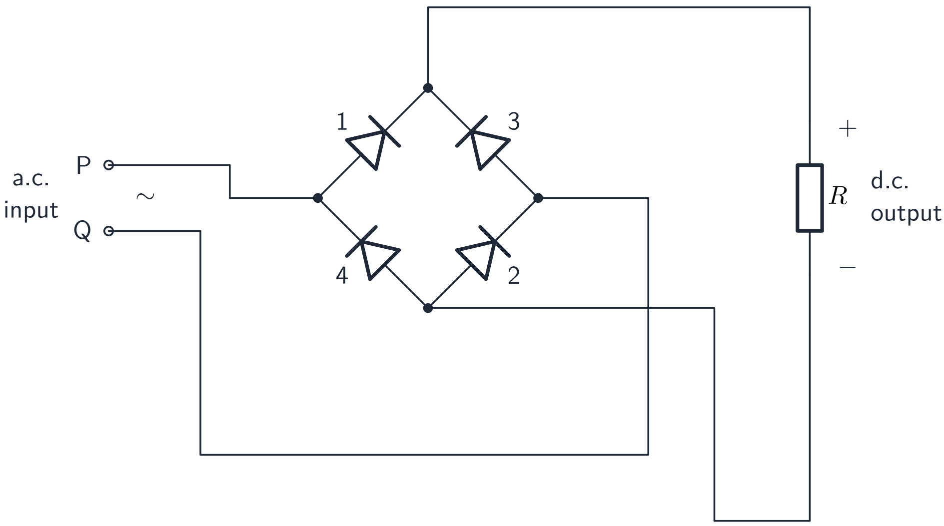
Output: positive half-waves with flat zero gaps. The mean output is $V_0/\pi \approx 0.32V_0$. Drawback: half the input is wasted and the output is very uneven.



In half-wave rectification a single diode passes only the positive half-cycles

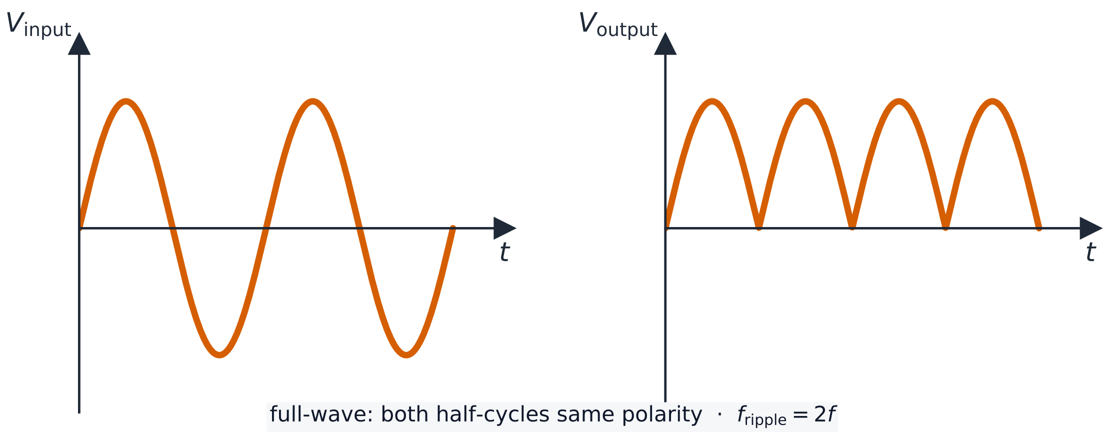
Full-wave rectification (bridge rectifier)

A **bridge rectifier** 桥式整流器 uses **four diodes** arranged so the current through the load always flows the same way, whichever a.c. terminal is positive — **full-wave rectification** 全波整流. On each half-cycle a different pair of diodes conducts, but the load always sees the same direction.



A four-diode bridge sends the load current the same way whichever a.c. terminal is positive

Output: a continuous run of positive half-waves (no gaps), at **twice** the input frequency. The mean output is $2V_0/\pi \approx 0.64V_0$ — double the half-wave value. It uses all the input and is smoother and easier to filter.



In full-wave rectification every half-cycle is used, giving a continuous run of positive humps

Drawing the diagrams

- half-wave: a.c. source — single diode — load R, in series.
- full-wave bridge: four diodes as the arms of a "diamond"; the a.c. input goes to one pair of opposite corners, the load R across the other pair. The diode directions make the load terminals keep the same polarity for either input polarity.

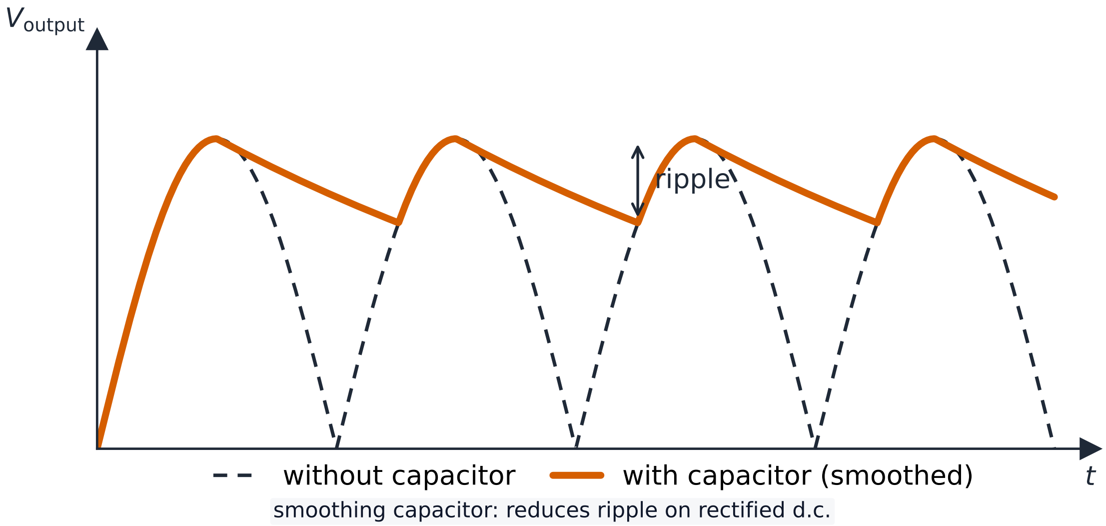
Smoothing with a capacitor

A rectifier's output is still bumpy. To smooth it, put a **capacitor** 电容器 C **in parallel with the load** R.

How it works

- on the **rising part** of each pulse, the capacitor charges up to near the peak.
- on the **falling part** (and any gap), the diodes are reverse-biased, so the capacitor **discharges through the load**, keeping current flowing. The voltage falls with **time constant** 时间常数 RC (Topic 19).
- at the next peak, the capacitor charges again, and the cycle repeats.

The output now sits near the peak with small dips. The size of the dips is the **ripple** 纹波 (this whole step is called **smoothing** 平滑).



A capacitor across the load smooths the rectified output, leaving only a small ripple

What reduces the ripple

- **larger** C → more stored charge → smaller dip between peaks → smaller ripple.
- **larger** R → smaller load current → slower discharge → smaller ripple.
- **higher rectified frequency** (full-wave is twice the input) → less time to discharge between peaks → smaller ripple.

In short, a large RC compared with the time between peaks gives a smoother output.

Purpose in summary

The smoothing capacitor reduces the ripple, giving a steadier d.c. voltage suitable for sensitive electronics.

Exam tips

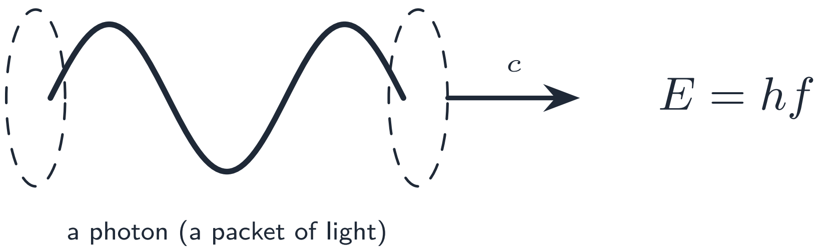
- $I_{rms} = I_0/\sqrt{2}$ for a sinusoidal current; use r.m.s. values for power ($P = I_{rms}^2 R$).
- Mean power in a resistor $= \frac{1}{2}$ of the peak power.
- Explain **rectification** (diode or bridge) and how a **smoothing capacitor** reduces the ripple.

Quantum physics

A-Level Physics

Photons: the particle nature of light

Electromagnetic radiation behaves like **particles** as well as like a wave. The particles of EM radiation are **photons** 光子 — small packets ("**quanta**" 量子) of EM energy that travel at the speed of light.



A photon is a packet of energy, $E = hf$

Energy of a photon

A photon of **frequency** 频率 f has energy

$$E = hf,$$

where $h = 6.63 \times 10^{-34}$ J s is the **Planck constant** 普朗克常量. Using $c = f\lambda$:

$$E = \frac{hc}{\lambda}.$$

Worked example. Find the energy of a photon of green light of wavelength 500 nm. ($h = 6.63 \times 10^{-34}$ J s, $c = 3.0 \times 10^{8}$ m s^{-1}.)

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3.0 \times 10^{8})}{500 \times 10^{-9}} \approx 4.0 \times 10^{-19} \text{ J } (\approx 2.5 \text{ eV}).$$

Higher-frequency (shorter-**wavelength** 波长) photons carry more energy: one γ-ray photon carries far more than one radio photon.

The electronvolt

The **electronvolt 电子伏特** (eV) is a handy energy unit on the atomic scale:

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}.$$

It is the **kinetic energy 动能** an **electron 电子** gains moving through a **potential difference 电势差** of 1 V. For example, a visible photon ($\lambda \approx 500$ nm) has energy ≈ 2.5 eV. To go eV $\to$ J multiply by 1.60×10^{-19}; J $\to$ eV divide.

Momentum of a photon

A photon also carries **momentum 动量**:

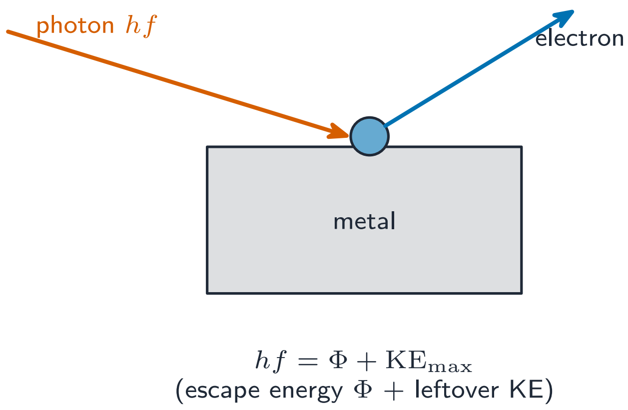
$$p = \frac{E}{c} = \frac{h}{\lambda}.$$

It has zero rest mass but a non-zero momentum E/c. Radiation pressure (photons pushing on a surface) follows from this.

