Week 51

A-Level Physics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

A-Level Physics Name: ______________ Score: ________

1. The mass-energy relation is:

☐ $E = mc^2$

☐ $E = mc$

☐ $E = \frac{1}{2}mc^2$

☐ $E = m^2c$

2. Energy is released when nuclei move:

☐ toward the iron peak (higher binding energy per nucleon)

☐ away from the iron peak

☐ to lower nucleon number, always

☐ nowhere — energy is never released

3. Select **all** the true statements about radioactive decay.

☐ it is spontaneous (no trigger needed)

☐ it is random

☐ the rate is unaffected by temperature

☐ you can predict exactly when one nucleus will decay

Tick every correct option.

4. A mass change of 1 u corresponds to about how many MeV?

______________ MeV

5. Fusion needs very high temperatures to overcome the electrostatic repulsion between nuclei.

☐ True ☐ False

6. The activity of a radioactive source is:

☐ $A = \lambda N$ (decays per second, in Bq)

☐ $A = \dfrac{N}{\lambda}$

☐ $A = \dfrac{\lambda}{N}$

☐ $A = \lambda + N$

7. A nucleus has ______ mass than its separate protons and neutrons added together.

____

8. The extra neutrons released in fission can trigger a ______ reaction.

   ____

9. A source has $\lambda = 0.010$ per second and 1.0×10^6 undecayed nuclei. What is its activity?

   __________________ Bq

10. The binding energy is the energy needed to pull a nucleus completely apart.

    ☐ True ☐ False

A-Level Physics

1. $E = mc^2$

   Mass and energy are equivalent: $E = mc^2$, so a mass change Δm releases $c^2 \Delta m$.

2. **toward the iron peak (higher binding energy per nucleon)**

   Climbing the B/A curve toward iron means the products are more tightly bound, so energy is released.

3. **it is spontaneous (no trigger needed); it is random; the rate is unaffected by temperature**

   It is spontaneous and random, and the rate does not depend on conditions. You can only give the probability of a single decay.

4. **931 MeV**

   1 u $= 1.66 \times 10^{-27}$ kg gives $\Delta E = c^2 \Delta m \approx 931$ MeV.

5. **True**

   The nuclei must get close enough for the strong force to act, despite repelling — hence the millions of kelvin in stars.

6. $A = \lambda N$ **(decays per second, in Bq)**

   Activity = decay constant × number of undecayed nuclei, measured in becquerel.

7. **less**

   That missing mass (the mass defect) was released as energy when the nucleus formed.

8. **chain**

   Each fission releases neutrons that cause more fissions — a chain reaction in a critical mass of fuel.

9. **10000 Bq**

   $A = \lambda N = 0.010 \times 1.0 \times 10^6 = 1.0 \times 10^4$ Bq.

10. **True**

    $B = \Delta m\, c^2$ — the energy released on forming the nucleus, which must be returned to separate it.

23.1 Mass defect and nuclear binding energy

Name: ____________ Class: ________ Date: __________ **Total: 27 marks**

KEY WORDS energy 能量 • mass defect 质量亏损 • binding energy 结合能 • binding energy per nucleon 比结合能 • nucleon number 核子数

Objective

Build the skills to answer exam questions on **mass defect and binding energy**.

You must be able to:

- use $E = mc^2$ and balance **nuclear equations**
- define **mass defect** and **binding energy**
- use the **binding energy per nucleon** curve for fusion and fission

1 Worked examples

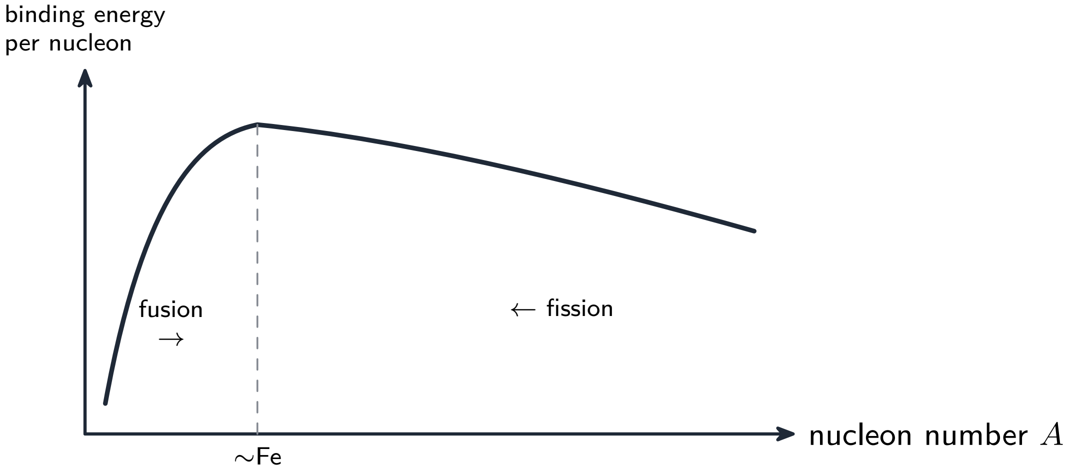
Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

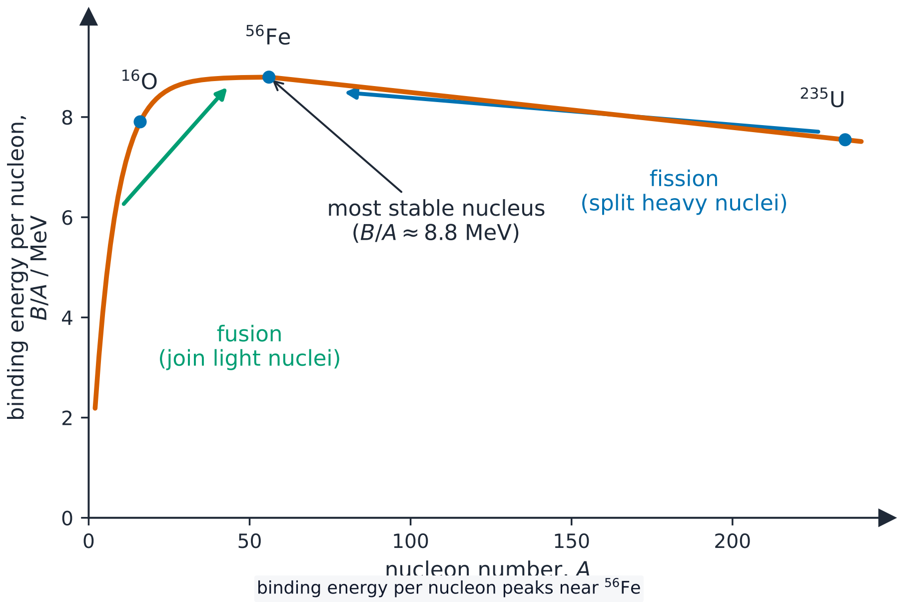
■ **Mass-energy and binding energy**

$$E = mc^2, \qquad \Delta E = c^2 \Delta m, \qquad 1\text{ u} \approx 931\text{ MeV}.$$

The **mass defect** is the difference between the mass of the separate nucleons and the mass of the nucleus; the **binding energy** is the energy equivalent of this ($\Delta E = c^2 \Delta m$) — the energy to split the nucleus apart.

■ **Binding energy per nucleon**





Binding energy per nucleon across the nuclides

The peak (near iron) is the **most stable**. **Fusion** of light nuclei and **fission** of heavy nuclei both move **towards the peak** — increasing binding energy per nucleon and **releasing energy**.

In nuclear equations, **nucleon number** and **charge** are conserved.

■ Energy released in a decay (from masses)

The energy released is the **mass lost** (parent mass − total product mass), converted with 1 u ≈ 931 MeV:

$$Q = \left(m_{\text{before}} - m_{\text{after}}\right) \times 931\ \text{MeV/u}.$$

Worked example. ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\alpha$, with masses (u): U = 238.05079, Th

$= 234.04360$, $\alpha = 4.00260$. Mass lost $\Delta m = 238.05079 - (234.04360 + 4.00260) = 0.00459$ u; energy released $Q = 0.00459 \times 931 = 4.3$ MeV.

Working **backwards**: if the released energy is known, $\Delta m = Q/931$ u, which fixes a missing mass.

2 Practice

Now apply the methods above.

2.1 State **Einstein's** mass–energy equation. [1]

2.2 Define **mass defect**. [1]

2.3 Define **binding energy**. [1]

2.4 In ${}^{14}_{7}\mathrm{N} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{A}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H}$, find the nucleon number A. [1]

3 Exam-style questions

3.1 The **most stable** nuclei have: [1]

- **A** the highest mass
- **B** the highest binding energy per nucleon
- **C** the lowest binding energy per nucleon
- **D** zero binding energy

3.2 A reaction has a mass defect of 0.030 u. Calculate the energy released. (1 u = 931 MeV.) [2]

3.3 Explain how the binding-energy-per-nucleon curve shows that **both** fusion of light nuclei **and** fission of heavy nuclei can release energy. [3]

3.4 Calculate the energy equivalent of a mass change of 1.0×10^{-29} kg. [2]

3.5 In the decay $^{210}_{84}\mathrm{Po} \rightarrow \ ^{206}_{82}\mathrm{Pb} + \ ^{4}_{2}\alpha$, the masses (u) are: Po = 209.98286, Pb = 205.97444, $\alpha = 4.00260$. Calculate the energy released, in MeV. (1 u = 931 MeV.) [3]

3.6 Deuterium nuclei fuse to form helium-3 in the reaction

$$^{2}_{1}\mathrm{H} + ^{2}_{1}\mathrm{H} \rightarrow \ ^{3}_{2}\mathrm{He} + ^{1}_{0}\mathrm{n}$$

The masses are: deuterium 2.01410 u, helium-3 3.01603 u, neutron 1.00867 u. Take 1 u $= 1.66 \times 10^{-27}$ kg, $c = 3.00 \times 10^{8}$ m s^{-1} and 1 MeV $= 1.60 \times 10^{-13}$ J.

(a) **State** what is meant by the **mass defect** of a nucleus. [1]

(b) **Determine** the mass difference of this reaction, in u. [2]

(c) **Determine** the energy released, in joules and in MeV. [3]

(d) **Determine** the energy released, in MeV, when 1.0 g of deuterium is fused this way. [3]

(e) **Explain** why fusion releases energy for light nuclei but fission releases energy for heavy ones, referring to **binding energy per nucleon**. [3]

Solutions

2.1 $E = mc^2$.