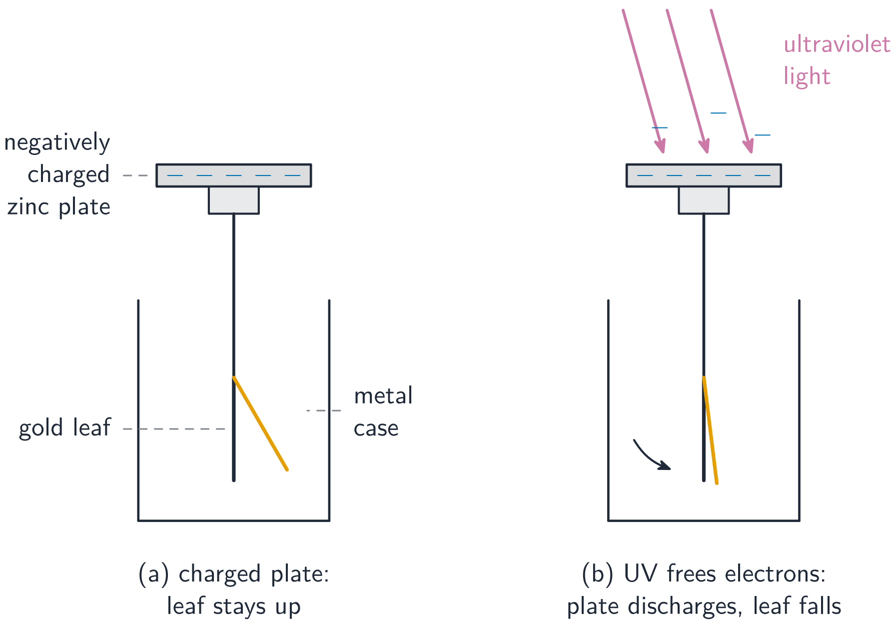
Photoelectric effect

Solar cells use the photoelectric effect to turn light into electricity.

When EM radiation of high enough frequency hits a metal, **electrons are emitted**. These are **photoelectrons 光电子**, and the effect is the **photoelectric effect 光电效应**.



One photon gives its energy hf to one electron: part frees it (the work function Φ), the rest is the electron's KE



A charged zinc plate loses its charge — the gold leaf falls — when ultraviolet light shines on it

Threshold frequency and work function

Each metal has a lowest photon frequency, the **threshold frequency** 极限频率 f_0, below which **no** electrons come out, however bright the light. The **work function** 逸出功 Φ is the **least energy** needed to free an electron from the surface:

$$\Phi = hf_0.$$

Different metals have different work functions (about 2–5 eV).

Einstein's photoelectric equation

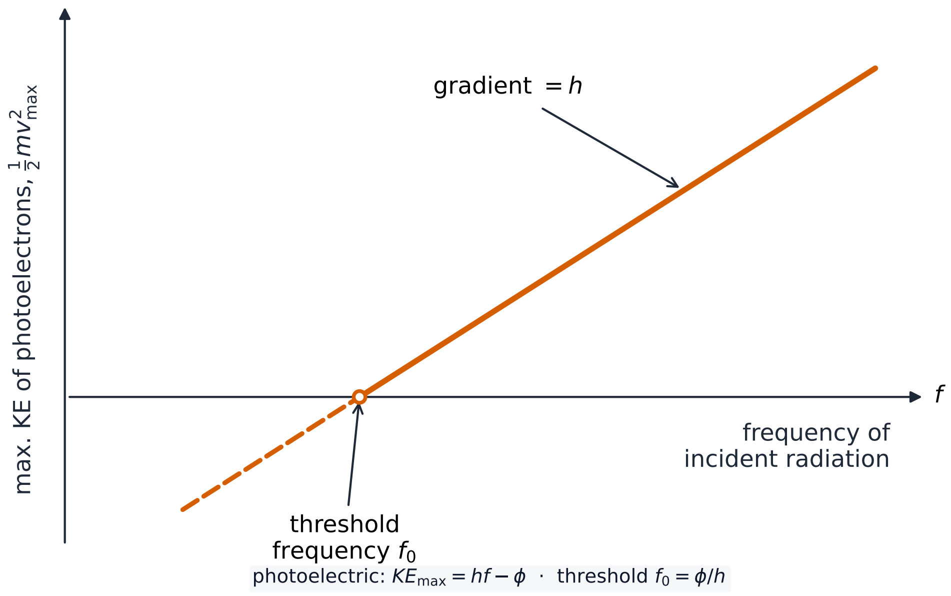
One photon gives all its energy to one electron. If the photon energy hf is more than the work function, the electron escapes with kinetic energy up to a maximum:

$$hf = \Phi + \tfrac{1}{2}mv_{\max}^2, \qquad \text{so} \qquad \tfrac{1}{2}mv_{\max}^2 = h(f - f_0).$$

Worked example. A metal has a work function of 2.0 eV. Light made of photons of energy 3.5 eV shines on it. Find the maximum kinetic energy of the photoelectrons.

$$\tfrac{1}{2}mv_{\max}^2 = hf - \Phi = 3.5 - 2.0 = 1.5\ \text{eV}\ (= 2.4 \times 10^{-19}\ \text{J}).$$

So the maximum KE of photoelectrons depends **linearly on frequency**, not on brightness.



The maximum kinetic energy of photoelectrons rises linearly with frequency, reaching zero at the threshold frequency f_0

Why the wave model fails

A wave model predicts that brightness should set the electrons' kinetic energy, and that emission should happen at any frequency given enough time. But experiments show:

- **no emission below the threshold frequency**, however bright.
- **immediate emission** at or above the threshold, even when dim.
- **maximum KE depends on frequency**, not brightness.
- the **number of photoelectrons** (the current) depends on brightness.

The photon model explains this: light arrives as photons each of energy hf. One photon–electron interaction either has enough energy to free the electron ($hf \geq \Phi$) or it does not.

Why max KE is fixed but current grows with brightness

A brighter beam of the **same frequency** has **more photons per second**, but each still carries hf. So the maximum KE of any electron is $hf - \Phi$ (set by f only), while the rate of emission (the current) grows with the number of photons, i.e. with brightness. Doubling the brightness doubles the current but does not change the maximum KE.

Wave–particle duality

The photoelectric effect is strong evidence for the **particle** nature of light. But **interference** 干涉 (Young's double slit, the **diffraction grating** 衍射光栅) and **diffraction** 衍射 show its **wave** nature. So light has **both** wave and particle sides — this is **wave–particle duality** 波粒二象性.

De Broglie hypothesis

If a wave can act like particles, perhaps particles can act like waves. De Broglie proposed that any moving particle has a **de Broglie wavelength** 德布罗意波长:

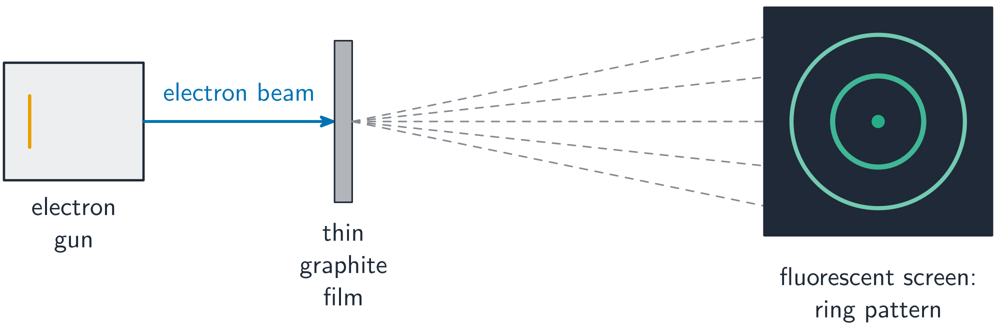
$$\lambda = \frac{h}{p},$$

where $p = mv$. Example: an electron at $v = 4.9 \times 10^7$ m s^{-1} has $p = 4.46 \times 10^{-23}$ kg m s^{-1}, so $\lambda = 1.49 \times 10^{-11}$ m ≈ 0.015 nm — close to atomic spacings.

Electron diffraction

When electrons are fired at a **crystal lattice** 晶格 (e.g. thin graphite), they make a **diffraction pattern** of bright rings on a screen — exactly what waves of wavelength $\lambda = h/p$ would do. This is direct evidence for the **wave nature of particles** (**electron diffraction** 电子衍射): only waves diffract, yet electrons do.

A faster electron has more momentum, so a **shorter** de Broglie wavelength, which diffracts less — the rings move **closer together**. Slowing the electrons spreads the rings apart. To calculate: $p = \sqrt{2mE_{\mathrm{K}}}$, and for an electron accelerated through p.d. V, $E_{\mathrm{K}} = eV$, so $\lambda = h/\sqrt{2m_e eV}$.



Electrons fired at graphite form a ring diffraction pattern — only waves diffract, so electrons behave as waves

Worked example. An electron is accelerated from rest through a p.d. of 2500 V. Find its de Broglie wavelength. ($m_e = 9.11 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J s.)

Its kinetic energy is $E_{\mathrm{K}} = eV$, so $\lambda = \dfrac{h}{\sqrt{2m_e eV}}$:

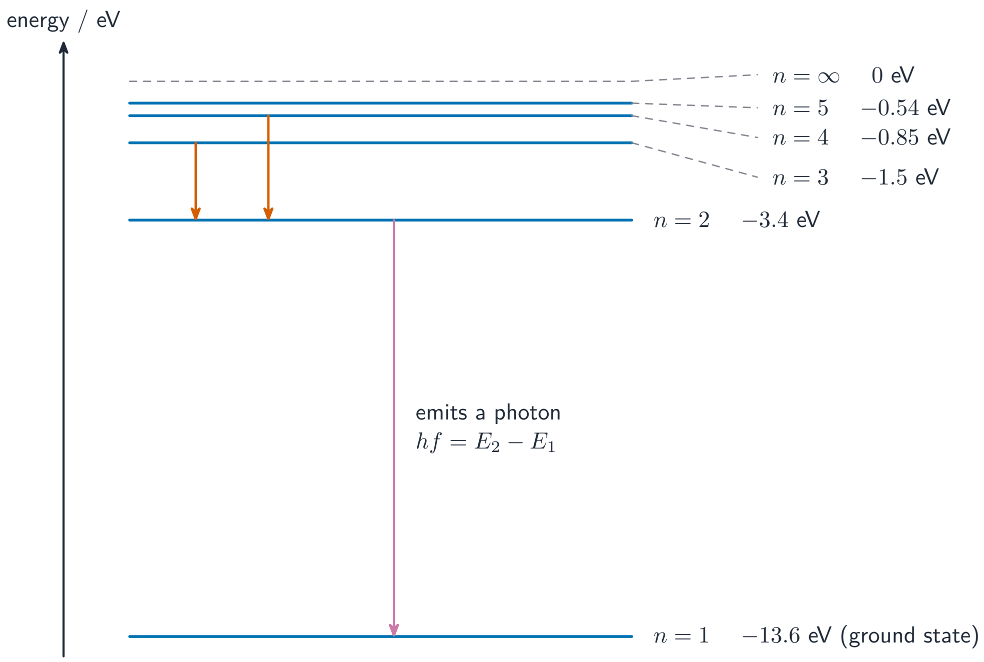
$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2(9.11 \times 10^{-31})(1.6 \times 10^{-19})(2500)}} \approx 2.5 \times 10^{-11} \text{ m}.$$

This is close to the spacing between atoms in a crystal, which is why the electrons diffract off the graphite.

Energy levels in atoms

In an isolated atom, electrons can only sit at certain **discrete 分立 energy levels 能级** — never in between. The lowest is the **ground state 基态**; the others are **excited states 激发态**.

By convention, energies are written **negative**, with $E = 0$ for an electron just free of the atom. For hydrogen the ground state is $E_1 = -13.6$ eV; higher states approach zero.