2.2 The difference between the **total mass of the separate nucleons** and the **mass of the nucleus**.

2.3 The energy needed to **separate a nucleus** into its individual nucleons (equivalent to the mass defect).

2.4 $14 + 4 = A + 1 \Rightarrow A = 17$.

3.1 **B** — the highest binding energy per nucleon.

3.2 $E = 0.030 \times 931 = 28$ MeV.

3.3 Both light nuclei (fusing) and heavy nuclei (splitting) move **towards the peak** of the curve; this **increases** the binding energy per nucleon; the products are more tightly bound, so energy is **released**.

3.4 $E = mc^2 = (1.0 \times 10^{-29})(3.0 \times 10^8)^2 = 9.0 \times 10^{-13}$ J.

3.5 Mass lost $\Delta m = 209.98286 - (205.97444 + 4.00260) = 0.00582$ u; energy released $Q = 0.00582 \times 931 = 5.4$ MeV.

3.6 (a) The **difference** between the total mass of the separate nucleons and the mass of the assembled nucleus.
(b) Left: $2 \times 2.01410 = 4.02820$ u; right: $3.01603 + 1.00867 = 4.02470$ u; difference $= 0.00350$ u.
(c) $\Delta m = 0.00350 \times 1.66 \times 10^{-27} = 5.81 \times 10^{-30}$ kg; $E = \Delta mc^2 = 5.81 \times 10^{-30} \times (3.00 \times 10^8)^2 = 5.2 \times 10^{-13}$ J; dividing by 1.60×10^{-13} gives 3.3 MeV.
(d) 1.0 g of deuterium contains $\dfrac{1.0 \times 10^{-3}}{2.01410 \times 1.66 \times 10^{-27}} = 3.0 \times 10^{23}$ nuclei; the reaction consumes **two** nuclei per event, so there are 1.5×10^{23} events; total $= 1.5 \times 10^{23} \times 3.3 = 4.9 \times 10^{23}$ MeV.
(e) Binding energy per nucleon rises steeply from hydrogen to a maximum near iron, then falls slowly for heavier nuclei. Energy is released whenever the products have a **greater** binding energy per nucleon than the reactants. So joining light nuclei moves up the curve and releases energy, while splitting a heavy nucleus moves up it from the other side and does the same.

9 **(a)** State what is meant by the mass defect of a nucleus.

....

....

.... [2]

(b) The nuclear fusion reaction for the formation of helium-4 from deuterium is represented by

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{4}_{2}He.$$

Table 9.1 shows the masses of the nuclides involved in this reaction.

Table 9.1

nuclide	nuclide mass / u
$^{2}_{1}H$	2.013 553
$^{4}_{2}He$	4.001 505

Calculate the energy released in the formation of 1.00 mol of helium-4.

energy = ... J [4]

(c) The star Sirius has a radius of 1.19×10^{9} m and loses mass due to nuclear fusion at a rate of 1.09×10^{11} kg s^{-1}. Assume that the power of the radiation emitted by the star is equal to the power released by this process.

(i) Determine a value for the luminosity of Sirius. Give a unit with your answer.

luminosity = ... unit .... [2]

(ii) Use your answer in **(c)(i)** to determine the surface temperature of Sirius.

surface temperature = ... K [2]

(d) Explain how cosmologists use standard candles to estimate the distance of a galaxy from the Earth.

....

....

....

....

.... [3]

[Total: 13]

11 The deuterium nucleus ($^{2}_{1}H$) has a mass defect of 0.002 388 u. The helium-4 nucleus ($^{4}_{2}He$) has a mass defect of 0.030 377 u. Helium-4 is formed from deuterium in a nuclear reaction that can be represented by the equation

$$3\,^{2}_{1}H \longrightarrow \,^{4}_{2}He + \,^{1}_{1}p + \,^{1}_{0}n.$$

(a) **(i)** State the name of this type of nuclear reaction.

.... [1]

(ii) Show that the energy released when one nucleus of helium-4 is formed from deuterium is 3.47×10^{-12} J.

[3]

(b) A star has a radius of 6.96×10^{8} m. Helium-4 is produced in this star, from deuterium, at a mass rate of $7.34 \times 10^{11}\,\text{kg}\,\text{s}^{-1}$. All the energy released from this process is radiated away from the star. All the energy that is radiated from the star is released by this process.

(i) Calculate the luminosity of the star.

luminosity = ... W [3]

(ii) Use your answer in **(b)(i)** to determine the surface temperature of the star.

temperature = ... K [2]

[Total: 9]

9 Polonium-193 ($^{193}_{84}Po$) is an unstable nuclide. A nucleus of polonium-193 decays to a nucleus of lead-189 ($^{189}_{82}Pb$) by emitting an alpha-particle.

(a) Radioactive decay is both random and spontaneous.

State what is meant by:

(i) random

....

.... [1]

(ii) spontaneous.

....

.... [1]

(b) Define half-life.

....

....

.... [1]

(c) Data for the binding energy per nucleon of the particles involved in the decay of a nucleus of polonium-193 are given in Table 9.1.

Table 9.1

particle	binding energy per nucleon / eV
$^{193}_{84}Po$	7.774
$^{189}_{82}Pb$	7.826
$^{4}_{2}\alpha$	7.074

Determine the energy, in eV, released when a nucleus of polonium-193 decays into a nucleus of lead-189.

ergy = ... eV [2]

(d) A pure sample of polonium-193 contains N_0 nuclei. After a time t the sample contains N nuclei of polonium-193. The variation of $\ln(N/N_0)$ with t is shown in Fig. 9.1.

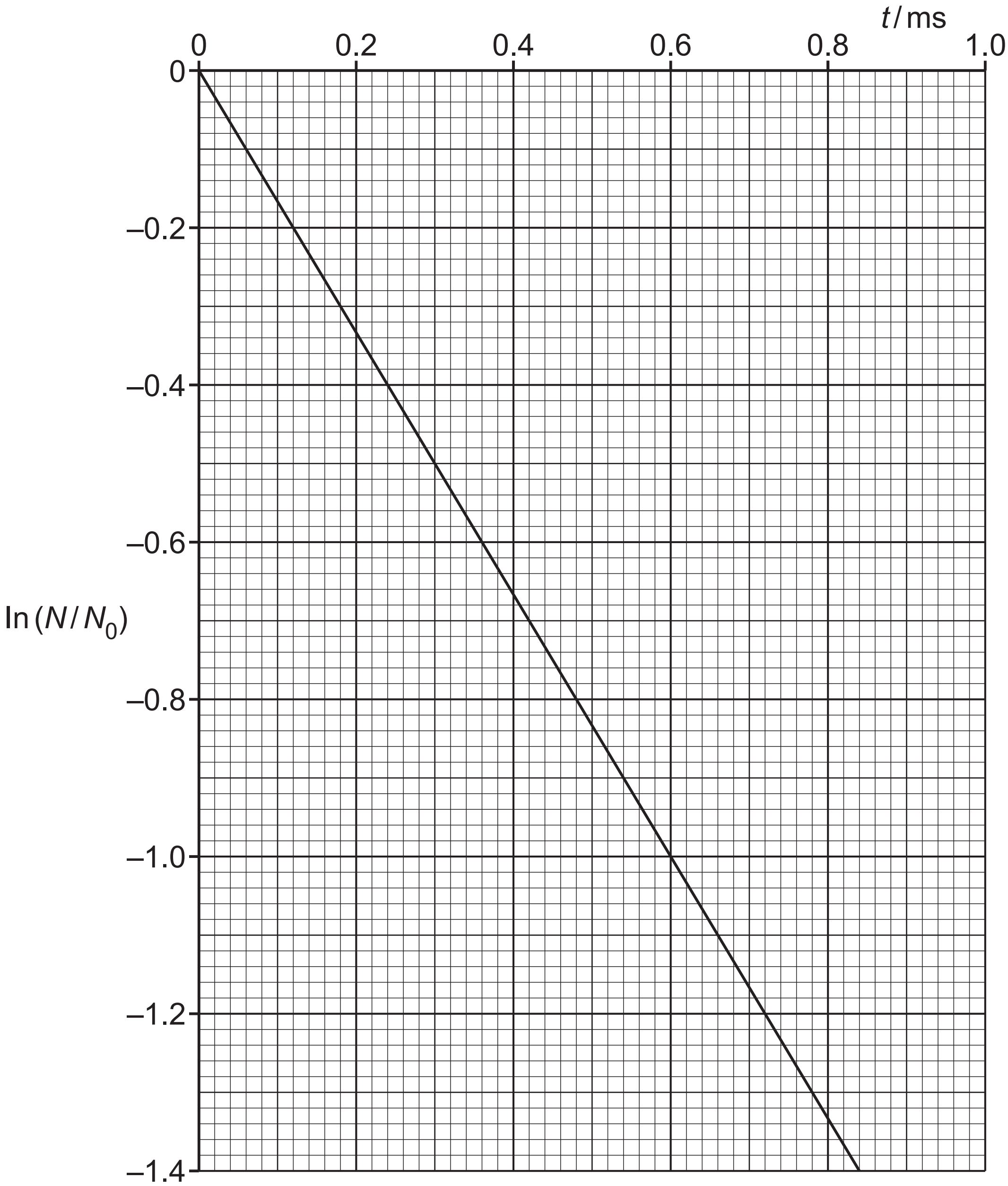


Fig. 9.1

(i) State the name of the quantity that is represented by the magnitude of the gradient of the line in Fig. 9.1.

.... [1]

(ii) Use Fig. 9.1 to determine the half-life, in ms, of polonium-193.

half-life = ... ms [2]

(e) Positron emission tomography (PET scanning) uses a radioactive tracer.

(i) State what happens to the positrons emitted by the tracer.

....

.... [1]

(ii) Explain why a tracer with a half-life of approximately 2 hours is a suitable tracer to use.

....

.... [1]

[Total: 10]

23.2 Radioactive decay

Name: ______________ Class: __________ Date: ____________ **Total: 32 marks**

KEY WORDS activity 活度 • half-life 半衰期 • spontaneous 自发 • decay constant 衰变常数 • random 随机

Objective

Build the skills to answer exam questions on **radioactive decay**.