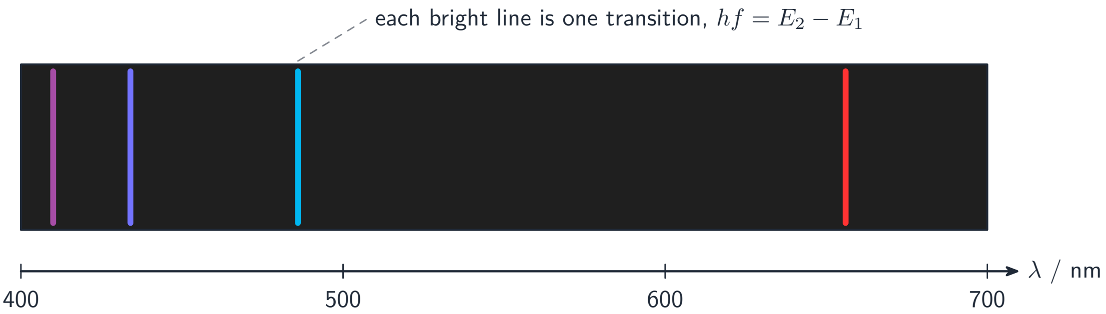
The electron energy levels of hydrogen are discrete and negative, with the ground state at -13.6 eV

Emission spectrum

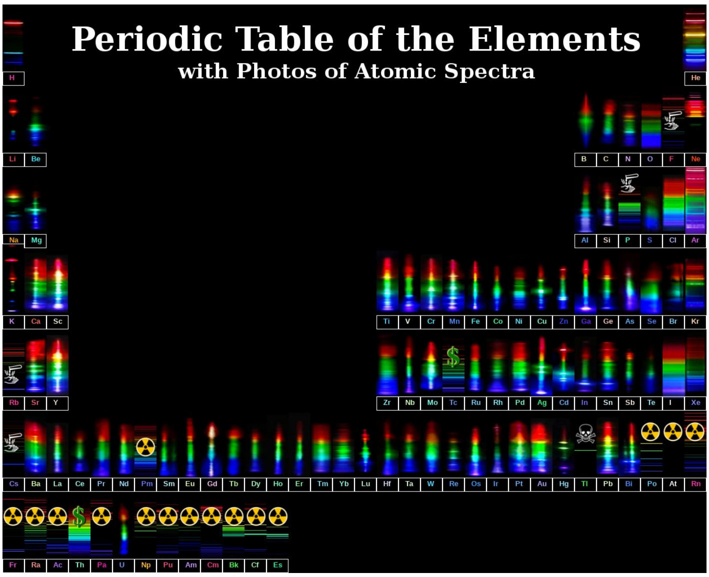
When an electron drops from a higher level E_2 to a lower level E_1, it emits one photon of energy

$$hf = E_2 - E_1.$$

(Both energies are negative; their difference is positive.) Because the levels are discrete, only certain photon energies — and so certain wavelengths — come out. The **emission spectrum** 发射光谱 is a set of **sharp bright lines** on a dark background, one line per **transition** 跃迁. The pattern is a "fingerprint" of the element.



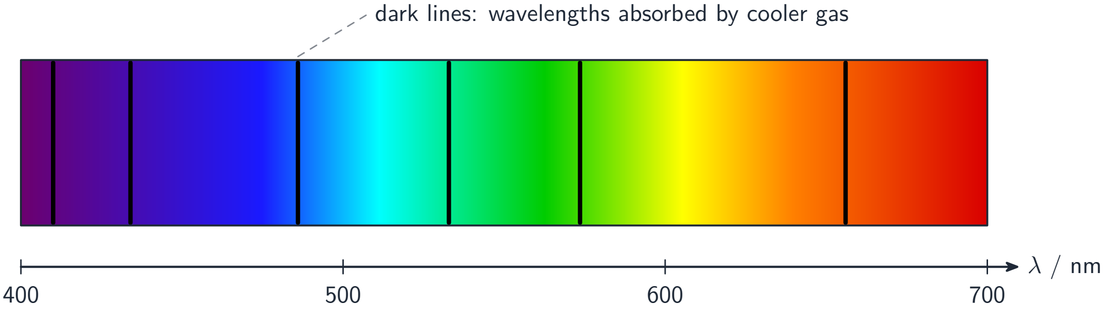
The emission spectrum of hydrogen is a set of sharp bright lines on a dark background



Real emission spectra of the elements: each one is a unique set of bright lines – a fingerprint of that element

Absorption spectrum

When white light passes through a cool gas, photons whose energy exactly matches an upward transition are absorbed. The light then shows **dark lines on a bright background** — the **absorption spectrum** 吸收光谱. The dark lines sit at the **same wavelengths** as the emission lines of the same gas.



Dark absorption lines in the Sun's spectrum mark the wavelengths absorbed by cooler gas

Calculations

For a transition between two known levels:

$$hf = E_2 - E_1, \qquad \lambda = \frac{hc}{E_2 - E_1}.$$

Work in consistent units — convert eV to joules ($\times\ 1.60 \times 10^{-19}$) before finding λ in metres, or use $hc \approx 1240$ eV nm for a quick estimate.

Exam tips

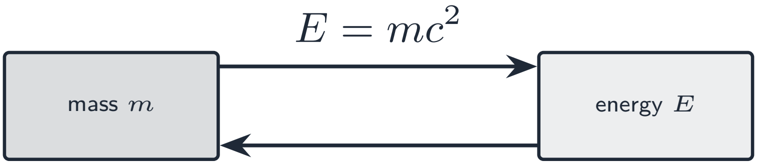
- Photon energy $E = hf = hc/\lambda$; the **photoelectric equation** is $hf = \Phi + \frac{1}{2}mv_{max}^2$.
- The photoelectric effect shows light is **quantised**: below the threshold frequency, no electrons are emitted whatever the intensity.
- **de Broglie** $\lambda = h/p$ gives particles a wavelength; electron diffraction is the evidence.
- Energy levels are discrete; an emitted photon's energy equals the **difference between two levels**.

Nuclear physics

A-Level Physics

Mass-energy equivalence

Einstein's special relativity gives the famous link (**mass-energy equivalence 质能等价**):



Mass and energy are equivalent and can change into each other

$$E = mc^2,$$

where $c = 3.00 \times 10^8$ m s^{-1}. A mass m matches an **energy 能量** E — the two can change into each other. For a mass change Δm:

$$\Delta E = c^2 \Delta m.$$

In nuclear physics the masses are tiny but c^2 is huge, so a small mass change means a large energy. A mass change of 1 u (1.661×10^{-27} kg) matches $\Delta E \approx 1.49 \times 10^{-10}$ J. This gives a conversion you will use again and again:

$$1 \text{ u} \longleftrightarrow 931 \text{ MeV}.$$

Nuclear reactions

A **nuclear reaction 核反应** is written like

$${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \to {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H},$$

with **nucleon number 核子数** conserved (top numbers: $14 + 4 = 17 + 1$) and charge conserved (bottom numbers: $7 + 2 = 8 + 1$) — this is **conservation of charge 电荷守恒**. Use these to fill in an unknown: identify the species, then balance the top and bottom numbers.

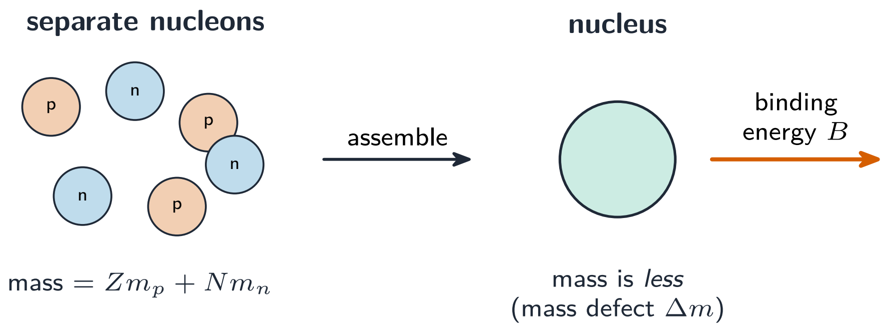
Mass defect and binding energy

The mass of a **nucleus** 原子核 is **less than** the total mass of its separate **protons** 质子 and **neutrons** 中子. The difference is the **mass defect** 质量亏损 Δm:

$$\Delta m = (Zm_\text{p} + Nm_\text{n}) - m_\text{nucleus}.$$

By $E = mc^2$, this "missing" mass was released as energy when the nucleus formed. To pull the nucleus fully apart you must put that energy back — the **binding energy** 结合能 B:

$$B = \Delta m \cdot c^2.$$



The assembled nucleus has less mass than its separate nucleons; the missing mass is released as binding energy

Worked example. A helium-4 nucleus has a mass defect of $\Delta m = 0.0304$ u. Find its binding energy. (1 u corresponds to 931 MeV.)

$$B = \Delta m \cdot c^2 = 0.0304 \times 931 \approx 28 \text{ MeV}.$$

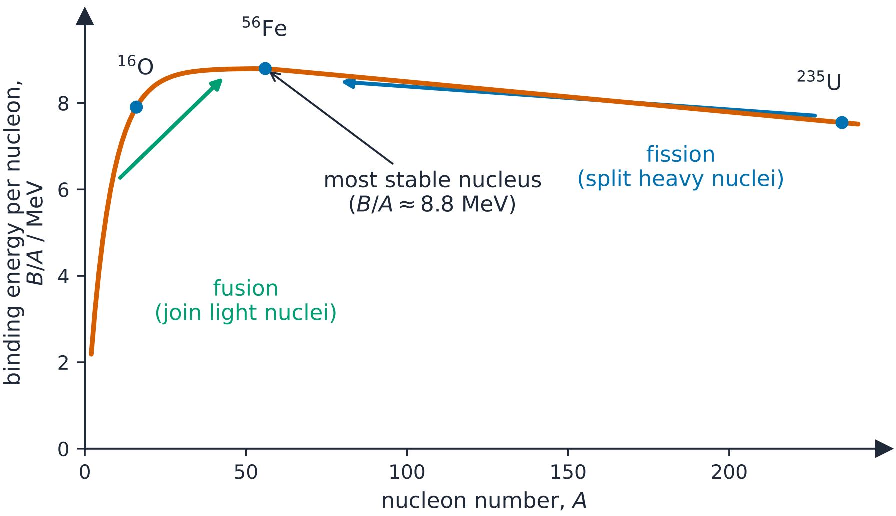
A more tightly bound nucleus has a larger mass defect and larger binding energy. The **binding energy per nucleon** 比结合能 is B/A (usually in MeV per nucleon) — a measure of how tightly each nucleon is held, useful for comparing nuclides.

Binding energy per nucleon vs nucleon number

A graph of B/A against A has a typical shape:

- for light nuclei ($A < 20$), B/A rises quickly (with a spike at the very stable $^4_2$He).
- around $A \sim 56$ (iron), B/A reaches its **maximum** of about 8.8 MeV. **Iron-56 is the most stable nucleus.**
- for heavy nuclei ($A > 100$), B/A falls slowly, to about 7.5 MeV for uranium.

So the curve is dome-shaped, rising to iron then falling.



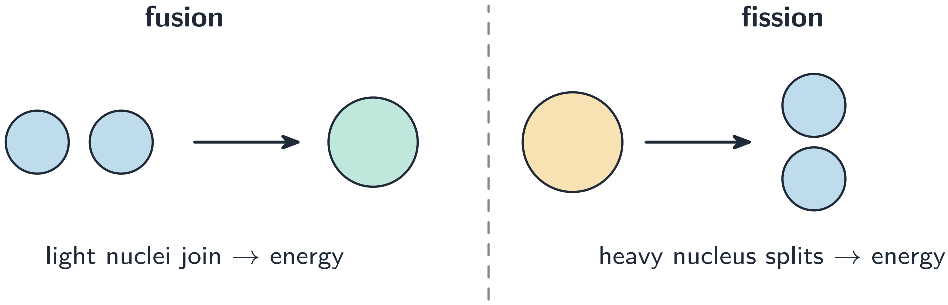
binding energy per nucleon peaks near ^{56}Fe

Binding energy per nucleon peaks near iron ($A \approx 56$); lighter and heavier nuclei are less tightly bound

Nuclear fusion and fission

A nuclear power station releases energy by nuclear fission.

Energy is **released** when nuclei move **towards** the iron peak — by joining light nuclei or splitting heavy ones.