You must be able to:

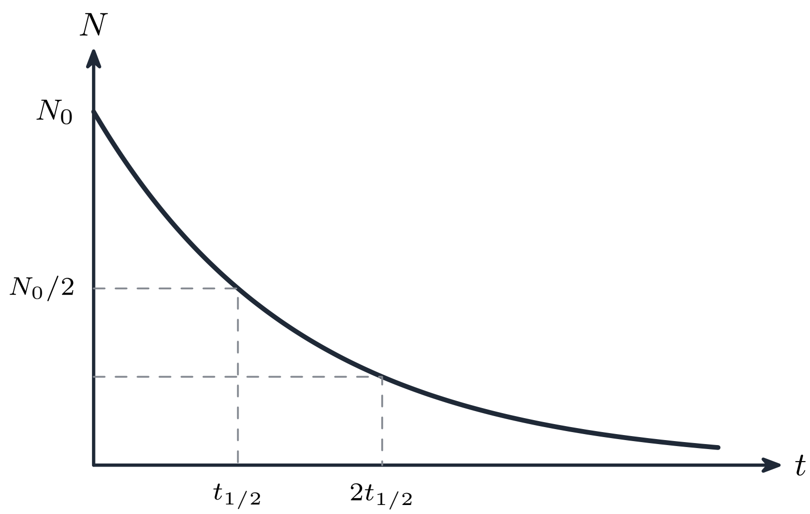
- describe decay as **spontaneous** and **random**
- use **activity** $A = \lambda N$ and $\lambda = 0.693/t_{1/2}$
- use the exponential decay $x = x_0 e^{-\lambda t}$ and **half-life**

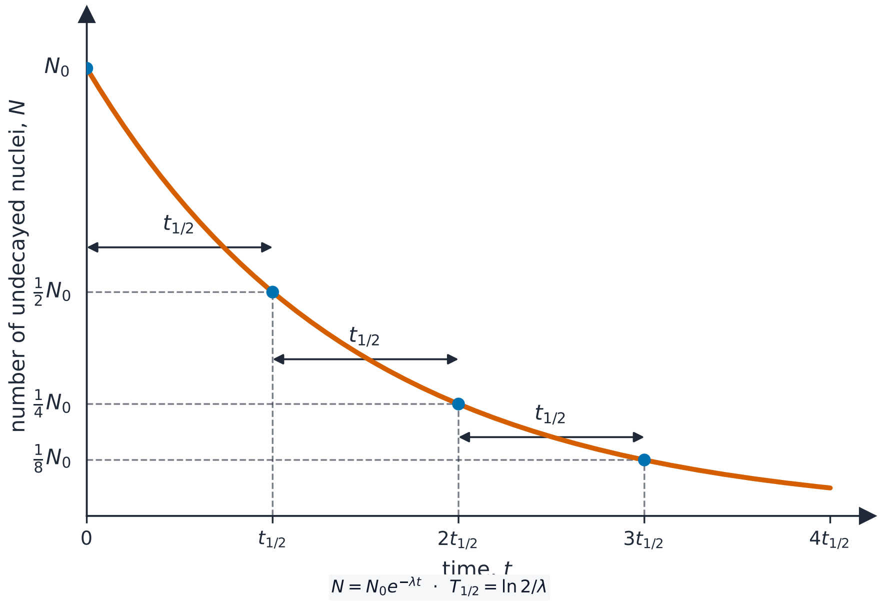
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Random and exponential

Decay is **spontaneous** (no trigger) and **random** (the count rate fluctuates). The number of undecayed nuclei falls **exponentially**, halving every **half-life**:





The exponential radioactive decay curve

$$A = \lambda N, \qquad \lambda = \frac{0.693}{t_{1/2}}, \qquad N = N_0 e^{-\lambda t}.$$

- **activity** A — decays per second (becquerel, Bq); **decay constant** λ — probability of decay per second.

The same exponential law applies to the **activity** and the **count rate**: $A = A_0 e^{-\lambda t}$.

■ Finding the time — take logs

For a **whole number** of half-lives, just halve repeatedly (after n half-lives a fraction $(\frac{1}{2})^n$ remains). For **any** time or fraction, take the natural log of $N = N_0 e^{-\lambda t}$:

$$\ln\frac{N}{N_0} = -\lambda t \quad \Longrightarrow \quad t = -\frac{1}{\lambda}\ln\frac{N}{N_0}.$$

Worked example. A source has half-life 8.0 days, so $\lambda = \dfrac{0.693}{8.0} = 0.0866\ \text{day}^{-1}$. The time for its activity to fall to 20% is $t = -\dfrac{1}{0.0866}\ln(0.20) = 18.6$ days.

2 Practice

Now apply the methods above.