Fusion joins light nuclei; fission splits a heavy nucleus — both release energy by moving towards the iron peak

Nuclear fusion

Nuclear fusion 核聚变 joins two light nuclei into one heavier nucleus:

$$ {}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \to {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} + \text{energy}. $$

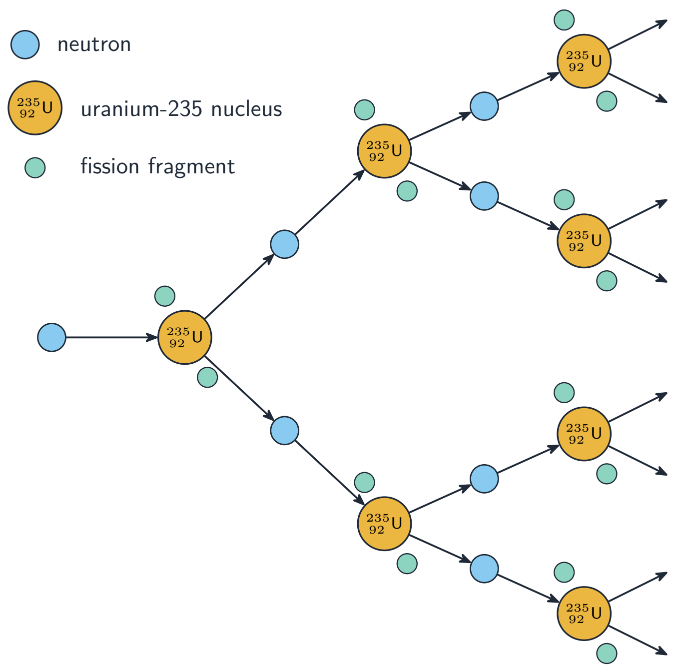
The product has greater binding energy per nucleon than the reactants, so energy is released. Fusion powers stars. It needs very high temperatures (millions of kelvin) so the nuclei have enough **kinetic energy** 动能 to beat their **electrostatic** 静电 repulsion and get close enough for the **strong nuclear force** 强核力 to take over.

Nuclear fission

Nuclear fission 核裂变 splits a heavy nucleus into two lighter ones:

$$ {}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \to {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n} + \text{energy}. $$

The products have higher binding energy per nucleon than $^{235}\mathrm{U}$, so energy is released. The extra neutrons can cause more fissions — a **chain reaction** 链式反应 in a large enough mass of fuel (the **critical mass** 临界质量). This is the basis of nuclear power and weapons.



In an uncontrolled chain reaction each fission of uranium-235 releases neutrons that cause more fissions

Calculating the energy released

1. find the total mass of the reactants.
2. find the total mass of the products.
3. mass change $\Delta m = m_{\text{reactants}} - m_{\text{products}}$ (positive when energy is released).
4. energy released $\Delta E = c^2 \Delta m$.

In kg this gives joules; in atomic mass units use ΔE (MeV) $= \Delta m$ (u) $\times$ 931.

Worked example. In a nuclear reaction the total mass decreases by 0.020 u. Find the energy released.

$$\Delta E = \Delta m \text{ (u)} \times 931 = 0.020 \times 931 \approx 19 \text{ MeV}.$$

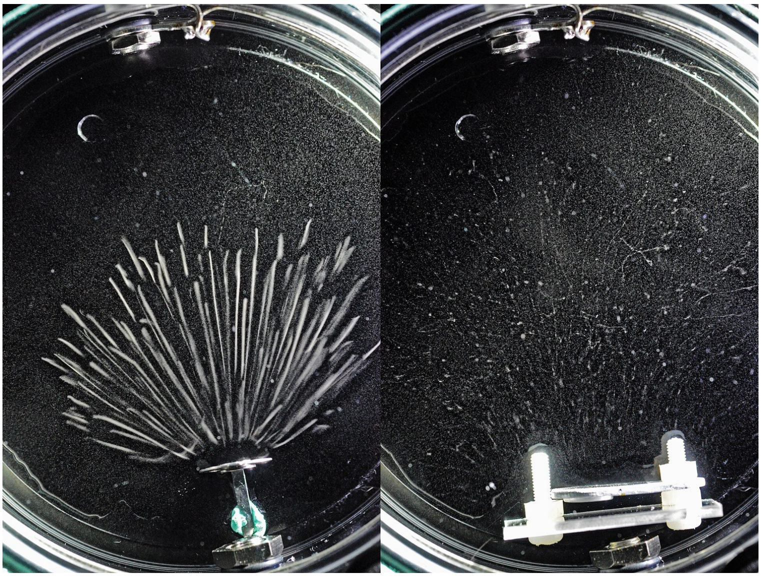
Radioactive decay

Random and spontaneous

Radioactive decay is:

- **spontaneous** 自发 — it happens with no outside trigger, and the rate is not changed by temperature, pressure or chemical state; and
- **random** 随机 — you cannot predict when a given nucleus will decay, only the probability that it decays in a time.

Evidence for randomness: the count rate **fluctuates**. A **Geiger counter** 盖革计数器 next to a source clicks at uneven intervals — never a steady stream — although the long-run mean rate is well-defined.



Each beta particle from the source leaves a thin track in a cloud chamber – direct evidence of separate, random decays

Activity and decay constant

For N undecayed nuclei of a **radionuclide** 放射性核素, the rate of decay is

$$A = \lambda N.$$

- A is the **activity** 活度 — decays per unit time. Unit: **becquerel** 贝克勒尔 (Bq) $= s^{-1}$.
- λ is the **decay constant** 衰变常数 — the probability per unit time that a nucleus decays. Unit: s^{-1}.

λ is fixed for a nuclide; a larger sample (larger N) has proportionally larger activity.

Exponential decay

Since λ is the fractional decay rate, $\dfrac{dN}{dt} = -\lambda N$, whose solution is an **exponential decay** 指数衰减:

$$N = N_0 e^{-\lambda t}.$$

Because $A = \lambda N$, the activity (and any **count rate** 计数率 proportional to it) decays the same way:

$$A = A_0 e^{-\lambda t}.$$

Why exponential? For each nucleus, λ is a fixed probability per unit time, independent of the others and of the nucleus's age. So the same **fraction** decays in each time interval, which gives exponential decay.

Half-life

The **half-life** 半衰期 $t_{1/2}$ is the time for the number of undecayed nuclei (or the activity, or the count rate) to fall to **half**. From $N = N_0 e^{-\lambda t}$ with $N = N_0/2$:

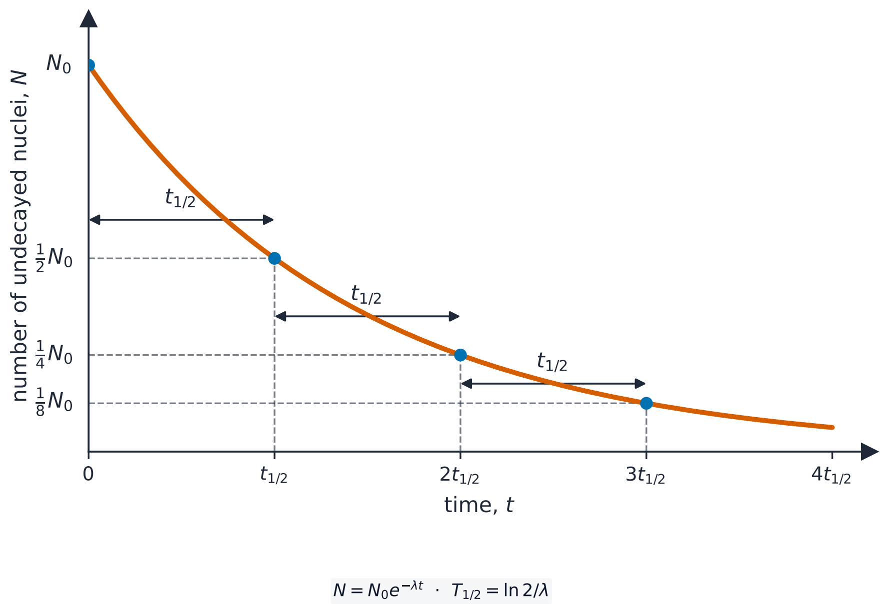
$$\ln 2 = \lambda t_{1/2}, \qquad \lambda = \frac{\ln 2}{t_{1/2}} \approx \frac{0.693}{t_{1/2}}.$$

Worked example. A radioactive isotope has a half-life of 6.0 hours. Find its decay constant.

Convert the half-life to seconds: 6.0 h = 21 600 s. Then

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{21\,600} \approx 3.2 \times 10^{-5}\ \text{s}^{-1}.$$

A larger decay constant means a shorter half-life. After n half-lives the surviving fraction is $(1/2)^n$; after 5 half-lives only about 3% remains.



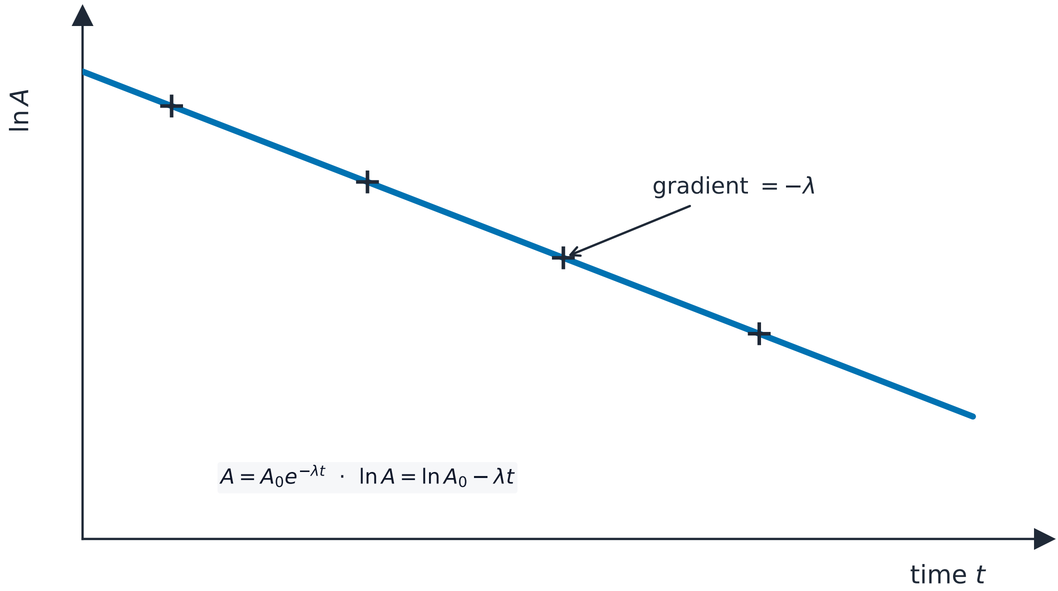
The number of undecayed nuclei falls by half in each half-life

Finding λ from data

Given A_0 and A at time t:

$$\lambda = \frac{1}{t}\ln\frac{A_0}{A}, \qquad t_{1/2} = \frac{\ln 2}{\lambda}.$$

Taking logs of $A = A_0 e^{-\lambda t}$ gives $\ln A = \ln A_0 - \lambda t$, so a plot of $\ln A$ against t is a straight line with gradient $-\lambda$. Use this with several data points.



A graph of $\ln A$ *against time is a straight line of gradient* $-\lambda$

Exam tips

- $E = mc^2$; the **mass defect** is the mass lost when nucleons bind, released as binding energy.
- **Binding energy per nucleon** peaks near iron — both fusion (light nuclei) and fission (heavy nuclei) move towards it and release energy.
- Activity $A = \lambda N$; decay is exponential $N = N_0 e^{-\lambda t}$; half-life $t_{1/2} = \ln 2/\lambda$.
- Balance nuclear equations by conserving **nucleon number and proton number**.

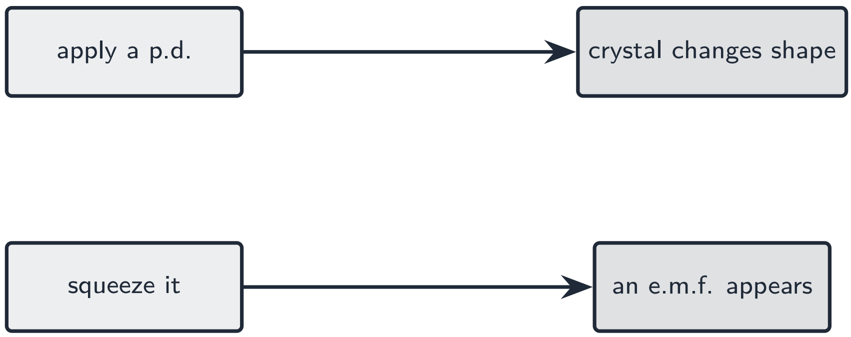
Medical physics

A-Level Physics

Ultrasound

Piezo-electric effect

A **piezo-electric** 压电 crystal changes shape a little when a p.d. is put across it, and the reverse: it makes an **electromotive force** 电动势 (e.m.f.) across itself when its shape is changed. Quartz and PZT are common examples. Two linked effects:



Applying a p.d. deforms the crystal; squeezing it makes a p.d.

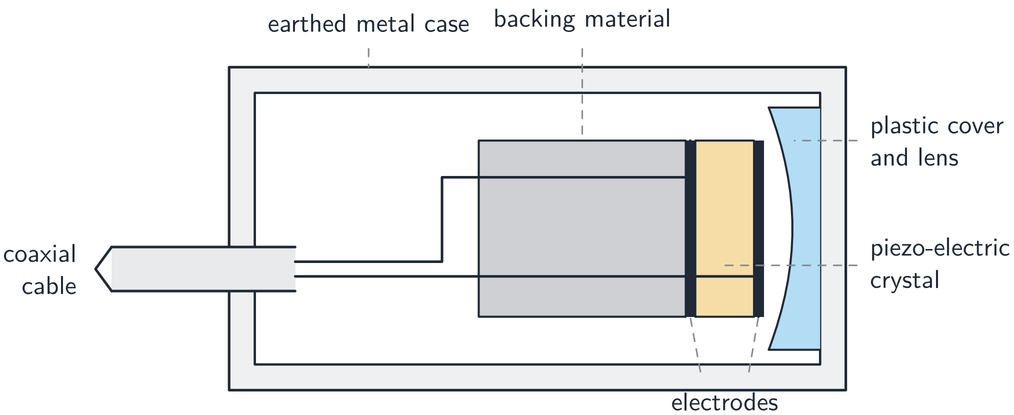
- **apply a p.d.** → the crystal changes shape (used to make vibrations).
- **change the shape** (a wave squeezes it) → an e.m.f. appears (used to detect vibrations).

Piezo-electric transducer

A **transducer** 换能器 uses this effect to both **make** and **detect ultrasound** 超声波.

- an alternating p.d. (a few MHz) makes the crystal vibrate at the same frequency, sending out **longitudinal** 纵波 waves above 20 kHz (1–10 MHz for medical imaging).
- the same crystal then detects: returning ultrasound makes it vibrate and produce an e.m.f.

So one transducer is both emitter and detector, switching between sending and listening.



A piezo-electric transducer both sends and detects ultrasound using a vibrating crystal

Pulse-echo imaging

To see inside the body (**pulse-echo** 脉冲回波 imaging):

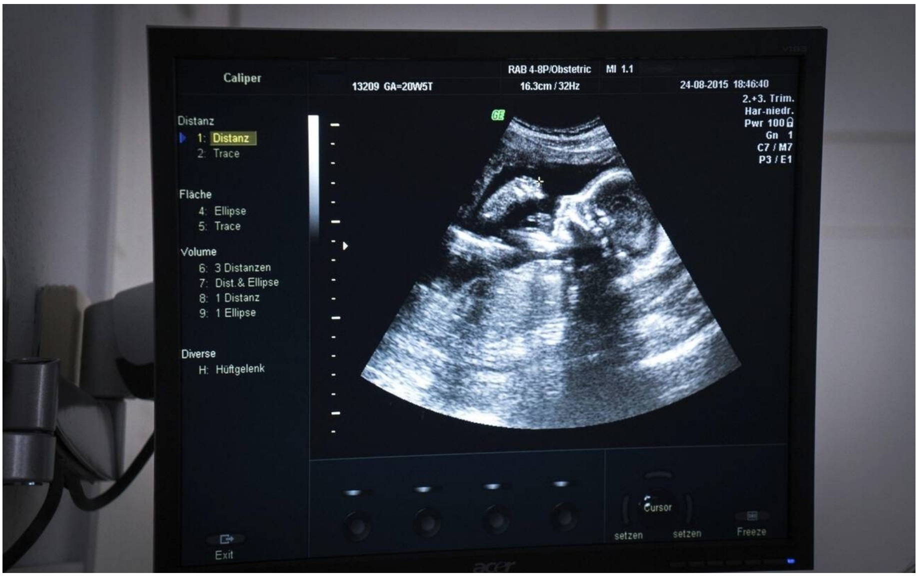
1. the transducer sends a **short pulse** into the body.
2. at each tissue boundary, part of the pulse is **reflected** and part goes on.
3. the transducer detects each reflected pulse.
4. the **time delay** gives the depth: $d = ct/2$ (there and back). The echo's amplitude gives the strength of the reflection.

Worked example. An ultrasound pulse returns to the transducer 60 μs after it was sent. The speed of sound in the tissue is 1500 m s^{-1}. Find the depth of the reflecting boundary.

The pulse travels there and back, so $d = \dfrac{ct}{2}$:

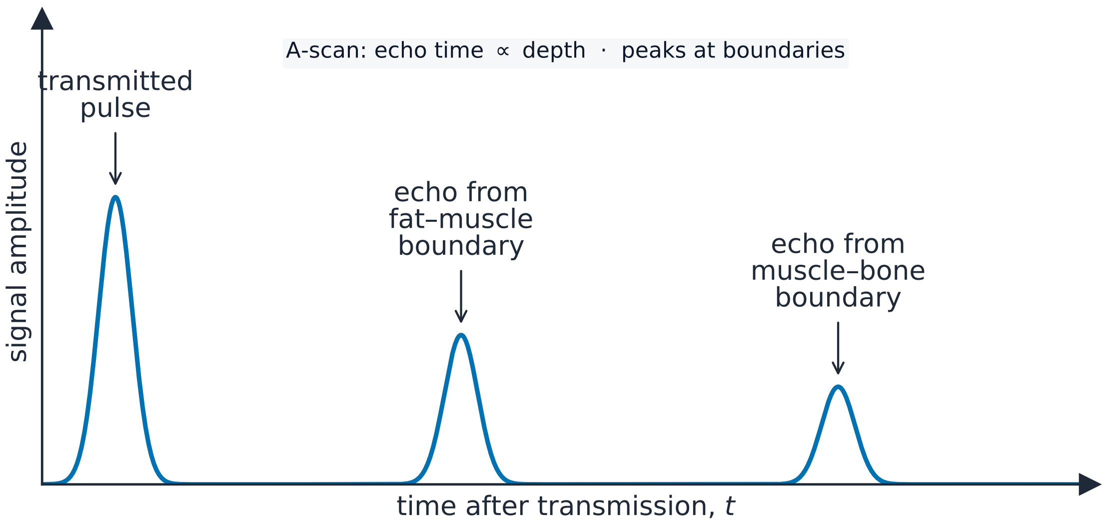
$$d = \frac{1500 \times 60 \times 10^{-6}}{2} = 0.045 \text{ m } (= 4.5 \text{ cm}).$$

1. sweeping across the body builds a 2-D image.



A real ultrasound image — the fan shape comes from the transducer sweeping across the body. Every bright speck is an echo from a boundary between tissues, and its depth was worked out from the echo's time delay, exactly as in the worked example above

A **coupling gel 耦合剂** is put between the transducer and the skin to push out the air; without it almost all the ultrasound would reflect at the skin–air boundary and never enter the body.



An A-scan shows the transmitted pulse and the echoes from each tissue boundary

Specific acoustic impedance

The **specific acoustic impedance** 声阻抗 of a medium is

$$Z = \rho c,$$

where ρ is the **density** 密度 and c the speed of sound. Unit: kg m^{-2} s^{-1}. Bone has large Z; air has small Z; soft tissue is in between.

Worked example. Find the specific acoustic impedance of soft tissue. (Density 1060 kg m^{-3}, speed of sound 1540 m s^{-1}.)

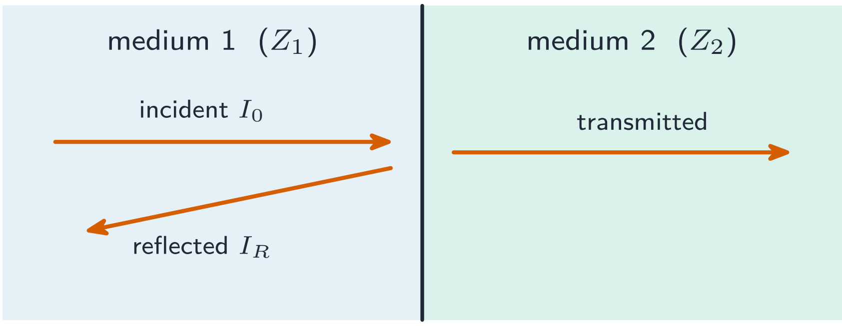
$$Z = \rho c = 1060 \times 1540 \approx 1.6 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-1}.$$

Reflection at a boundary

At a boundary between media of impedance Z_1 and Z_2, the **intensity reflection coefficient** 强度反射系数 (fraction reflected) is

$$\frac{I_\mathrm{R}}{I_0} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2.$$

- very different impedances: almost all is reflected (skin/air — hence the gel).
- very similar impedances: almost nothing is reflected, so the boundary cannot be seen.
- best for imaging: different enough to give an echo, but not so different that nothing passes on.



$\dfrac{I_R}{I_0} = \left(\dfrac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$ — a bigger impedance difference gives a bigger echo

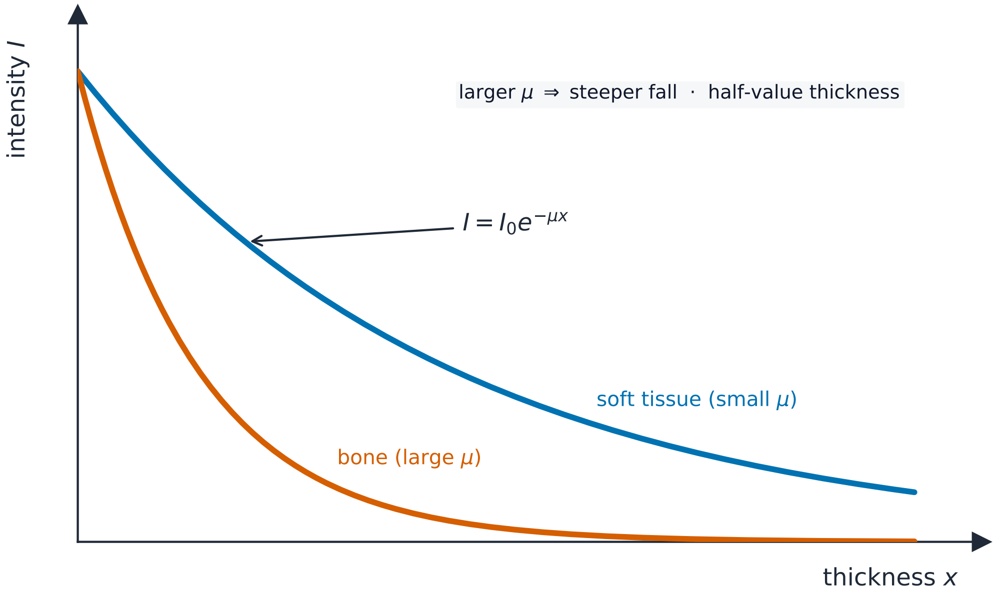
At a boundary, part of the pulse reflects and part transmits — a bigger impedance difference gives a bigger echo

Attenuation

As ultrasound goes through tissue, its intensity falls with distance:

$$I = I_0 e^{-\mu x},$$

where μ is the **attenuation coefficient** 衰减系数 (unit m^{-1}). The same form applies to X-rays.