2.1 State **two** features of radioactive decay. [2]

2.2 Define **activity** and the **decay constant**. [2]

2.3 Define **half-life**. [1]

2.4 Write the equation linking **activity**, **decay constant** and **number** of nuclei. [1]

3 Exam-style questions

3.1 The **half-life** is the time for: [1]

- **A** all the nuclei to decay
- **B** half the (undecayed) nuclei to decay
- **C** the activity to reach exactly zero
- **D** the decay constant to halve

3.2 A source has a decay constant of 0.040 s^{-1}. Calculate its **half-life**. [2]

3.3 A sample has 2.0×10^{20} undecayed nuclei and a decay constant of 1.5×10^{-9} s^{-1}. Calculate its **activity**. [2]

3.4 A radioactive isotope has a half-life of 6.0 hours. What **fraction** of the original nuclei remains after 18 hours? [2]

3.5 A radioactive source has a half-life of 8.0 days. Calculate the **time** for its activity to fall to 20% of its initial value. [4]

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(a) **State** what is meant by a **tracer**. [2]

(b) The decay is ${}^{15}_{8}\text{O} \rightarrow {}^{P}_{Q}\text{N} + {}^{R}_{S}\text{Z}$. **State** the values of P, Q, R and S, and **name** the particle Z. [3]

(c) **Define** the **activity** of a radioactive sample. [1]

(d) **Determine** the decay constant of oxygen-15. Give a **unit** with your answer. [2]

(e) **Determine** the initial activity of the sample. [2]

(f) **Determine** the activity 10 minutes after preparation. [3]

(g) **Suggest** why a tracer with a half-life of a few **days** would be unsuitable for this use. [2]

Solutions

2.1 It is **spontaneous** (unaffected by external conditions) and **random** (cannot predict which nucleus decays when).

2.2 Activity — the number of decays per unit time; **decay constant** — the probability per unit time that a nucleus decays.

2.3 The time for **half** the undecayed nuclei (or the activity) to decay.

2.4 $A = \lambda N$.

3.1 B — half the undecayed nuclei to decay.

3.2 $t_{1/2} = \dfrac{0.693}{\lambda} = \dfrac{0.693}{0.040} = 17$ s.

3.3 $A = \lambda N = (1.5 \times 10^{-9})(2.0 \times 10^{20}) = 3.0 \times 10^{11}$ Bq.

3.4 18 h = 3 half-lives; fraction $= (\frac{1}{2})^3 = \frac{1}{8}$.

3.5 $\lambda = \dfrac{0.693}{t_{1/2}} = \dfrac{0.693}{8.0} = 0.0866\ \text{day}^{-1}$; $t = -\dfrac{1}{\lambda} \ln \dfrac{A}{A_0} = -\dfrac{1}{0.0866} \ln(0.20) = 18.6$ days.

3.6 (a) A radioactive substance introduced into a system whose progress can be followed from outside by detecting the radiation it emits.
(b) $P = 15$, $Q = 7$, $R = 0$, $S = +1$; Z is a **positron**.
(c) The **rate of decay** — the number of nuclei decaying per unit time.
(d) $\lambda = \dfrac{\ln 2}{t_{1/2}} = \dfrac{0.693}{122} = 5.7 \times 10^{-3}\ \text{s}^{-1}$.
(e) $A = \lambda N = 5.68 \times 10^{-3} \times 4.8 \times 10^{14} = 2.7 \times 10^{12}$ Bq.
(f) 10 minutes is 600 s; $A = A_0 e^{-\lambda t} = 2.73 \times 10^{12} \times e^{-5.68 \times 10^{-3} \times 600} = 2.73 \times 10^{12} \times 0.0332 = 9.1 \times 10^{10}$ Bq. (That is about five half-lives, so roughly 1/32 of the start.)
(g) The patient would keep receiving **ionising radiation** long after the scan is over, raising the dose and the risk of cell damage; the tracer should decay away soon after the measurement, which a half-life of minutes achieves and one of days does not.

8 Oxygen-15 $\left({}^{15}_{8}\text{O}\right)$ is radioactive and has a half-life of 2.04 minutes.

The decay of oxygen-15 produces positrons. For this reason, oxygen-15 is sometimes used as a tracer in positron emission tomography (PET scanning).

(a) State what is meant by a tracer.

....

....

.... [2]

(b) The equation for the decay of oxygen-15 is

$${}^{15}_{8}\text{O} \rightarrow {}^{P}_{Q}\text{X} + {}^{R}_{S}\beta^{+} + \text{Z}$$

where X is the nucleus formed during the decay and Z is another particle.

(i) State the values of the integers P, Q, R and S.

P = ... R = ...

Q = ... S = ...

[2]

(ii) State the name of particle Z.

.... [1]

(c) **(i)** Define the activity of a sample.

....

.... [1]

(ii) Calculate the decay constant of oxygen-15. Give a unit with your answer.

decay constant = ... unit .... [2]

(iii) Determine the rate at which positrons are produced in a sample of oxygen-15 that has a mass of 2.85×10^{-6} kg.

rate = ... s^{-1} [4]

(d) The particles that are emitted from the body and detected outside it during PET scanning are not positrons but another type of particle.

(i) State the name of the particles that are detected.

.... [1]

(ii) Explain how these particles are formed inside the body.

....

....

....

.... [2]

[Total: 15]

9 **(a)** Define activity of a radioactive sample.

....

.... [1]

(b) Explain why the variation with time of the activity of a radioactive sample is exponential in nature.

....

....

....

....

.... [3]

(c) A sample contains a single radioactive isotope that decays to form a stable isotope.

The sample has an activity of 180 Bq at time $t = 0$.
At a time 8.4 minutes later, the activity is 120 Bq.

(i) Determine the decay constant, in min^{-1}, of the radioactive isotope.

decay constant = ... min^{-1} [2]

(ii) Use your answer in **(c)(i)** to determine the half-life, in min, of the radioactive isotope.

half-life = ... min [1]

(iii) On Fig. 9.1, sketch the variation of the activity A of the sample with t for values of t between $t = 0$ and $t = 24$ min.