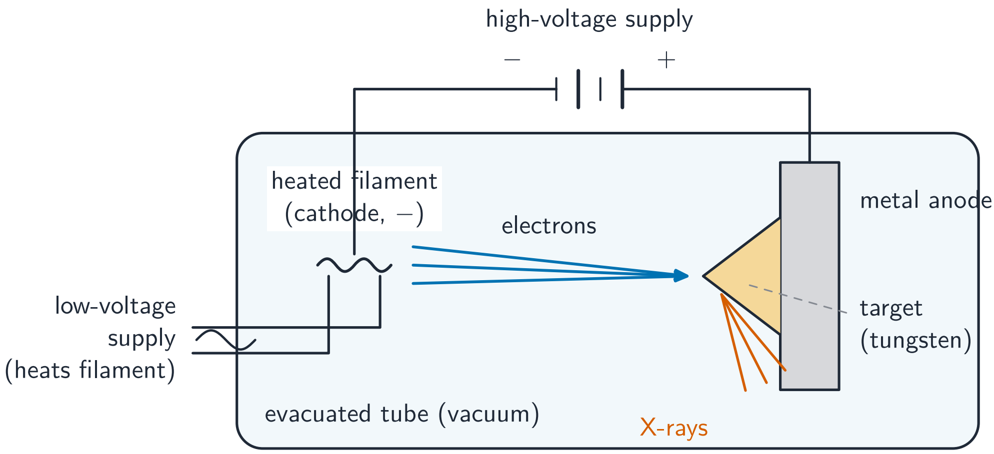
Intensity falls exponentially with thickness; a larger attenuation coefficient (bone) falls off faster

X-rays

Production

X-rays come from an **X-ray tube** X 射线管:

1. a heated **cathode** 阴极 emits electrons by **thermionic emission** 热电子发射.
2. a high p.d. (tens to hundreds of kV) accelerates the electrons across a **vacuum** 真空 to a metal **target** 靶 (the **anode** 阳极, often **tungsten** 钨).
3. the electrons hit the target and slow sharply. Most of their **kinetic energy** 动能 becomes heat; a small part is emitted as X-ray **photons** 光子 (**Bremsstrahlung** 轫致辐射, "braking radiation"). Some electrons knock out inner electrons of the metal atoms, and the refilling emits **characteristic** 特征 X-ray lines.



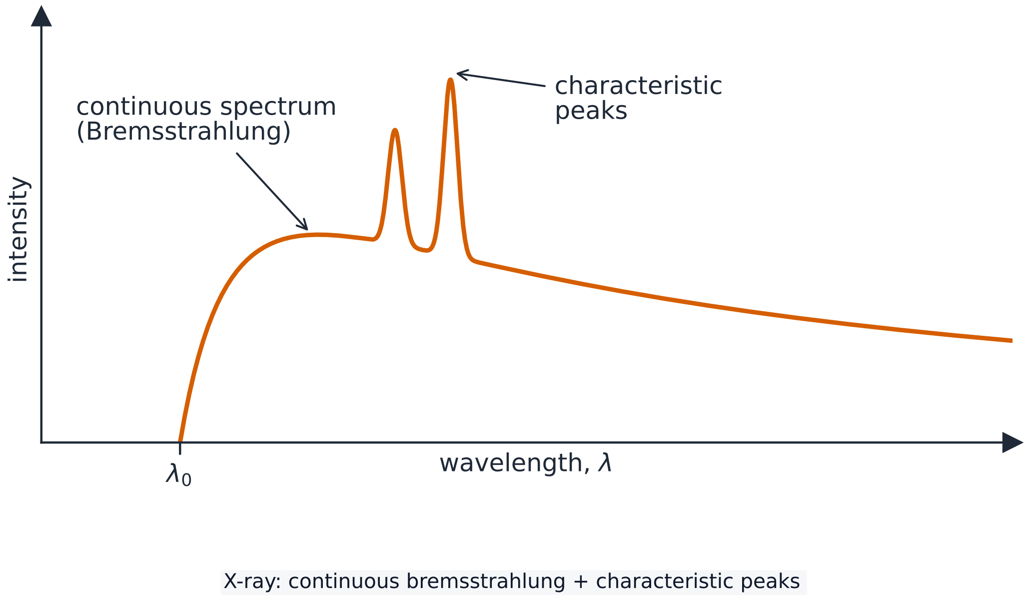
In an X-ray tube electrons from the heated cathode are accelerated onto a metal target anode

Minimum wavelength

The most energy one X-ray photon can have is the full kinetic energy of one accelerated electron, lost in a single event. For accelerating p.d. V, the KE is eV, so

$$hf_{\text{max}} = eV, \qquad \lambda_{\text{min}} = \frac{hc}{eV}.$$

This is the short-wavelength cut-off. The continuous Bremsstrahlung spectrum tails off above λ_{min}, with sharp characteristic peaks set by the target metal.

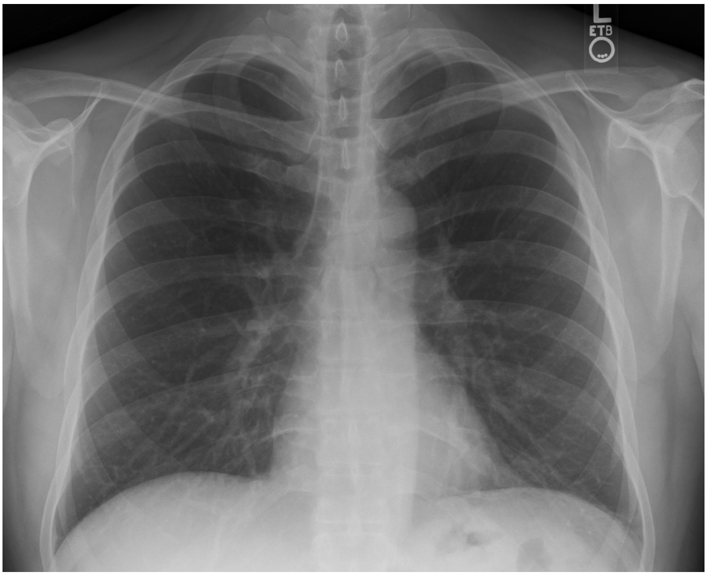


A typical X-ray spectrum — a continuous Bremsstrahlung curve cut off at λ_0, with sharp characteristic peaks

Imaging with X-rays

X-rays pass through the patient onto a detector. Tissues that **attenuate** 衰减 more (bone, high Z) cast a stronger shadow and look lighter; tissues that attenuate less (soft tissue, lung) look darker.

The **contrast** 对比度 is the difference in attenuation between tissues. A **contrast medium** 造影剂 (e.g. a barium meal) can be given to make soft tissues stand out.



A real chest X-ray: dense bone absorbs more X-rays and looks white; the air-filled lungs let X-rays through and look dark

Attenuation law

$$I = I_0 e^{-\mu x}.$$

Higher-energy X-rays penetrate further (smaller μ); bone has a much larger μ than soft tissue. To find the thickness for a given fraction, take logs: $x = \dfrac{1}{\mu}\ln\dfrac{I_0}{I}$. The **half-value thickness** 半值厚度 $x_{1/2} = \ln 2/\mu$ halves the intensity (like half-life in decay).

Worked example. X-rays pass through 3.0 cm of tissue with attenuation coefficient $\mu = 40\ \text{m}^{-1}$. Find the fraction of the intensity that gets through.

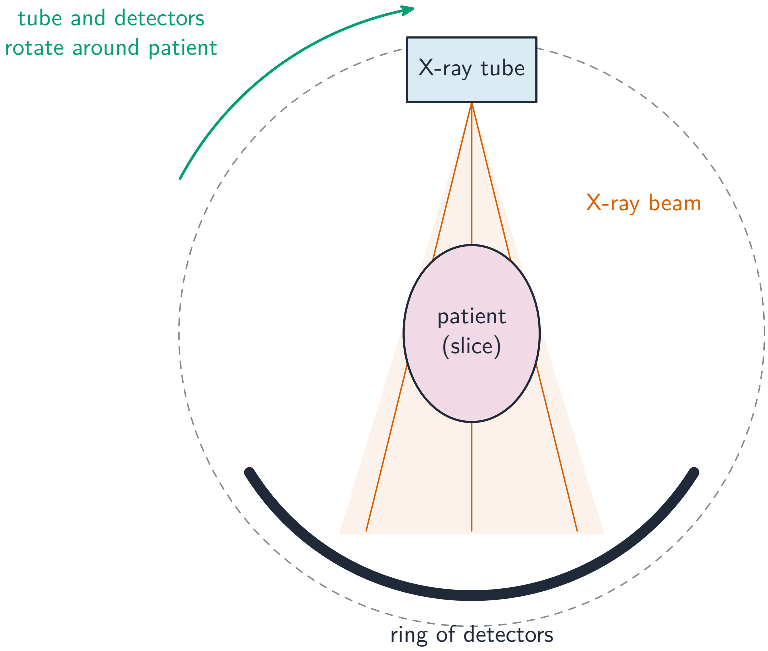
$$\frac{I}{I_0} = e^{-\mu x} = e^{-40\times 0.030} = e^{-1.2} \approx 0.30.$$

Computed tomography (CT)

A **computed tomography** 计算机断层扫描 (CT) scan builds a 3-D image:

1. the tube and detectors rotate around the patient, taking many images of one thin slice from **different angles**.
2. a computer combines these into a 2-D cross-section of the slice.
3. the patient is moved along, and the next slice is imaged.
4. the slices are stacked into a 3-D image.

CT shows far more than a single X-ray, because overlapping soft tissues are separated by the many-angle reconstruction.



In a CT scan the X-ray tube and detectors rotate around the patient to image a slice from many angles

PET scanning

Tracer

A **tracer** 示踪剂 is a substance with radioactive nuclei put into the body. It is taken up more by the tissue being studied (e.g. a tumour takes up more glucose-tagged tracer due to its high **metabolism** 代谢). Its decay is detected from outside.

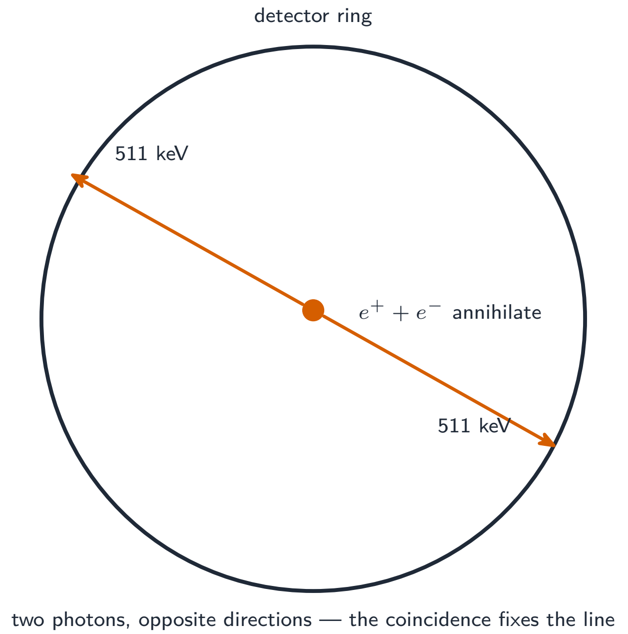
In **positron emission tomography** 正电子发射断层扫描 (PET), the tracer is a β^+

emitter — it gives out a **positron** 正电子. A common one is fluorine-18 on a glucose analogue (FDG).

Annihilation

When a particle meets its **antiparticle** 反粒子 they **annihilate** 湮灭: their mass turns into electromagnetic energy. In PET:

- a positron travels a few mm before meeting an **electron** 电子.
- they annihilate. Energy and **momentum** 动量 are conserved.
- since the total momentum is about zero, two photons are produced going in **opposite directions**, each 511 keV ($= m_e c^2$).



PET: annihilation gives two 511 keV photons in opposite directions; a coincidence fixes the line

Energy of the annihilation photons

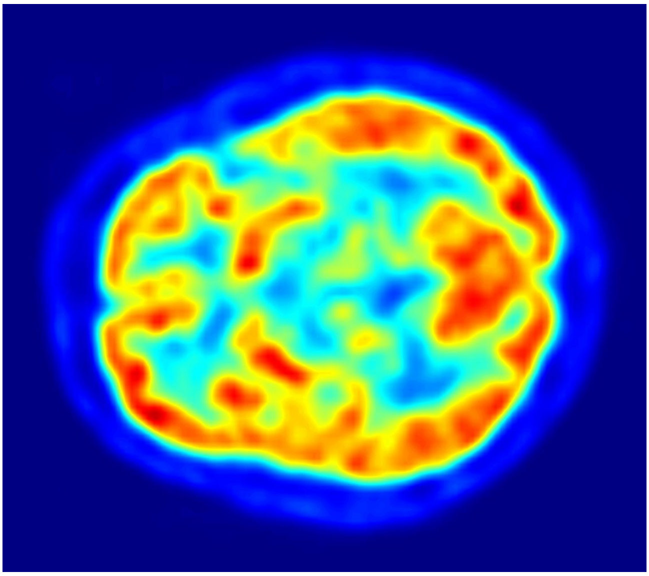
By energy conservation, the total photon energy equals the pair's rest **energy** 能量:

$$2hf = 2m_e c^2, \qquad hf = m_e c^2.$$

Each photon has $hf = m_e c^2 \approx 8.2 \times 10^{-14}$ J ≈ 0.51 MeV, with $\lambda \approx 2.4 \times 10^{-12}$ m.

Reconstructing the image

The two photons leave the body in opposite directions and hit **detector rings** around the patient. Recording the **two simultaneous arrivals** (a "coincidence") fixes the line the annihilation happened on. Many coincidences from many angles let the computer build a 3-D map of the tracer — showing tissues with high metabolic activity. Comparing the two **arrival times** can refine the position along that line (time-of-flight PET).



A finished PET image of the brain: warm colours (red, yellow) mark where the tracer collected – the most active tissue

Exam tips

- **Ultrasound** reflects at boundaries (acoustic impedance); a coupling gel reduces reflection at the skin.
- **X-rays** are attenuated as $I = I_0 e^{-\mu x}$; contrast media (barium, iodine) absorb strongly.
- **PET** uses a positron emitter; annihilation gives two γ-rays in opposite directions, located by the detector ring.

Astronomy and cosmology

A-Level Physics

Luminosity and radiant flux intensity

The **luminosity** 光度 L of a star is the **total power** 功率 of radiation it gives out — the **energy** 能量 radiated per second in all directions. Unit: watt (W).

At distance d, this power has spread over a sphere of area $4\pi d^2$. The **radiant flux intensity** 辐射通量密度 F (power per unit area) at distance d is

$$F = \frac{L}{4\pi d^2}.$$

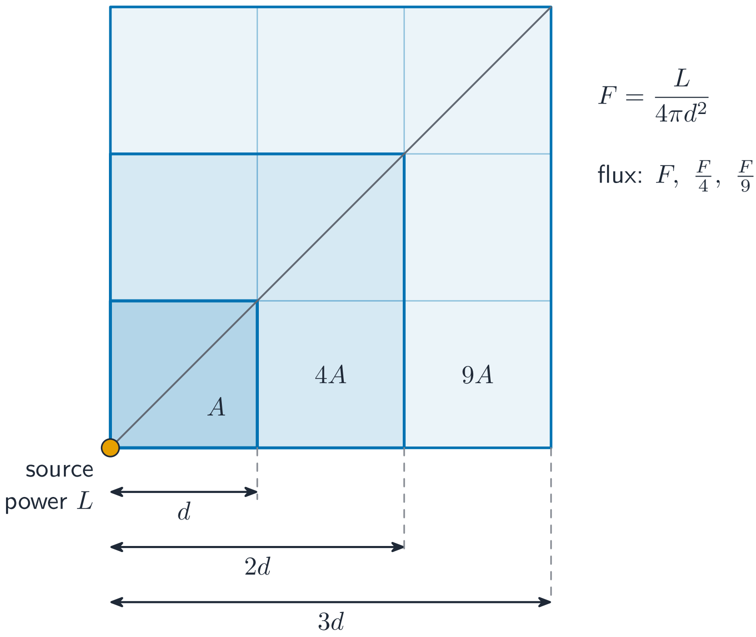
Worked example. The Sun's luminosity is $L = 3.8 \times 10^{26}$ W. Find the radiant flux intensity at the Earth, a distance $d = 1.5 \times 10^{11}$ m away.

$$F = \frac{L}{4\pi d^2} = \frac{3.8 \times 10^{26}}{4\pi(1.5 \times 10^{11})^2} \approx 1.4 \times 10^3 \text{ W m}^{-2}.$$

Unit: W m^{-2}. This is the **inverse-square law** 平方反比定律 for flux: doubling the distance cuts the flux to a quarter. A telescope measures F; if L is known, the distance follows:

$$d = \sqrt{\frac{L}{4\pi F}}.$$

The four units of ESO's Very Large Telescope in Chile, used to measure the flux from distant stars



The same power spreads over a larger area as distance grows, so flux falls as $1/d^2$

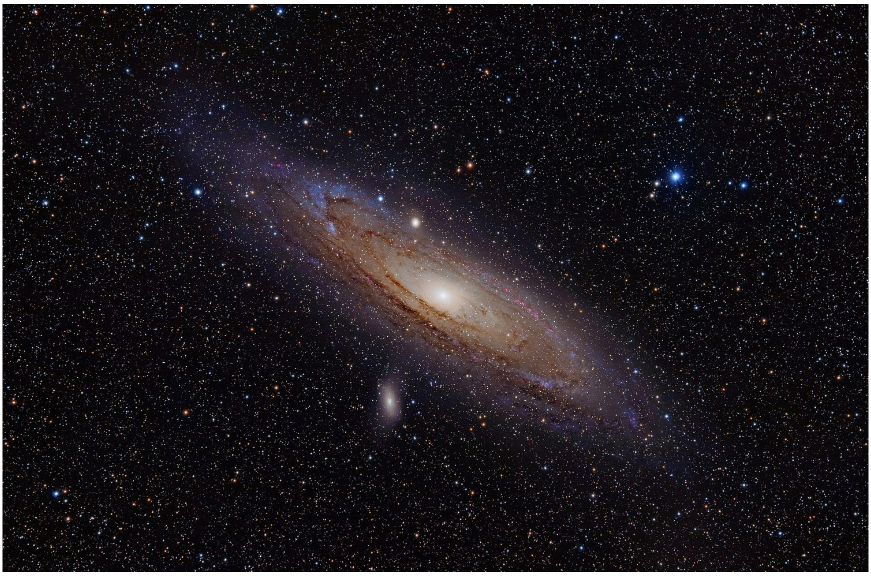
Standard candles

A **standard candle** 标准烛光 is an object whose **luminosity is known** from its type. Once you find one in a distant galaxy and measure the flux F from it, you get its distance from $d = \sqrt{L/(4\pi F)}$.

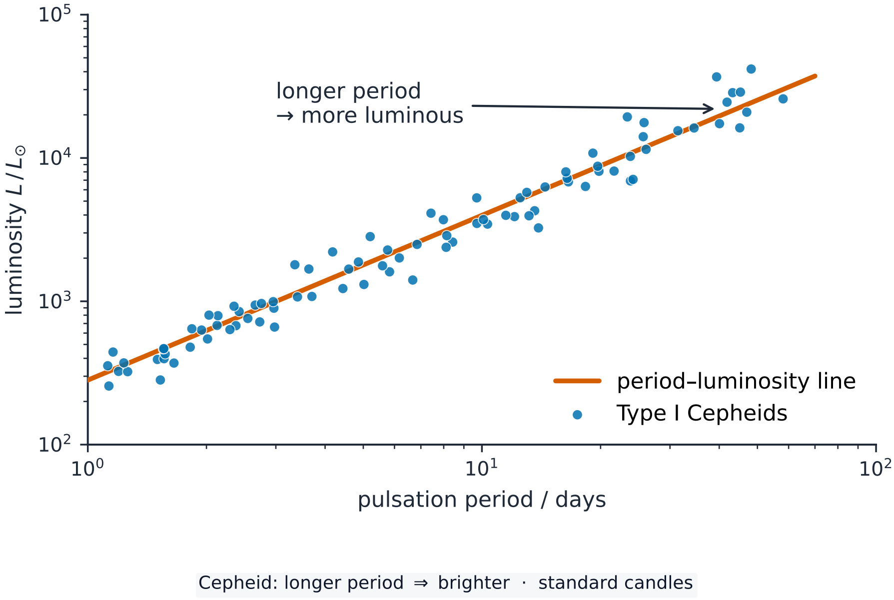
Examples:

- **Cepheid variables** 造父变星 (pulsating stars) — the pulsation period is tightly linked to the luminosity, so the period gives L.
- **Type Ia supernovae** 超新星 — a white dwarf reaching a critical mass and exploding always has about the same peak luminosity.

A standard candle gives L without first knowing the distance, so it reaches galaxies far beyond **parallax** 视差.



The Andromeda Galaxy, our nearest large galaxy, about 2.5 million light-years away — Cepheids in it are standard candles



For Cepheid variables the pulsation period sets the luminosity, making them standard candles

Stellar surface temperature

Wien's displacement law

A hot body gives out a continuous (**blackbody** 黑体) spectrum with a peak at a **wavelength** 波长 $\lambda_{\max}$ set by its **temperature** 温度. **Wien's displacement law** 维恩位移定律:

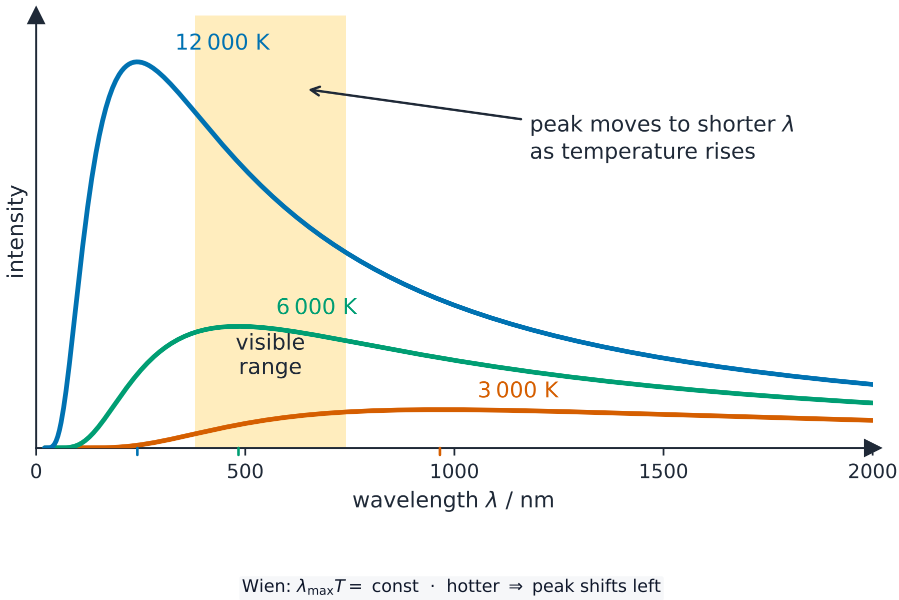
$$\lambda_{\max} T = \text{constant}, \qquad b \approx 2.90 \times 10^{-3}\ \text{m K}.$$

Worked example. A star's blackbody spectrum peaks at $\lambda_{\max} = 500$ nm. Find its surface temperature. ($b = 2.90 \times 10^{-3}$ m K.)

$$T = \frac{b}{\lambda_{\max}} = \frac{2.90 \times 10^{-3}}{500 \times 10^{-9}} \approx 5800\ \text{K}.$$

Hotter stars peak at **shorter** wavelengths: a cool red star (~ 3000 K) peaks in the infrared; the Sun (~ 5800 K) peaks near 500 nm; a hot blue-white star ($\sim 20{,}000$ K) peaks in the ultraviolet. Measuring $\lambda_{\max}$ gives the surface temperature.