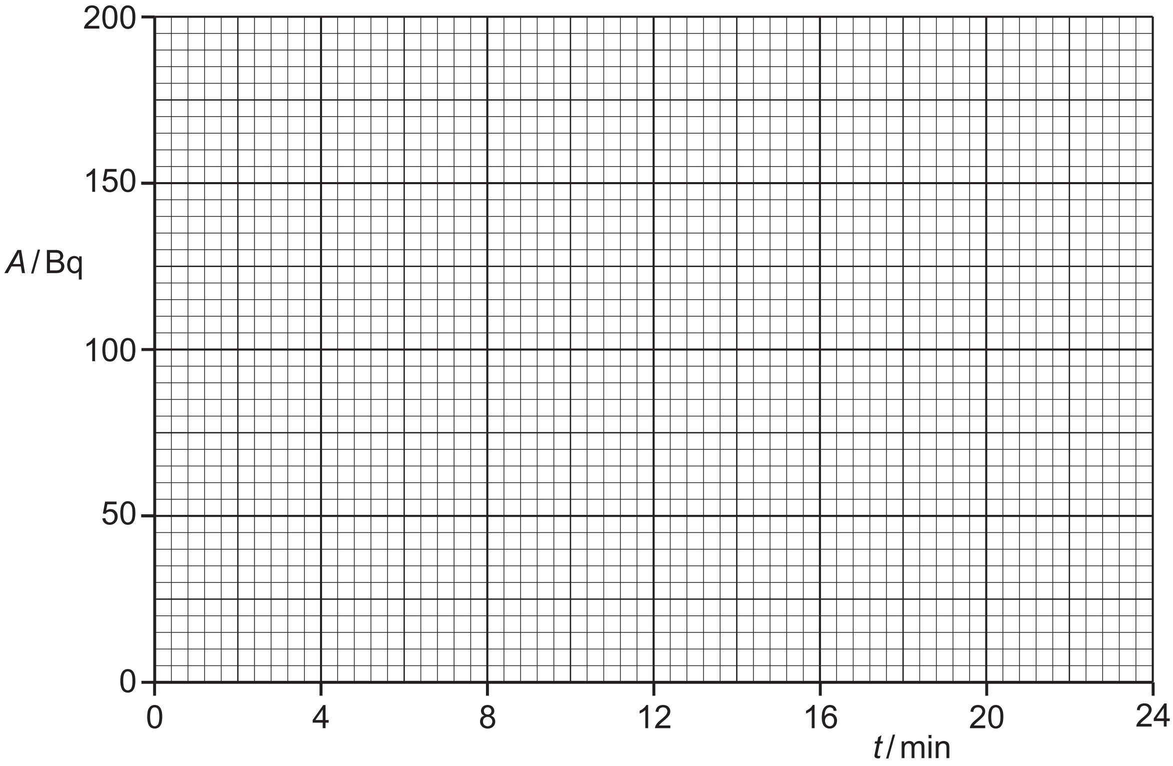


Fig. 9.1

[3]

[Total: 10]

10 (a) Radioactive decay is a spontaneous process.

State the meaning, in this context, of the term spontaneous.

....

.... [1]

(b) Two radioactive isotopes X and Y each decay to form a stable isotope.
A sample initially contains only atoms of isotope X. At this time, its activity is $4A$.
Another sample initially contains only atoms of Y. At this time, its activity is A.

Fig. 10.1 shows the variation of the activity of each sample with time t between $t = 0$ and $t = 6T$.