The Pillars of Creation in the Eagle Nebula — clouds of gas and dust lit by hot, newly formed stars



A hotter black body radiates more, and its peak wavelength shifts towards the blue (Wien's law)

Stefan–Boltzmann law

A star, treated as a blackbody sphere of radius r and surface temperature T, has luminosity

$$L = 4\pi\sigma r^2 T^4,$$

where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is the **Stefan–Boltzmann constant** 斯特藩-玻尔兹曼常量 (the **Stefan–Boltzmann law** 斯特藩-玻尔兹曼定律). Two strong dependences:

- $L \propto r^2$ — twice the radius, four times the luminosity (same T).
- $L \propto T^4$ — twice the temperature, sixteen times the luminosity (same r).

Worked example. A star has radius $r = 7.0 \times 10^8$ m and surface temperature $T = 5800$ K. Find its luminosity. ($\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}.)

$$L = 4\pi\sigma r^2 T^4 = 4\pi(5.67 \times 10^{-8})(7.0 \times 10^8)^2(5800)^4 \approx 3.9 \times 10^{26} \text{ W}.$$

Estimating a star's radius

Combine the two laws:

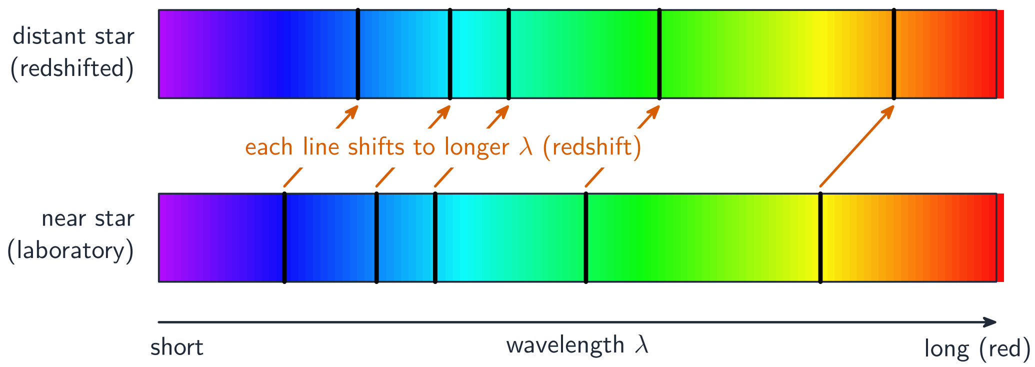
1. measure $\lambda_{\text{max}} \to$ get T from Wien's law.
2. find L (e.g. from flux F and distance d: $L = 4\pi d^2 F$).
3. solve the Stefan–Boltzmann law for r: $r = \sqrt{L/(4\pi\sigma T^4)}$.

This is how astronomers estimate radii of stars they cannot see as a disc.

Redshift, Hubble's law and the Big Bang

Cosmological redshift

The **spectral lines** 谱线 of light from distant galaxies are seen at **longer wavelengths** than their known laboratory values — the whole spectrum is stretched towards the red. This is **redshift** 红移.



The hydrogen absorption lines of a distant star are shifted to longer wavelengths — a redshift

Reading it as a Doppler shift, the galaxy is moving **away**. For $v \ll c$:

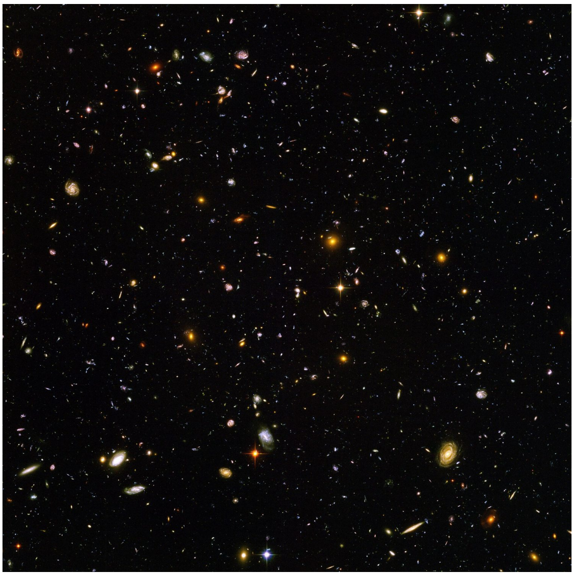
$$\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c},$$

where $\Delta\lambda = \lambda_{\text{observed}} - \lambda_{\text{emitted}}$ and v is the speed of **recession** 退行. Example: light emitted at 4.62×10^{-7} m but seen at 4.91×10^{-7} m gives $\Delta\lambda = 0.29 \times 10^{-7}$ m and

$$v \approx \frac{\Delta\lambda}{\lambda_{\text{em}}} c \approx 1.9 \times 10^{7} \text{ m s}^{-1}.$$

Why redshift means an expanding Universe

Almost every distant galaxy is redshifted (a few near ones are **blueshifted** 蓝移 by local motion). So galaxies are, on average, moving apart — not just from us but from each other. The Universe is **expanding**, with the space between galaxies stretching. More distant galaxies are redshifted more.



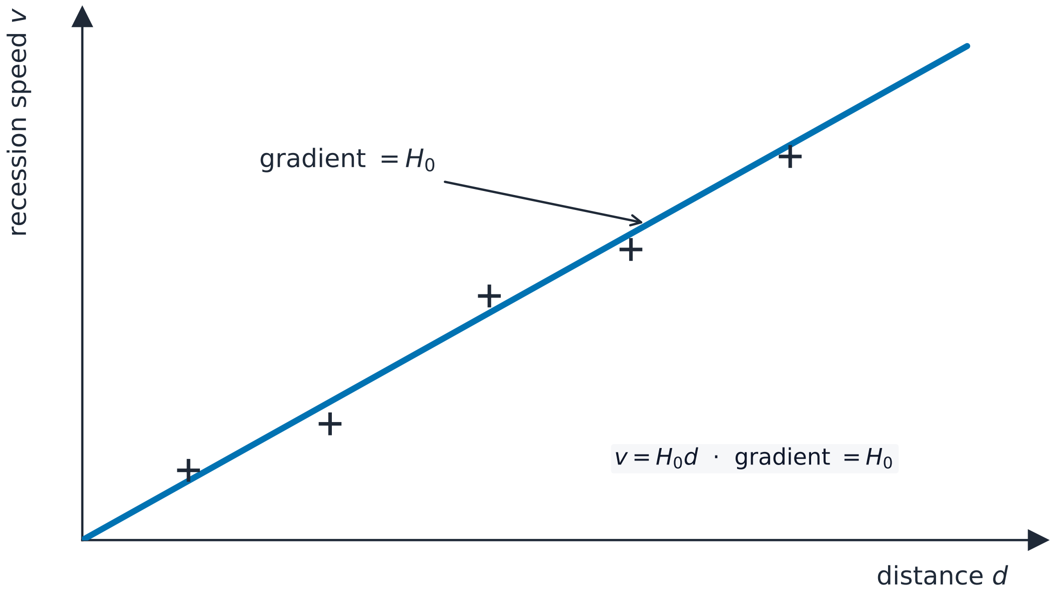
The Hubble Ultra Deep Field — almost every point of light is a whole galaxy, most of them redshifted and receding

Hubble's law

The link between recession speed v and distance d is **Hubble's law** 哈勃定律:

$$v \approx H_0 \cdot d,$$

where H_0 is the **Hubble constant** 哈勃常数 ($\approx 2.3 \times 10^{-18}$ s^{-1}). Always use SI units. Example: a galaxy receding at 1.9×10^7 m s^{-1} is at $d = v/H_0 \approx 8.3 \times 10^{24}$ m.



Hubble's law: a galaxy's recession speed is proportional to its distance, $v = H_0 d$

From Hubble's law to the Big Bang

Hubble's law means the galaxies were once together. Running the expansion backwards, all distances shrink to zero at $t = -1/H_0$ — the Universe was once a tiny, hugely dense, hot point. This is the **Big Bang** 大爆炸. The age of the Universe (for steady expansion) is about

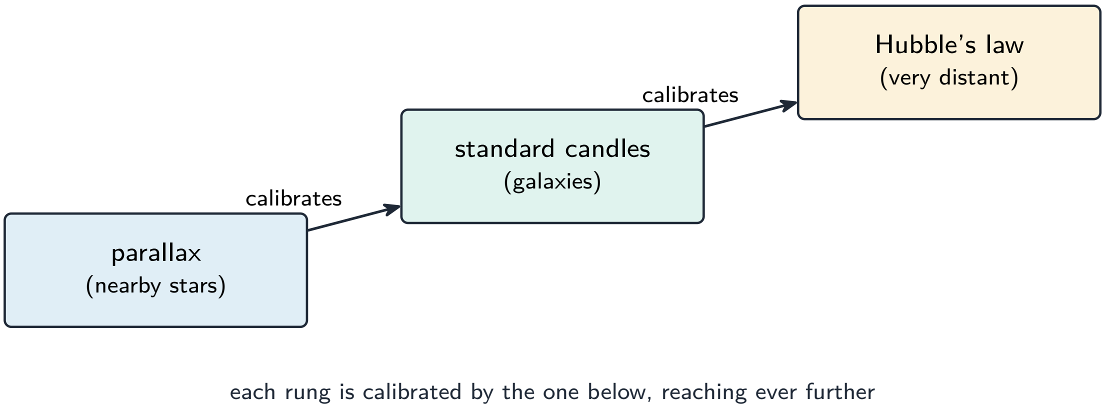
$$T_{\text{age}} \approx \frac{1}{H_0} \approx 4.3 \times 10^{17} \text{ s} \approx 14 \text{ billion years.}$$

The expansion, the redshift of galaxies, the **cosmic microwave background** 宇宙微波背景, and the hydrogen/helium abundances are the main evidence for the Big Bang.

Distance ladder

Astronomers combine methods, each calibrated by the one below:

1. **parallax** — for nearby stars.
2. **standard candles** (Cepheids, Type Ia supernovae) — for galaxies.
3. **Hubble's law** ($d = v/H_0$, with v from redshift) — for very distant galaxies.



The distance ladder: each method is calibrated by the one below and reaches further out

Exam tips

- $F = L/4\pi d^2$ (inverse-square): a standard candle has known luminosity L, so the distance follows from the measured flux F.
- **Wien's law** ($\lambda_{max} \propto 1/T$) gives surface temperature; **Stefan's law** $L = 4\pi r^2 \sigma T^4$.
- **Redshift** $z = \Delta\lambda/\lambda \approx v/c$; Hubble's law $v = H_0 d$ leads to the age of the universe (evidence for the Big Bang).