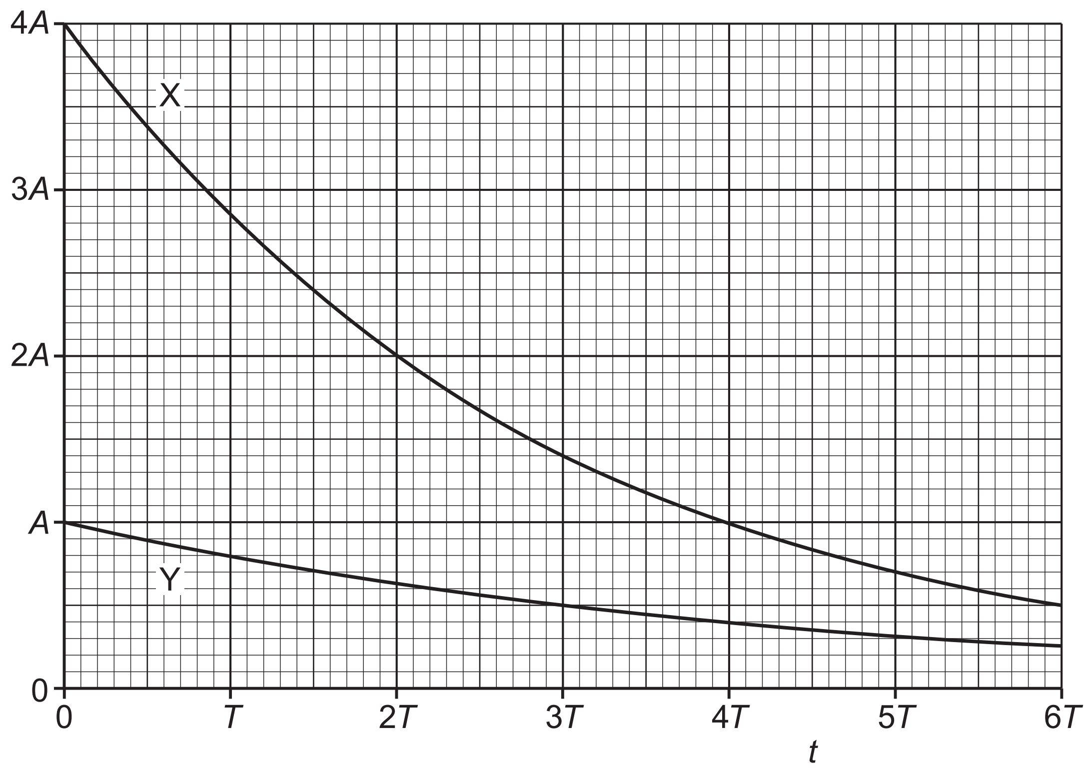


Fig. 10.1

(i) Complete Table 10.1 to give expressions, in terms of either or both of A and T, for the quantities indicated for each of the samples.

Table 10.1

sample	half-life	decay constant	initial activity	initial number of nuclei
X			$4A$	
Y			A	

[3]

(ii) Determine, in terms of T, the time at which the two samples will have equal activities.

time = ... T [3]

(c) A radiation detector is placed near to one of the samples in **(b)**.

Explain why the count rate measured by the detector is less than the activity of the sample.

....

....

.... [2]

[Total: 9]

9 **(a)** Define activity of a radioactive sample.

....

.... [1]

(b) Explain why the variation with time of the activity of a radioactive sample is exponential in nature.

....

....

....

....

.... [3]

(c) A sample contains a single radioactive isotope that decays to form a stable isotope.

The sample has an activity of 180 Bq at time $t = 0$.
At a time 8.4 minutes later, the activity is 120 Bq.

(i) Determine the decay constant, in min^{-1}, of the radioactive isotope.

decay constant = ... min^{-1} [2]

(ii) Use your answer in **(c)(i)** to determine the half-life, in min, of the radioactive isotope.

half-life = ... min [1]

(iii) On Fig. 9.1, sketch the variation of the activity A of the sample with t for values of t between $t = 0$ and $t = 24$ min.

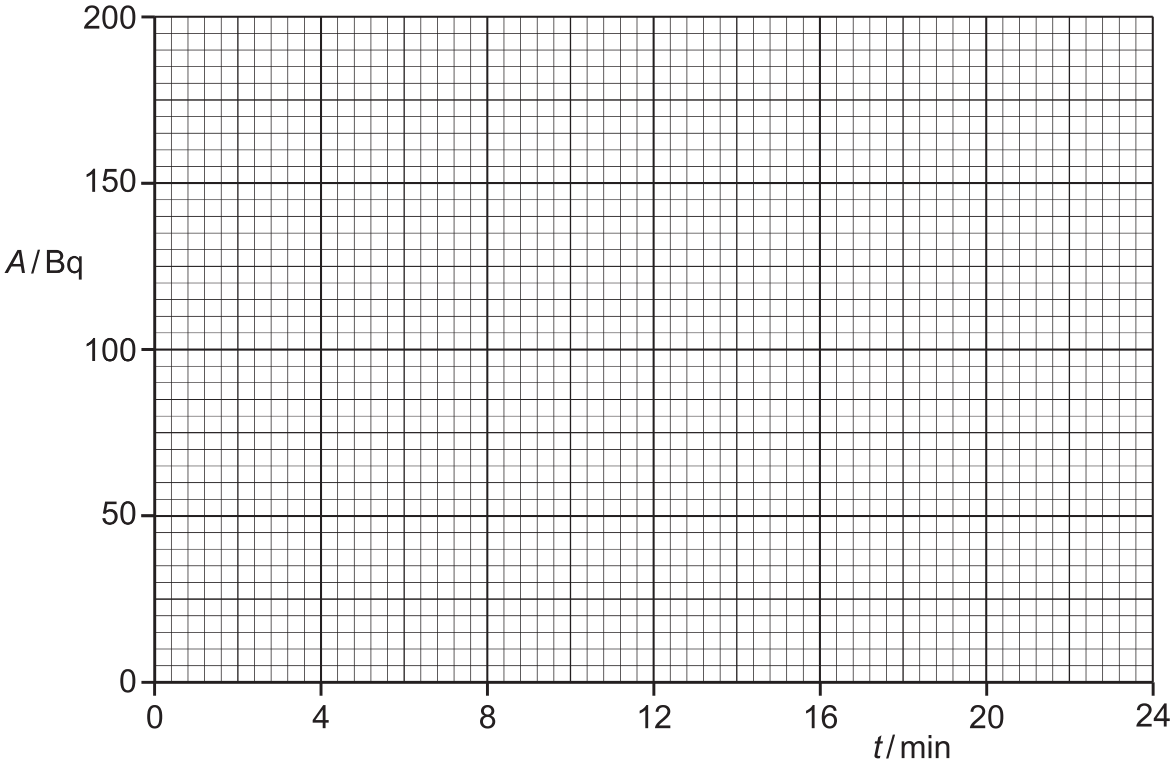


Fig. 9.1

[3]

[Total: 10]